Chapter 33 Electromagnetic (EM) Waves

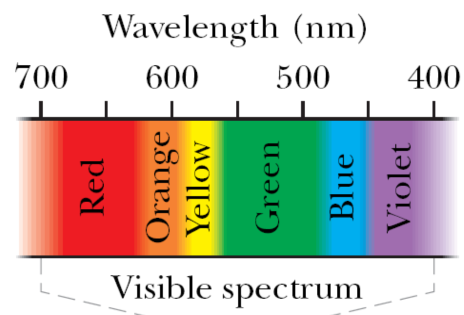
Maxwell's Rainbow

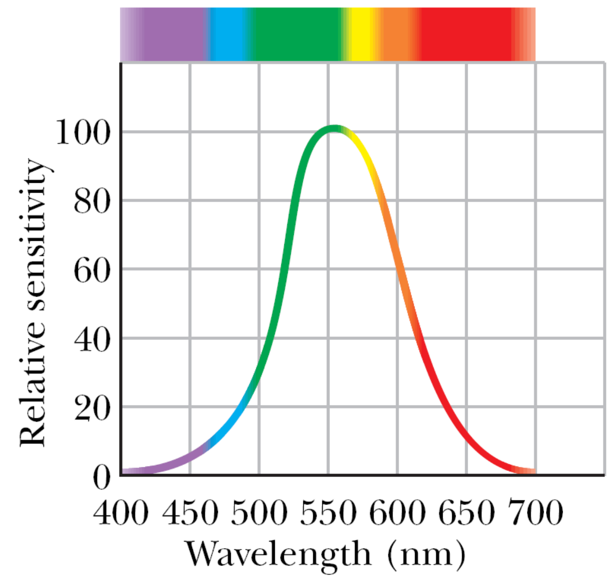
- Maxwell showed that a beam of light is a traveling wave of electric and magnetic fields--an EM wave--and the study of visible light is a branch of electromagnetism.

- Hertz discovered radio waves and verified that they are also EM wave.

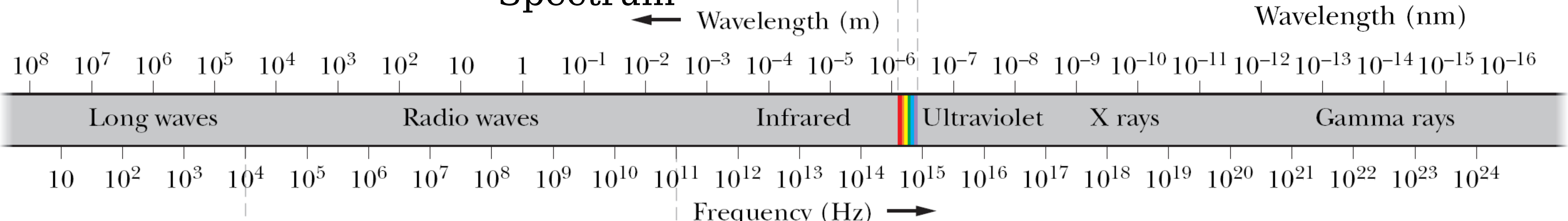
- All EM waves, no matter where they lie in the spectrum, travel through vacuum with the same speed c.

visible spectrum

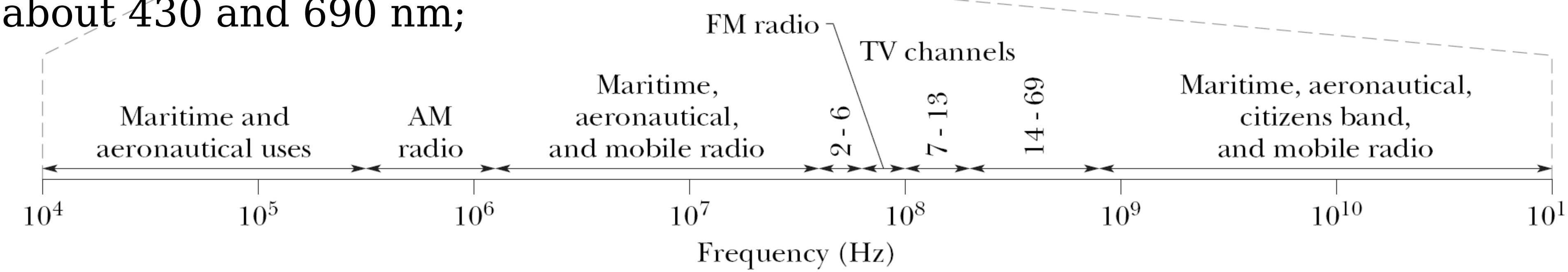




Spectrum



- The limits for human eye are about 430 and 690 nm;



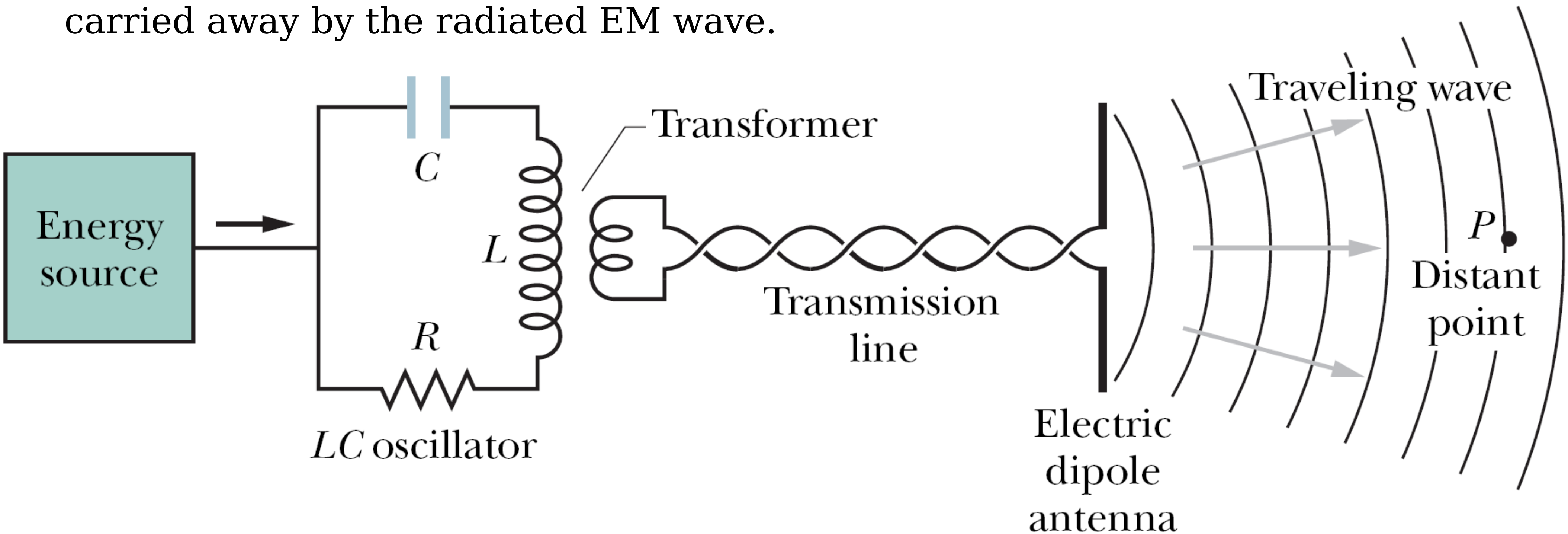
The Traveling Electromagnetic Wave

- Some EM waves (x rays, γ rays, visible light) are *radiated* from sources that are of atomic or nuclear size, where quantum physics rules.

- Here we only discuss how other EM waves are generated with the source in macroscopic and of manageable dimensions (wavelength $\lambda \approx 1\text{m}$).

- The heart of such wave generation is an LC oscillator, which establishes an angular frequency ω ($= \dfrac{1}{\sqrt{LC}}$).

- An external source — possibly an ac generator — must be included to supply energy to compensate both for thermal losses in the circuit and for energy carried away by the radiated EM wave.



- The LC oscillator is coupled by a transformer and a transmission line to an *antenna*, which consists essentially of 2 thin, solid, conducting rods.

- Through this coupling, the sinusoidally varying current in the oscillator causes charge to oscillate sinusoidally along the rods of the antenna at the angular freqency ω.

- The antenna has the effect of an electric dipole whose electric dipole moment varies sinusoidally in magnitude and direction along the antenna.

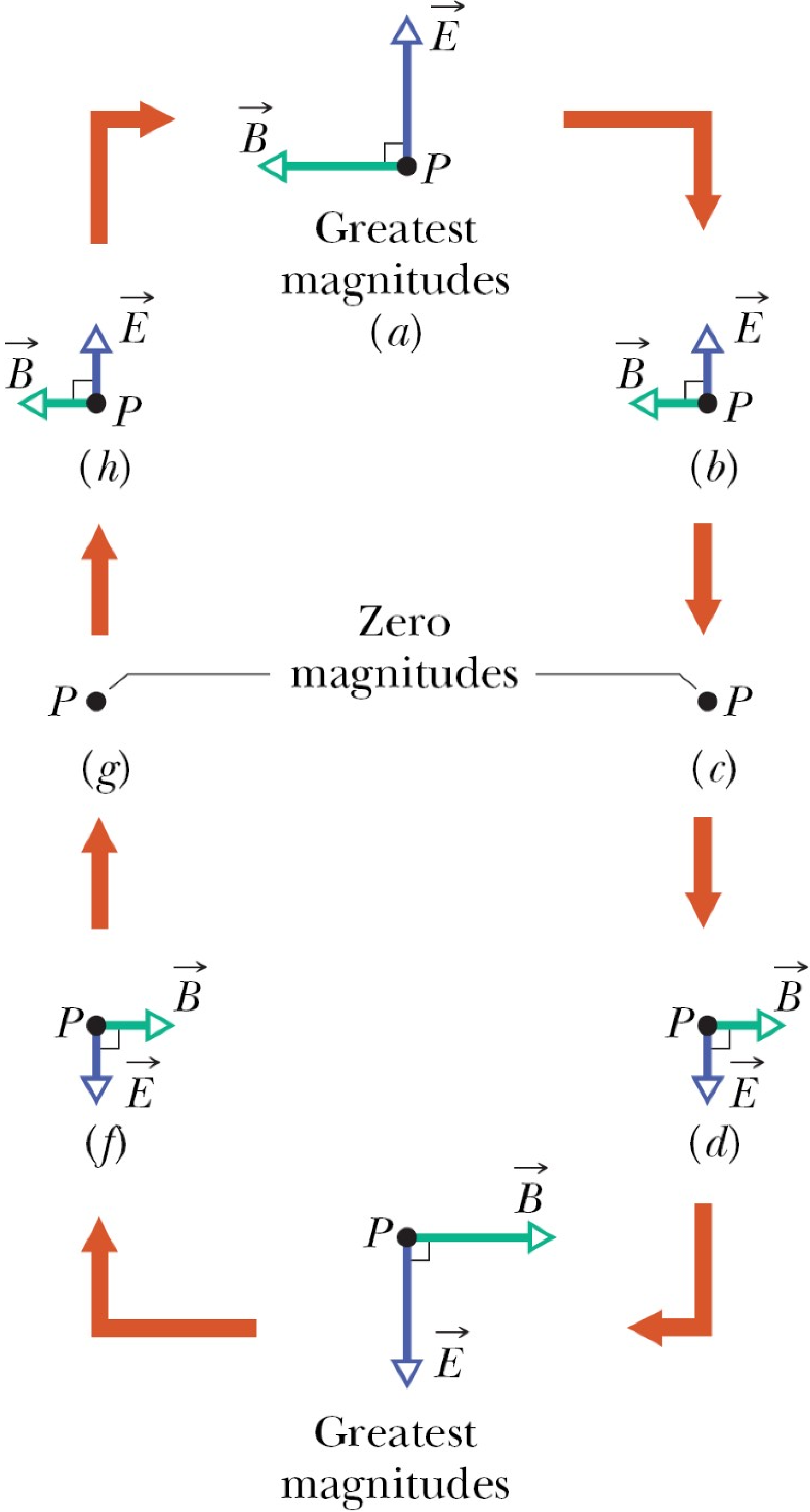
- Because the dipole moment varies in magnitude and direction, the electric field produced by the dipole varies in magnitude and direction. Also, because the current varies, the magnetic field produced by the current varies in magnitude and direction.

- Together the changing fields form an EM wave that travels away from the antenna at speed c with the angular frequency ω.

Electromagnetic Wave

1. The electric and magnetic fields are always perpendicular to the direction in which the wave is traveling. Thus, the wave is a transverse wave.

2. The electric field is always $\perp$ the magnetic field.



$\vec{E}$
$\vec{B}$
P
Greatest magnitudes
(a)
$\vec{E}$
$\vec{B}$
P
(h)
$\vec{E}$
$\vec{B}$
P
(b)
Zero magnitudes
P
(g)
P
(c)
P
$\vec{B}$
$\vec{E}$
(f)
P
$\vec{B}$
$\vec{E}$
(d)
P
$\vec{B}$
$\vec{E}$
Greatest magnitudes

3. The cross product $\vec{E} \times \vec{B}$ always gives the direction in which the wave travels.

4. The fields always vary sinusoidally. Moreover, the fields vary with the same frequency and in phase (in step) with each other.

- write the electric and magnetic fields as sinusoidal functions of position x (along the path of the wave) and time t:

$$E = E_m \sin(k\,x - \omega\,t)\,, \quad B = B_m \sin(k\,x - \omega\,t)$$

- Not only do the 2 fields form the EM wave but each also forms its own wave. One gives the *electric wave component* of the EM wave, and the other gives the *magnetic wave component*.

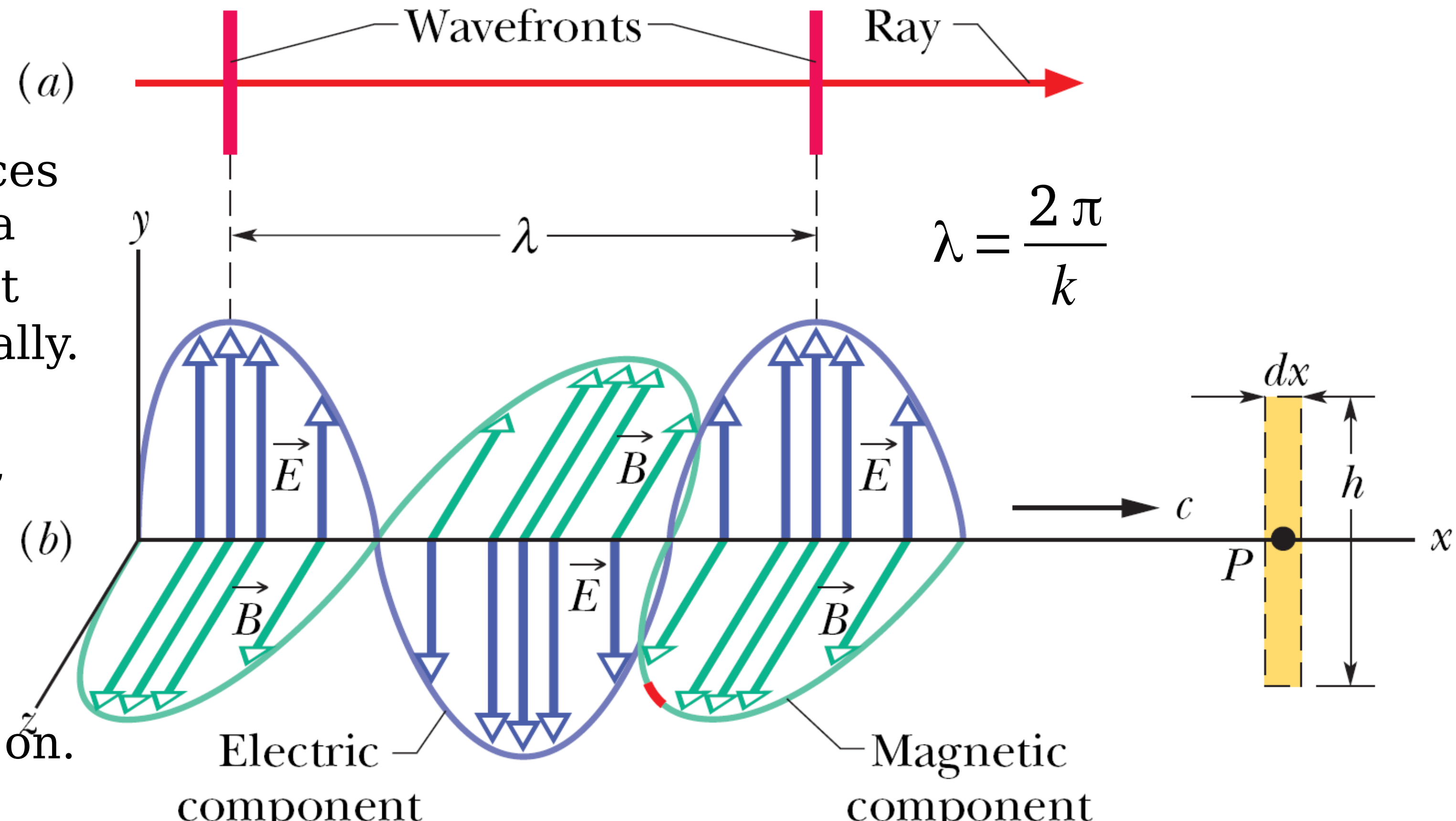
- $c = \dfrac{\omega}{k} = \dfrac{1}{\sqrt{\mu_0\,\epsilon_0}} = 3 \times 10^8\ \mathrm{m/s}$ wave speed

All EM waves, including visible light, have the same speed c in vacuum.

- We can represent the EM wave with a *ray* (a directed line showing the wave's direction of travel) or with *wavefronts* (imaginary surfaces over which the wave has the same magnitude of electric field), or both.

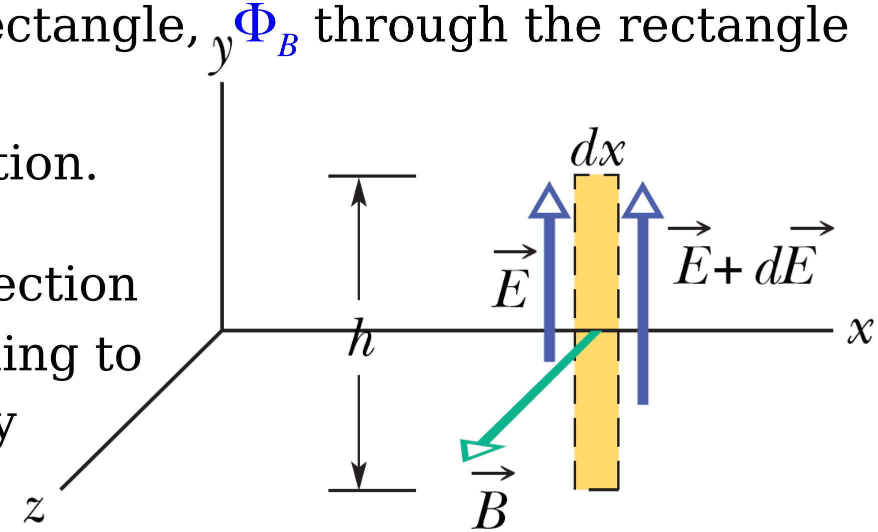
- $\vec{E} \perp \vec{B}\,, \quad \vec{E} \perp \vec{k}\,, \quad \vec{B} \perp \vec{k}$

- Because *B* varies sinusoidally, it induces (via Faraday's law) a perpendicular *E* that also varies sinusoidally. And because *E* is varying sinusoidally, it induces (via Maxwell's law) a perpendicular *B* that also varies sinusoidally. And so on.



- The 2 fields continuously create each other via induction, and the resulting sinusoidal variations in the fields travel as a wave — the electromagnetic wave.

- As the EM wave moves rightward past the rectangle, Φ_B through the rectangle changes and induced *E* appear throughout the rectangle, according to Faraday's law of induction.



- If *B* through the rectangle points in the *z* direction and decreases, then Φ_B also decreases. According to Faraday's law, this change in flux is opposed by induced *E*, which produce *B* in the *z* direction.

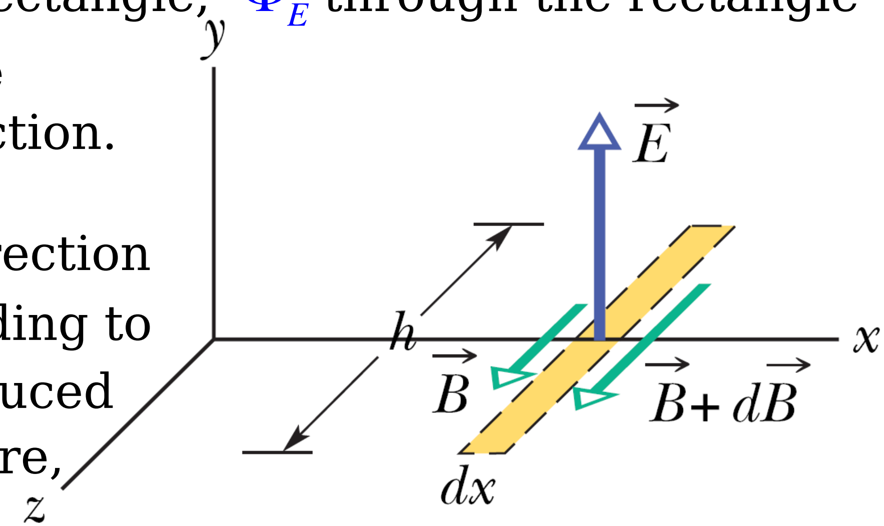
- Therefore, $E + \mathrm{d}E > E$.
- Apply Faraday's law of induction: $-\frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = \oint \vec{E}\cdot\mathrm{d}\vec{s} = (E+\mathrm{d}E)h - Eh = h\,\mathrm{d}E$

$$\Phi_B = Bh\,\mathrm{d}x \quad\Rightarrow\quad \frac{\mathrm{d}\Phi_B}{\mathrm{d}t} = h\,\mathrm{d}x\frac{\partial B}{\partial t} \quad\Rightarrow\quad h\,\mathrm{d}E = -h\,\mathrm{d}x\frac{\partial B}{\partial t} \quad\Rightarrow\quad \frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}$$

$$\frac{\partial E}{\partial x} = kE_m\cos(kx-\omega t), \quad \frac{\partial B}{\partial t} = -\omega B_m\cos(kx-\omega t)$$

$$\Rightarrow\quad kE_m\cos(kx-\omega t) = \omega B_m\cos(kx-\omega t) \quad\Rightarrow\quad \frac{E_m}{B_m} = c \quad \text{amplitude ratio}$$

- As the EM wave moves rightward past the rectangle, Φ_E through the rectangle changes and induced B appear throughout the rectangle, according to Maxwell's law of induction.



- If E through the rectangle points in the y direction and decreases, then Φ_E also decreases. According to Maxwell's law, this change in flux gives an induced B, which produce E in the y direction. Therefore, $B + \mathrm{d}B > B$.
- Apply Faraday's law of induction: $\frac{\mathrm{d}\Phi_E}{\mathrm{d}t} = \mu_0\epsilon_0\oint \vec{B}\cdot\mathrm{d}\vec{s} = \mu_0\epsilon_0[Bh - (B+\mathrm{d}B)h]$

$$= -\mu_0\epsilon_0 h\,\mathrm{d}B$$

$$\Phi_E = E\,h\,\mathrm{d}x \quad \Rightarrow \quad \frac{\mathrm{d}\Phi_E}{\mathrm{d}t} = h\,\mathrm{d}x\,\frac{\partial E}{\partial t} \quad \Rightarrow \quad -h\,\mathrm{d}B = \mu_0\,\epsilon_0\,h\,\mathrm{d}x\,\frac{\partial E}{\partial t}$$

$$\Rightarrow \quad -\frac{\partial B}{\partial x} = \mu_0\,\epsilon_0\,\frac{\partial E}{\partial t} \quad \Rightarrow \quad -k\,B_m\cos(k\,x-\omega\,t) = -\mu_0\,\epsilon_0\,\omega\,E_m\cos(k\,x-\omega\,t)$$

$$\Rightarrow \quad \frac{E_m}{B_m} = \frac{k}{\mu_0\,\epsilon_0\,\omega} = \frac{1}{\mu_0\,\epsilon_0\,c} \quad \Rightarrow \quad c = \frac{1}{\sqrt{\mu_0\,\epsilon_0}} \quad \text{wave speed}$$

- The magnitudes of the fields at every instant and at any point are related by

$$\boxed{\frac{E}{B} = c \quad \text{magnitude ratio}}$$

A Most Curious Wave

- An EM wave is different in that it requires no medium for its travel. It can travel through a medium such as air/glass, it can also travel through the vacuum.

- Light has the same speed regardless of the frame of reference from which it is measured.

$$c = 299{,}792{,}458\ \mathrm{m/s} \qquad \text{in vacuum}$$

Energy Transport and the Poynting Vector

- An EM wave can transport energy and the rate of energy transport per unit area in such a wave is described by the **Poynting vector**,

$$\boxed{\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B} \quad \text{Poynting vector}} \quad \Rightarrow \quad S = \frac{\text{energy/time}}{\text{area}} = \frac{\text{power}}{\text{area}}$$

the SI unit for $\vec{S}$ is the watt per square meter (W/m^2).

The direction of the Poynting vector $\vec{S}$ of an EM wave at any point gives the wave's direction of travel and the direction of energy transport at that point.

- $\vec{E} \perp \vec{B}$ in an EM wave $\Rightarrow$ $|\vec{E}\times\vec{B}| = E\,B$ $\Rightarrow$ $S = \dfrac{1}{\mu_0}E\,B$

$$\Rightarrow \boxed{S = \frac{1}{\mu_0 c}E^2 \quad \text{instantaneous energy flow rate}}$$

- More useful in practice is the average energy transported over time, ie, the time-averaged value of S, ie, $<S>$ and called the **intensity** I of the wave,

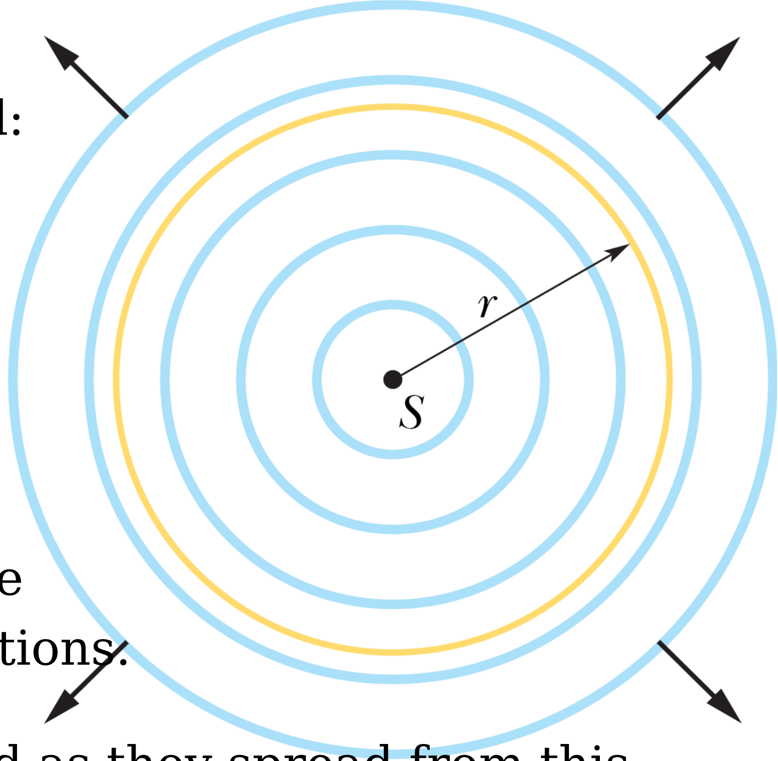
$$I = \langle S\rangle = \left\langle \frac{\text{energy/time}}{\text{area}}\right\rangle = \left\langle \frac{\text{power}}{\text{area}}\right\rangle = \frac{1}{\mu_0 c}\langle E^2\rangle = \frac{1}{\mu_0 c}\langle E_m^2 \sin^2(k\,x+\omega\,t)\rangle$$

$$\Rightarrow \boxed{I = \frac{1}{\mu_0 c}E_{\text{rms}}^2} \quad \Leftarrow \quad E_{\text{rms}} = \frac{E}{\sqrt{2}}$$

- The 2 energies of E & B in an EM wave are equal:

$$E = c\,B \text{ in an EM wave} \quad + \quad c^2 = \frac{1}{\mu_0\,\epsilon_0}$$

$$\Rightarrow \quad u_E = \frac{\epsilon_0}{2}E^2 = \frac{\epsilon_0}{2}(c\,B)^2 = \frac{\epsilon_0}{2}\frac{1}{\mu_0\,\epsilon_0}B^2 = \frac{B^2}{2\,\mu_0} = u_B$$



Variation of Intensity with Distance

- Assume the source is a *point source* that emits the light *isotropically*— with equal intensity in all directions.

- Assume that the energy of the waves is conserved as they spread from this source. Thus, the rate at which energy passes through the sphere via the radiation must equal the rate at which energy is emitted by the source—that is, the source power P_s

$$\boxed{I = \frac{\text{power}}{\text{area}} = \frac{P_s}{4\,\pi\,r^2}} \Rightarrow \quad I \propto \frac{1}{r^2}$$

Problem 33.1

Radiation Pressure

- EM waves have linear momentum and thus can exert a pressure on an object when shining on it, although the pressure is small.

- Shine a beam of EM radiation on an object for a time interval Δt. Let the object free to move and that the radiation is entirely absorbed by the object. This means that during Δt, the object gains an energy ΔU from the radiation

$$\text{Momentum change } \Delta p = \frac{\Delta U}{c} \quad \text{total absorption}$$

- The direction of Δp of the object is the one of the *incident* (incoming) beam.

- Instead of being absorbed, the radiation can be **reflected** by the object. If the radiation is entirely reflected back along its original path, then

$$\Delta p = \frac{2\Delta U}{c} \quad \text{total reflection back along path}$$

- If the incident radiation is partly absorbed and partly reflected, the momentum change of the object is between $\Delta U/c$ and $2\Delta U/c$.

- Intensity $I = \dfrac{\text{power}}{\text{area}} = \dfrac{\text{energy/time}}{\text{area}} \Rightarrow$ force $F \equiv \dfrac{\Delta p}{\Delta t} = \begin{cases} \dfrac{IA}{c} & \text{total absorption} \\ 2\dfrac{IA}{c} & \text{total reflection back along path} \end{cases}$

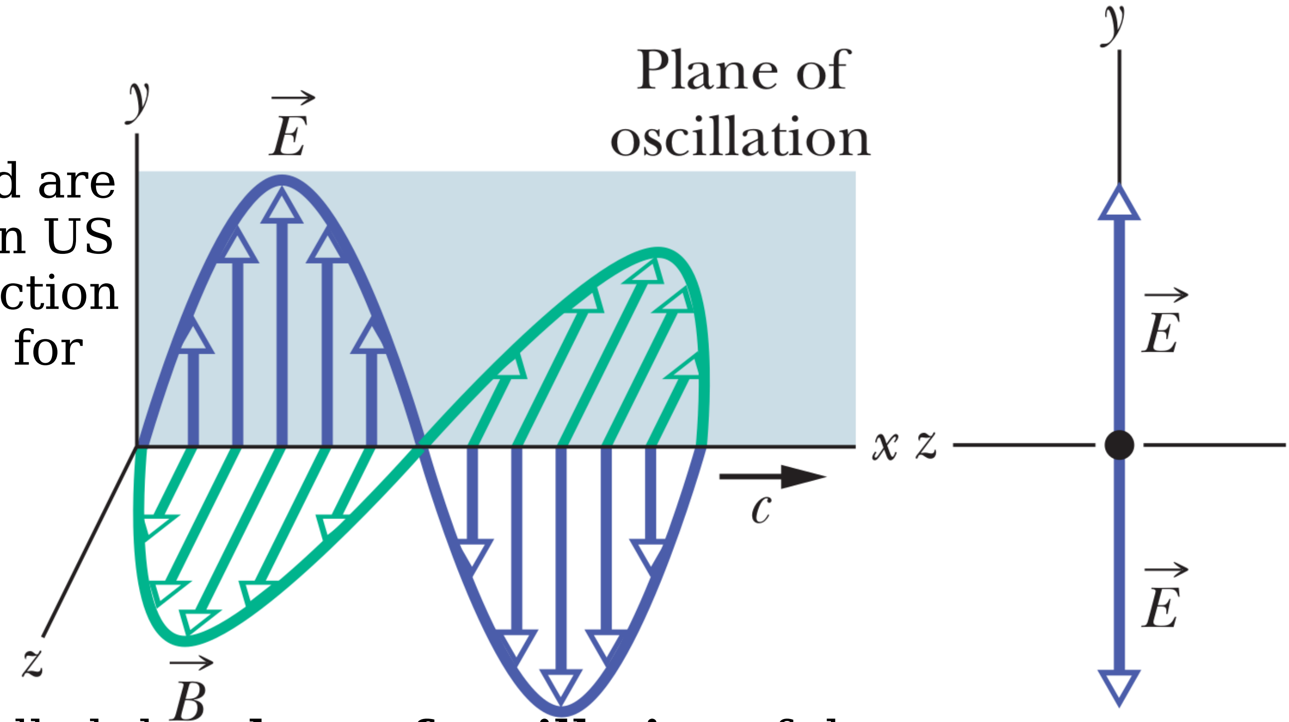
$$\Delta U = I A \Delta t$$

- If the radiation is partly absorbed and partly reflected, the magnitude of the force on area A is between the values of IA/c and $2IA/c$.
- Radiation pressure: $$p_r \equiv \frac{F}{A} = \begin{cases} \dfrac{I}{c} & \text{total absorption} \\ 2\dfrac{I}{c} & \text{total reflection back along path} \end{cases}$$
- The development of laser technology has permitted researchers to achieve radiation pressures easier. This is because a beam of laser light can be focused to a tiny spot. This permits the delivery of great amounts of energy to small objects placed at that spot.

Polarization

- VHF TV antennas in England are oriented vertically, but those in US are horizontal, due to the direction of oscillation of the EM waves for the TV signal.

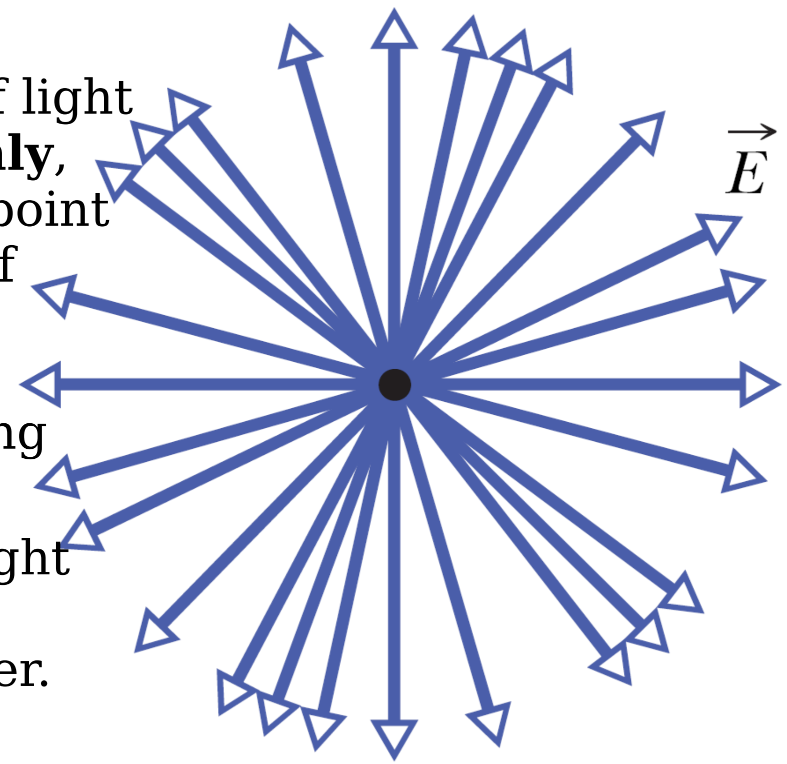
- EM waves are **polarized** vertically if their electric field oscillates vertically.



- The plane containing $\vec{E}$ is called the **plane of oscillation** of the wave.

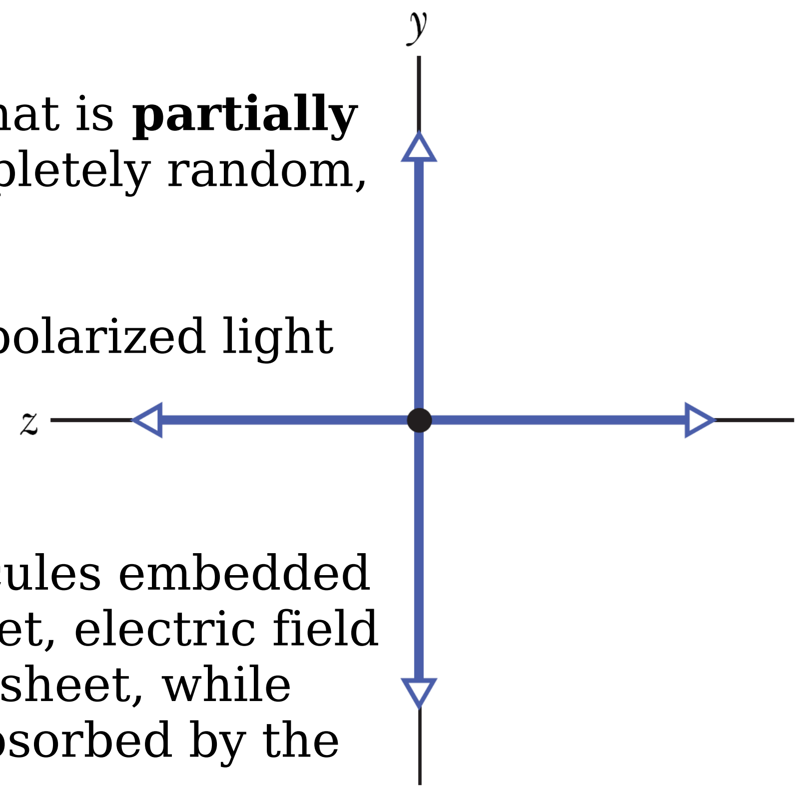
Polarized Light

- The EM waves emitted by any common source of light (such as the Sun or a bulb) are **polarized randomly**, or **unpolarized**, ie, the electric field at any given point is always perpendicular to the direction of travel of the waves but changes directions randomly.



- In principle, we can simplify the mess by resolving each unpolarized $\vec{E}$ into y and z components. In doing all this, we effectively change unpolarized light into the superposition of 2 polarized waves whose planes of oscillation are perpendicular to each other.

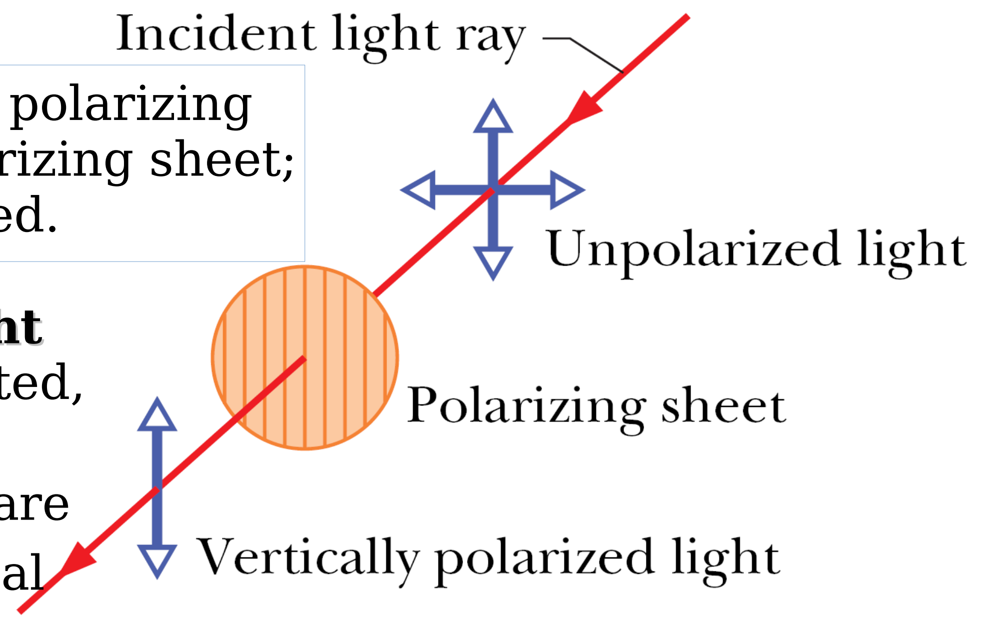
- We can draw similar figures to represent light that is **partially polarized** for which field oscillations are not completely random, nor are they parallel to a single axis.



- We can transform unpolarized visible light into polarized light by sending it through a *polarizing sheet*, known as Polaroids or Polaroid filters.

- A polarizing sheet consists of certain long molecules embedded in plastic. When light is then sent through the sheet, electric field components along one direction pass through the sheet, while components perpendicular to that direction are absorbed by the molecules and disappear.

> An electric field component parallel to the polarizing direction is passed (transmitted) by a polarizing sheet; a component perpendicular to it is absorbed.



Intensity of Transmitted Polarized Light

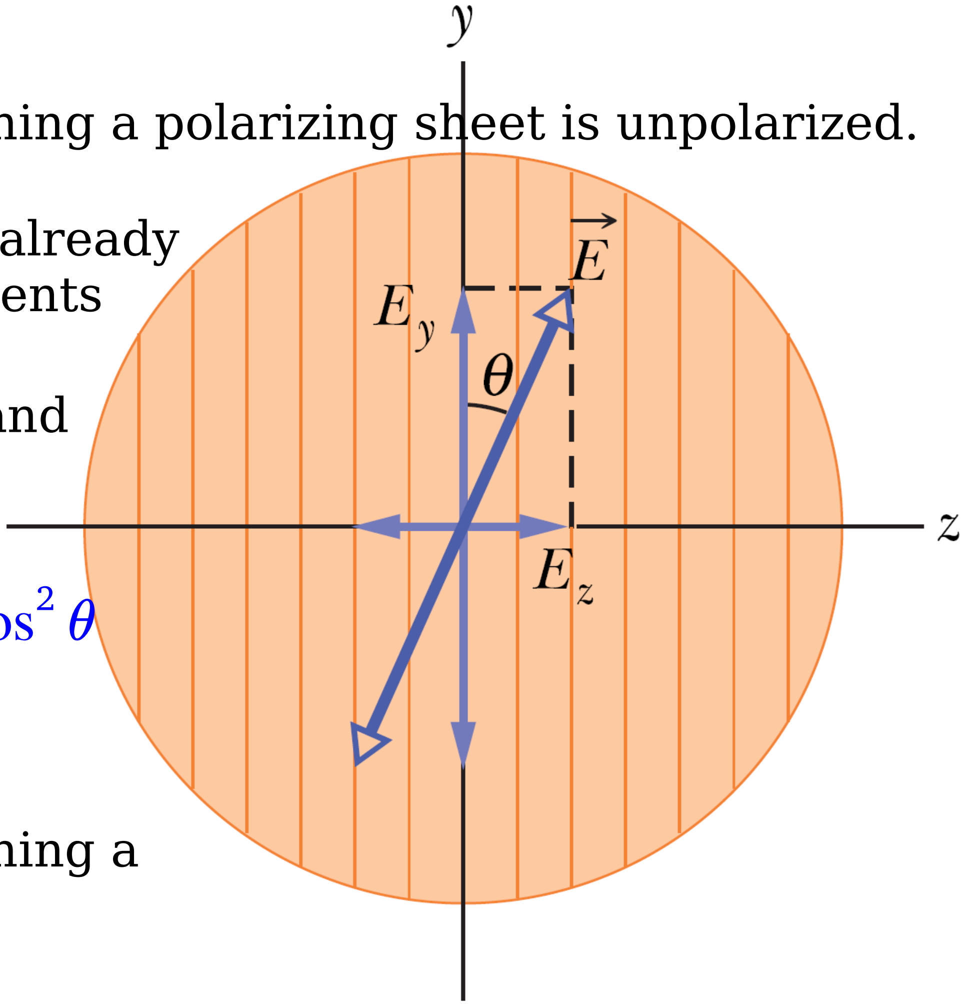
- If the original waves are randomly oriented,

$\sum E_y = \sum E_z$. When the z components are absorbed, half the intensity I_0 of the original light is lost.

$$I = \frac{I_0}{2} \quad \text{one-half rule}$$

- We use this rule *only* when the light reaching a polarizing sheet is unpolarized.

- If the light reaching a polarizing sheet is already polarized, we can resolve $\vec{E}$ into 2 components relative to the polarizing direction: parallel component E_y is transmitted by the sheet, and perpendicular component E_z is absorbed:



$$E_y = E\cos\theta \;\Rightarrow\; I = \frac{E_{y,\,\mathrm{rms}}^2}{c\,\mu_0} = \frac{E_{\mathrm{rms}}^2}{c\,\mu_0}\cos^2\theta = I_0\cos^2\theta$$

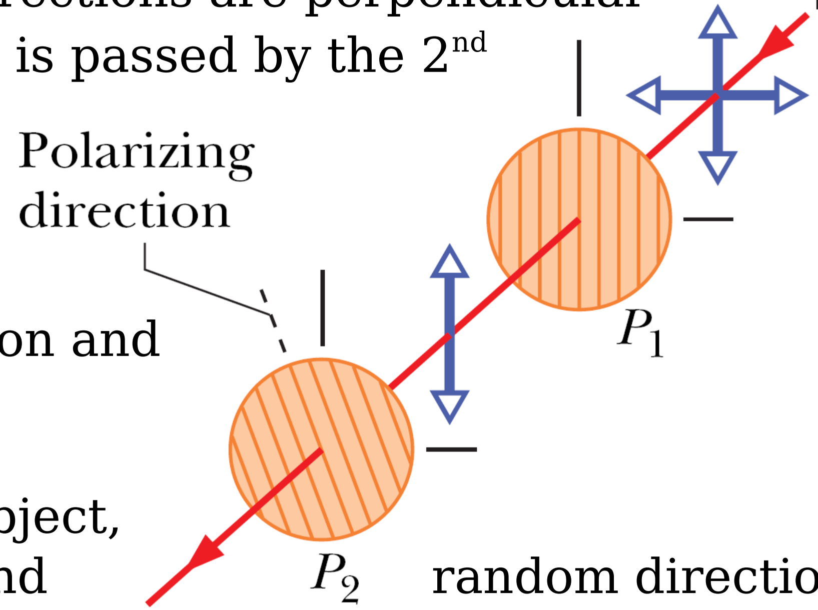
$$\Rightarrow \boxed{I = I_0\cos^2\theta \quad \text{cosine-squared rule}}$$

- We use this rule *only* when the light reaching a polarizing sheet is already polarized.

- The transmitted intensity I is a maximum and is equal to the original intensity I_0 when the original wave is polarized parallel to the polarizing direction of the sheet ($\theta = 0°$ or $180°$). The transmitted intensity is 0 when the original wave is polarized perpendicular to the polarizing direction of the sheet ($\theta = 90°$).

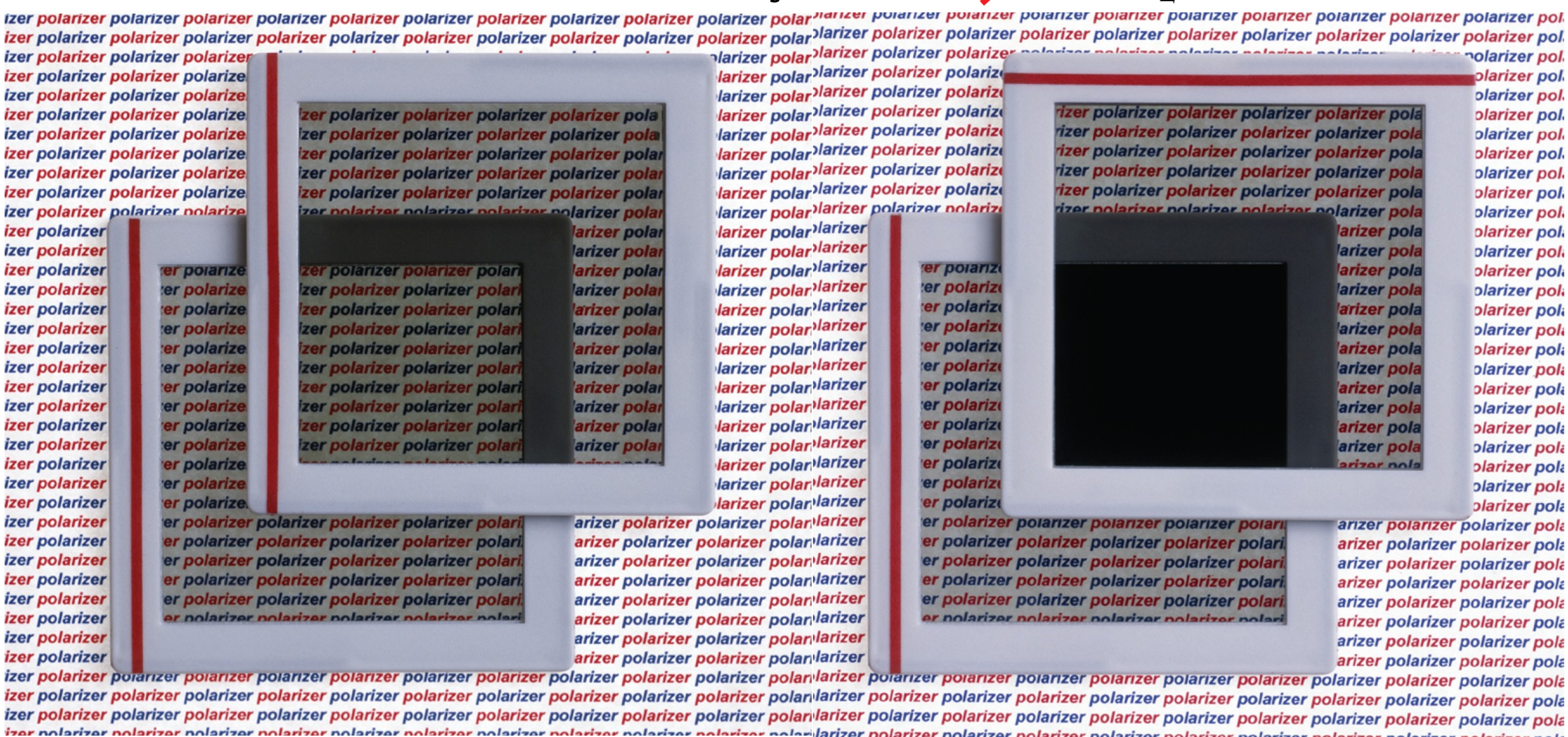
- For initially unpolarized light sent through two polarizing sheets P_1 and P_2, the 1st sheet is called the *polarizer*, and the 2nd the *analyzer*.

- If their polarizing directions are parallel, all the light passed by the 1st sheet is passed by the 2nd sheet. If those directions are perpendicular (the sheets are said to be *crossed*), no light is passed by the 2nd sheet. Finally, if the 2 polarizing directions gives $0° < \theta < 90°$, the transmission relations is as described above.



- Light can also be polarized by by reflection and by scattering from atoms or molecules.

- In *scattering*, light is intercepted by an object, such as a molecule, and sent off in many and random directions.



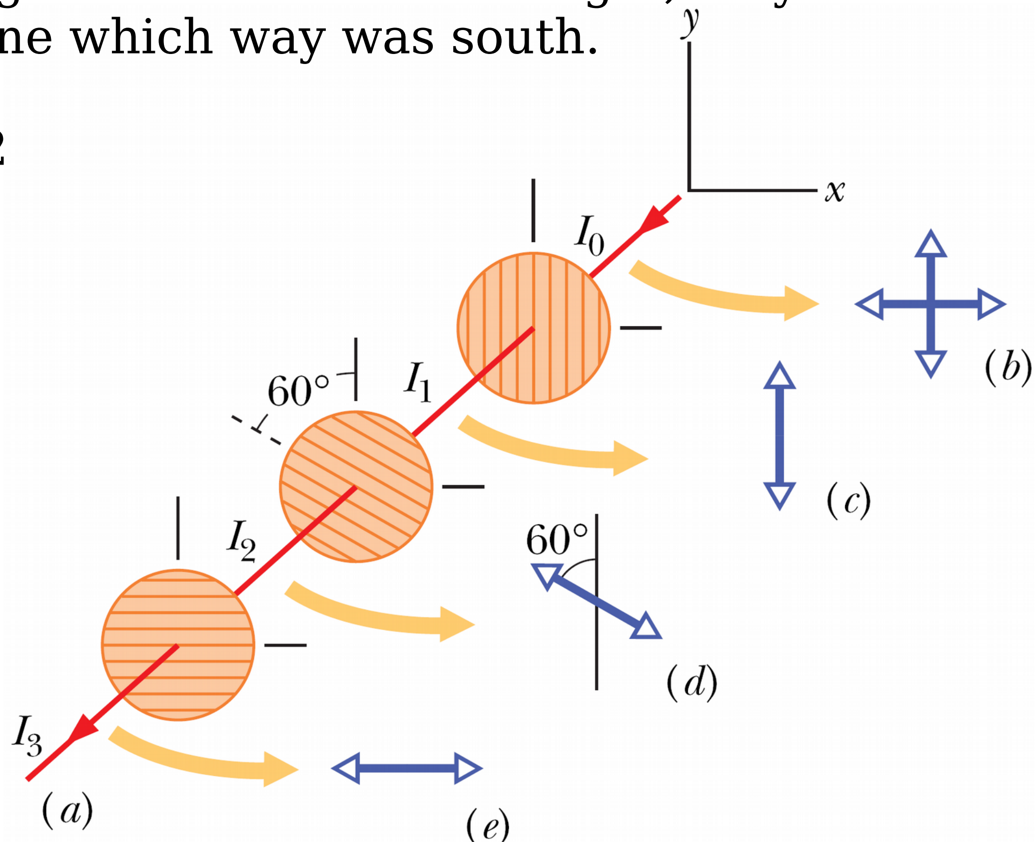
- The scattering of sunlight by molecules in the atmosphere gives the sky its general glow.

- Although direct sunlight is unpolarized, light from much of the sky is at least partially polarized by such scattering.

- Bees use the polarization of sky light in navigating to and from their hives.

- The Vikings used it to navigate across the North Sea when the daytime Sun was below the horizon. They had discovered that crystals cordierite changes color when rotated in polarized light. By looking at the sky through such a crystal while rotating it about their line of sight, they could locate the hidden Sun and thus determine which way was south.

Problem 33.2



Reflection and Refraction

- The study of of light waves under that approximation of its traveling as in a straight line is called *geometrical optics*.

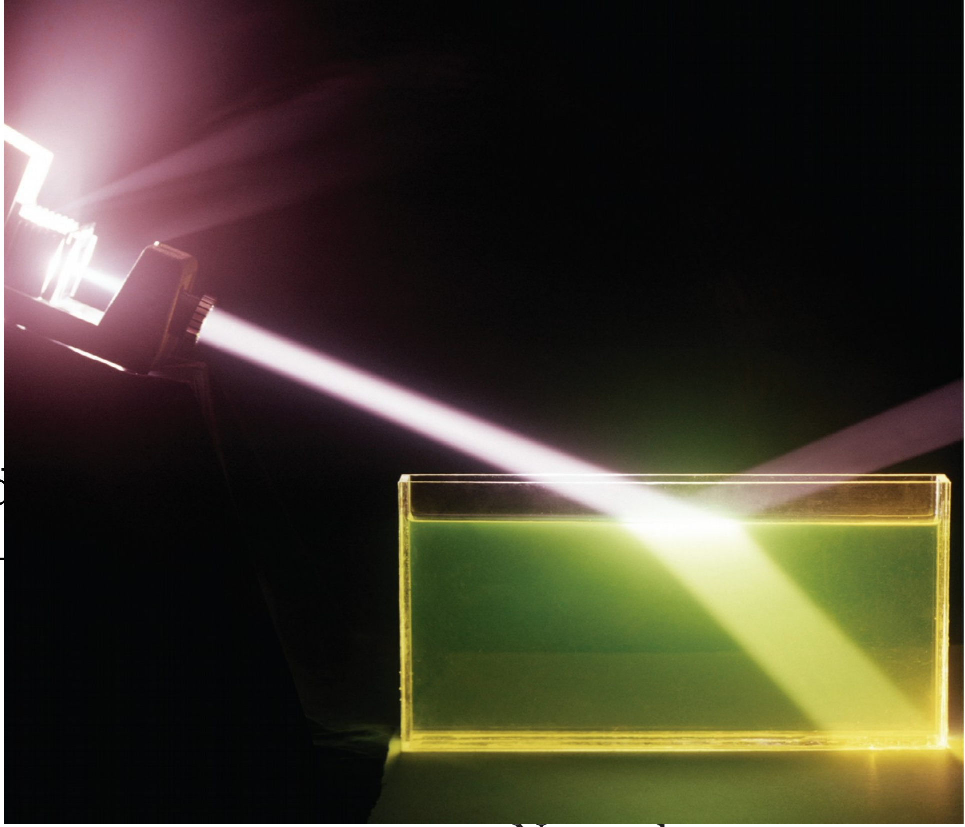
- For an *incident* beam, part of the light is **reflected**, forming a beam directed upward toward the right, traveling as if the original beam had bounced from the surface.

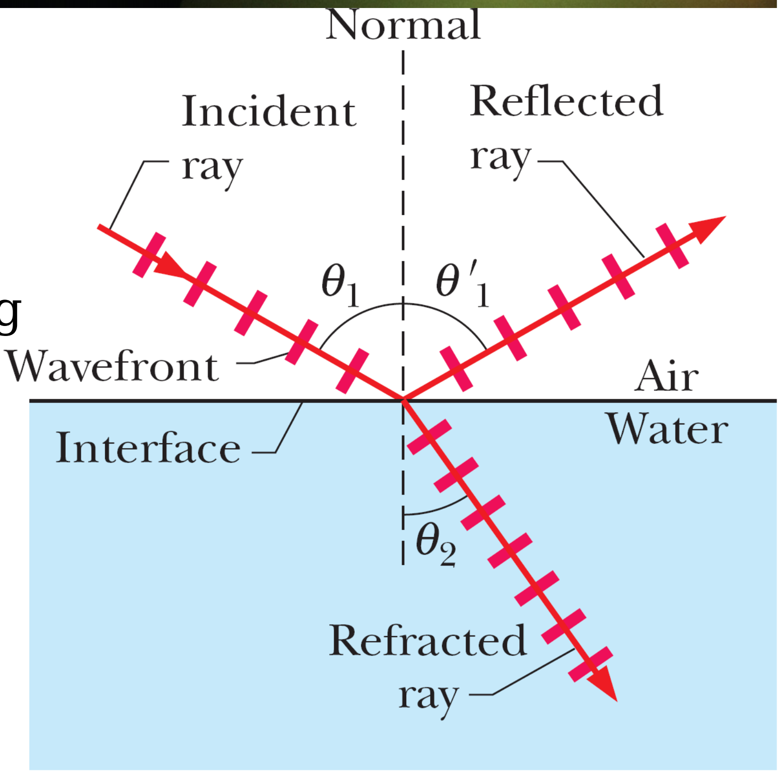
- The rest of the light travels through the surface and into the water, forming a beam directed downward to the right.

- Water is said to be *transparent* because light can travel through it.

- The travel of light through an *interface* separating 2 media is called **refraction**, and the light is said to be *refracted*.

- The **angle of incidence** is θ_1, the **angle of reflection** is θ'_1, and the angle of refraction is θ_2, all measured *relative to the normal*.





- The plane containing the incident ray and the normal is the *plane of incidence*.

Law of reflection: A reflected ray lies in the plane of incidence and has an angle of reflection equal to the angle of incidence, ie, $\theta'_1 = \theta_1$ reflection

Law of refraction: A refracted ray lies in the plane of incidence and has an angle of refraction θ_2 that is related to the angle of incidence θ_1 by

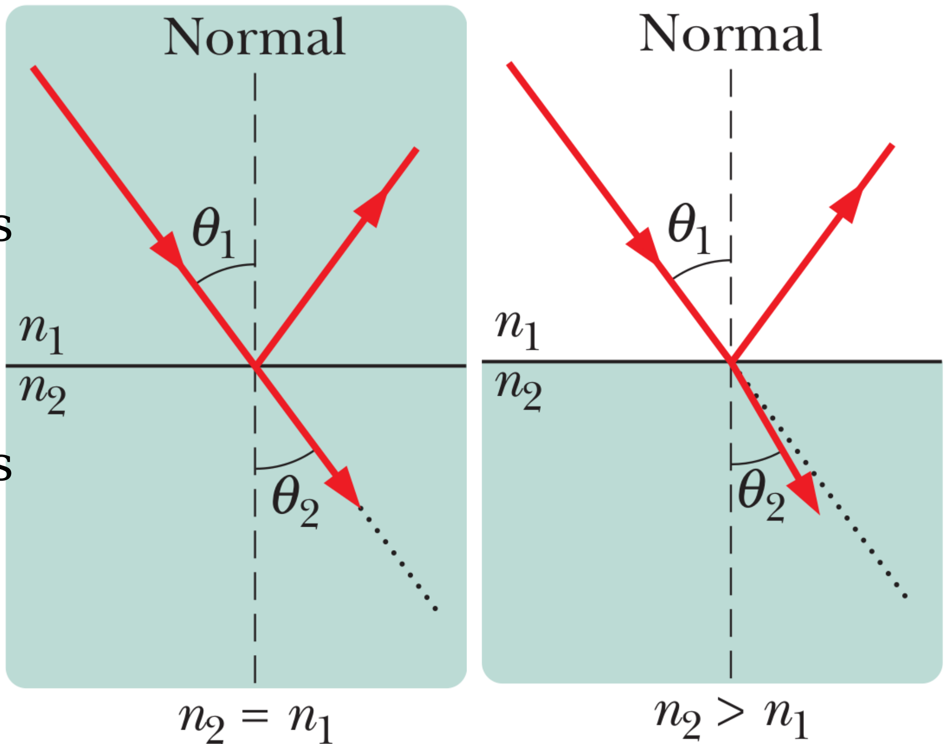
$$n_2 \sin\theta_2 = n_1 \sin\theta_1 \quad \text{refraction (Snell's law)} \quad \Leftarrow \quad n_1,\ n_2 \geq 1 \ : \text{index of refraction}$$

$$\Rightarrow \quad \sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1$$

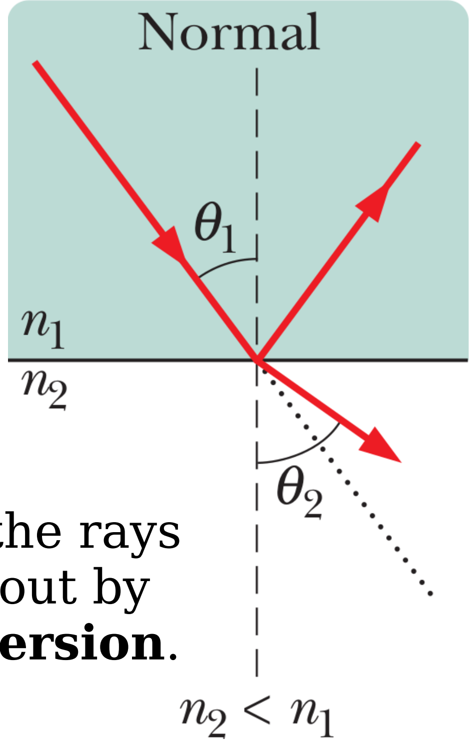
1. If $n_2 = n_1$, then $\theta_2 = \theta_1$ and refraction does not bend the light beam, which continues in the *undeflected direction*.

2. If $n_2 \geq n_1$, then $\theta_2 \leq \theta_1$, and refraction bends the light beam away from the undeflected direction and toward the normal.

3. If $n_2 \leq n_1$, then $\theta_2 \geq \theta_1$, and refraction bends the light beam away from the undeflected direction and away from the normal.



- Refraction *cannot* bend a beam so much that the refracted ray is on the same side of the normal as the incident ray.



Chromatic Dispersion

- The index of refraction n encountered by light in any medium except vacuum depends on the wavelength of the light.

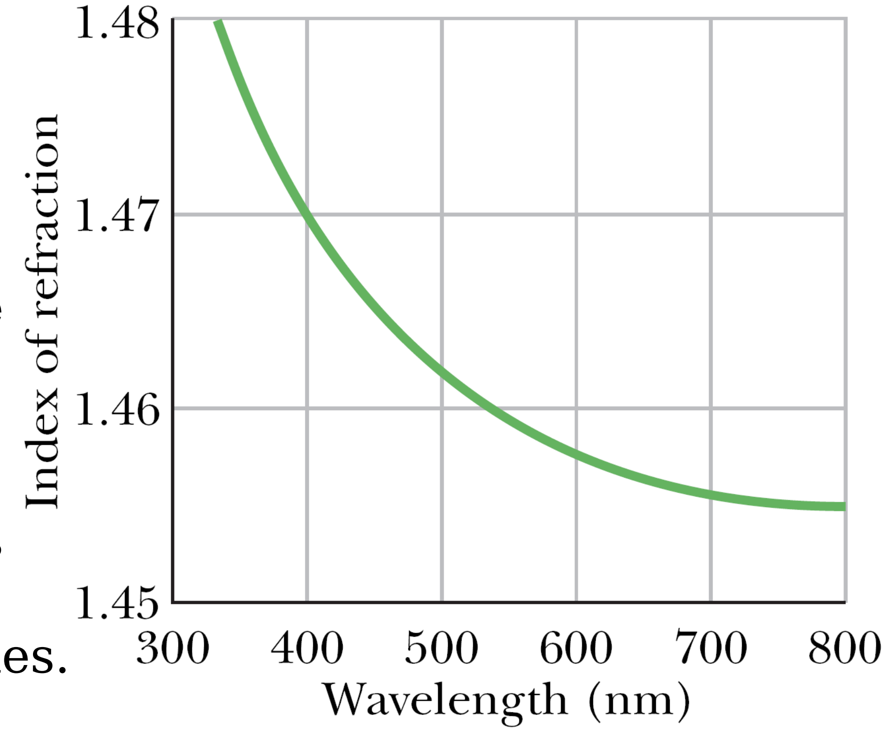
- When a light beam consists of rays of different wavelengths, the rays will be refracted at different angles, ie, the light will be spread out by the refraction. This spreading of light is called **chromatic dispersion**.

- There is no chromatic dispersion if the beams are *monochromatic*.

- The index of refraction of a given medium is usually greater for a shorter wavelength (blue light) than for a longer wavelength (red light).

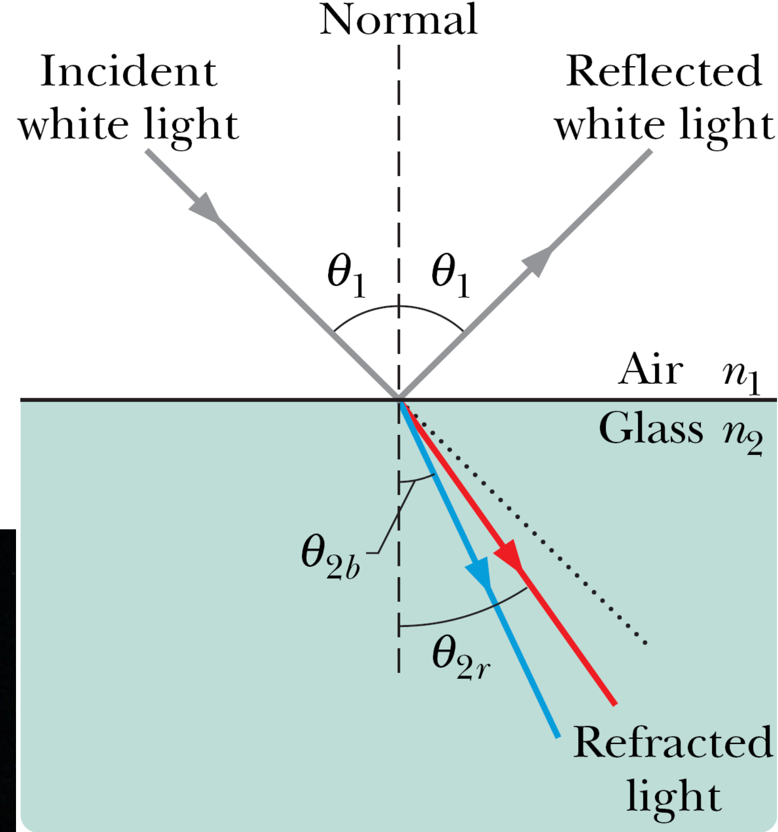
- When a beam made up of waves of both blue & red light is refracted through a surface, the blue component bends more than the red one.

- A beam of *white light* consists of components of all (or nearly all) the colors in the visible spectrum with approximately uniform intensities.

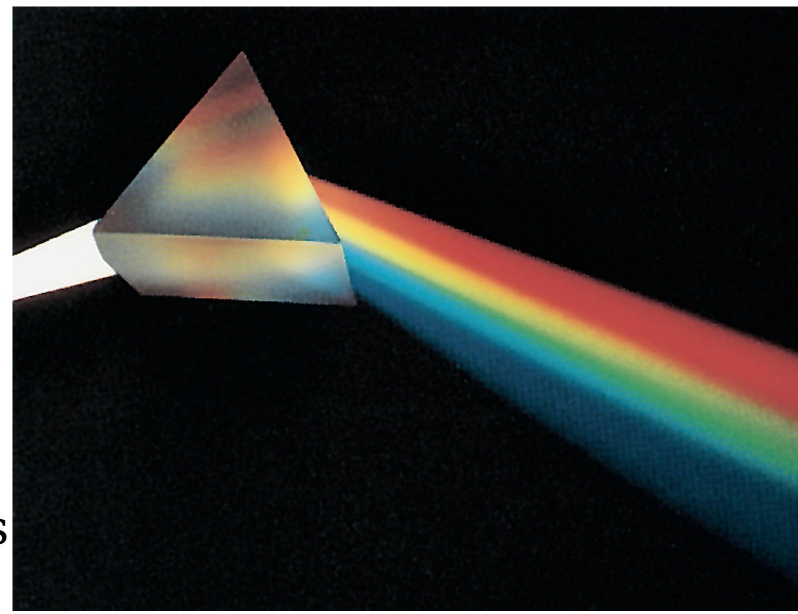


- For a beam of white light in air being incident on a glass surface, the blue component is bent more than the red component, thus $\theta_{2b} \leq \theta_{2r}$.

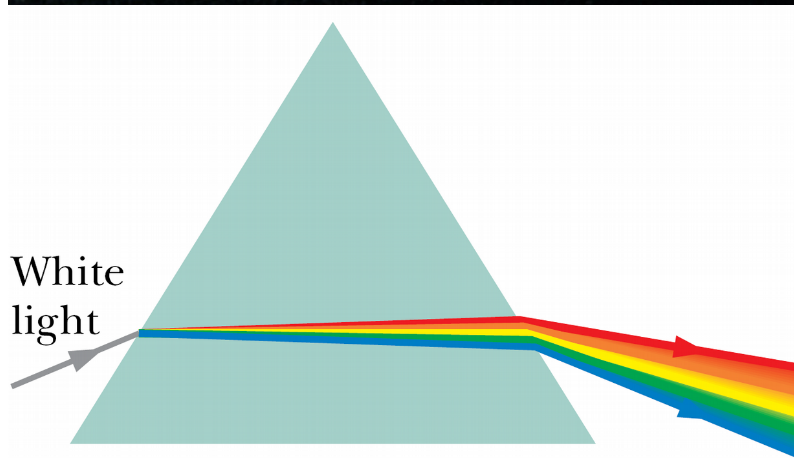
- For a ray of white light in glass is incident on a glass – air interface, the blue component is bent more than the red component, thus now $\theta_{2b} \geq \theta_{2r}$.

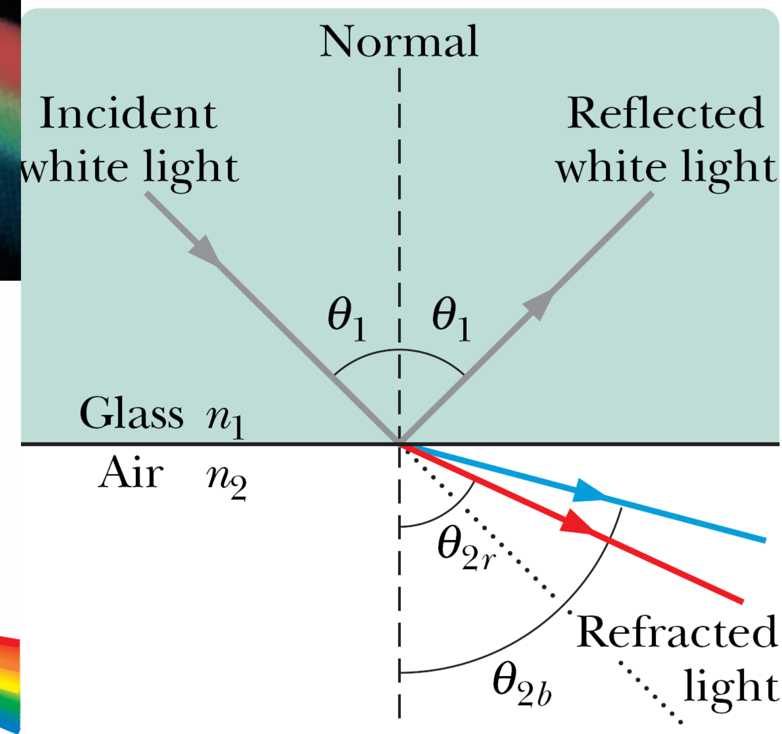


- To increase the color separation, we can use a solid glass prism with a triangular cross section.



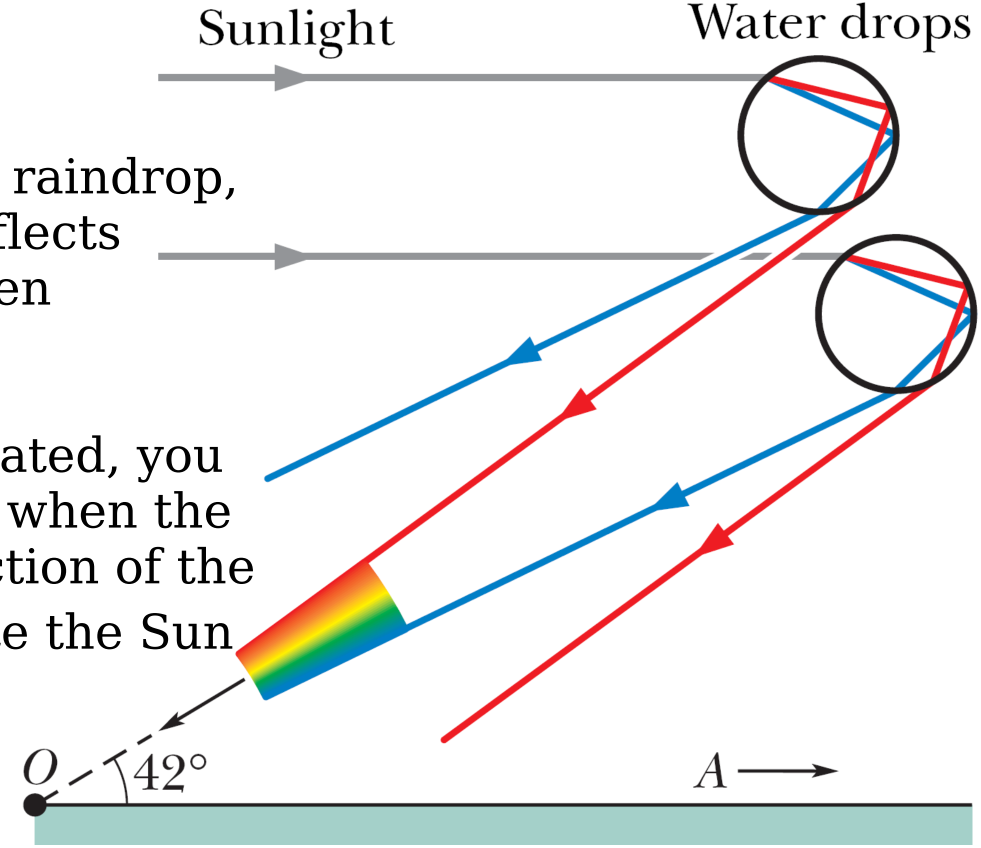
- The dispersion at the 1st surface is then enhanced by the dispersion at the 2nd surface.





Rainbows

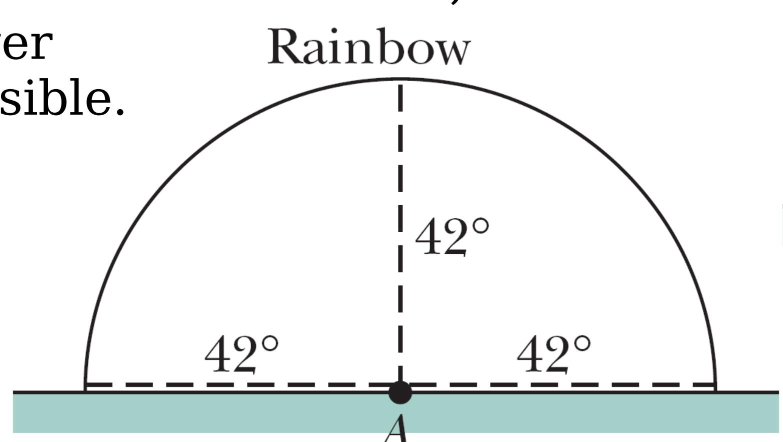
- When sunlight is intercepted by a falling raindrop, some of the light refracts into the drop, reflects once from the drop's inner surface, and then refracts out of the drop.

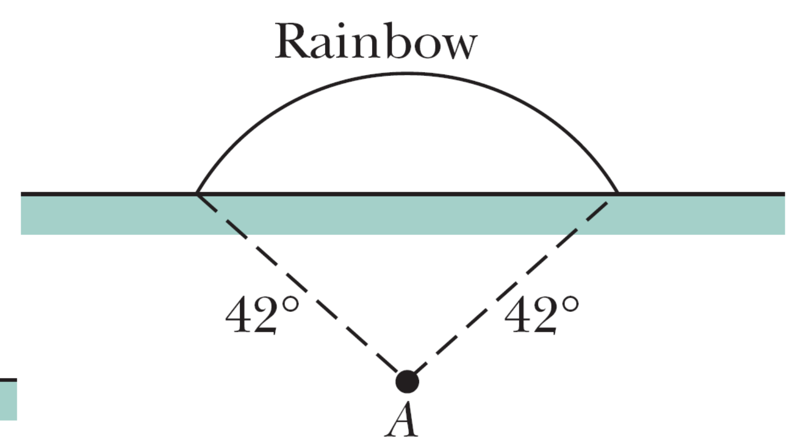


- If many falling drops are brightly illuminated, you can see the separated colors they produce when the drops are at an angle of 42° from the direction of the *antisolar point A*, the point directly opposite the Sun in your view.

- Because any drop at an angle of 42° in any direction from *A* can contribute to the rainbow, the rainbow is always a 42° circular arc around *A* and the top of a rainbow is never more than 42° above the horizon.

- When the Sun is above the horizon, the direction of *A* is below the horizon, and only a shorter, lower rainbow arc is possible.

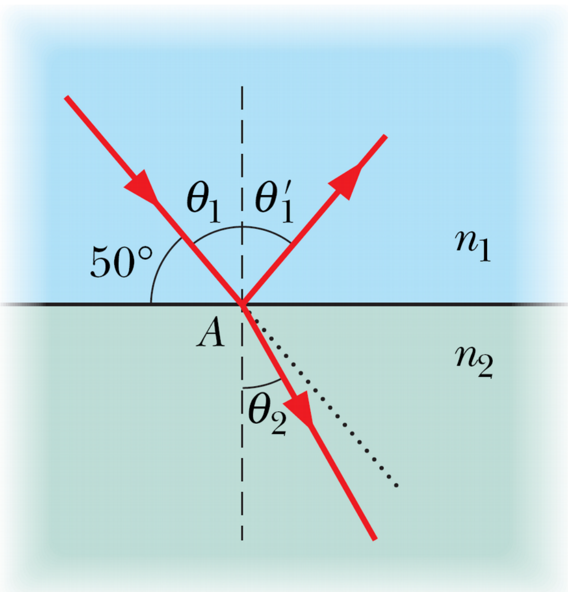




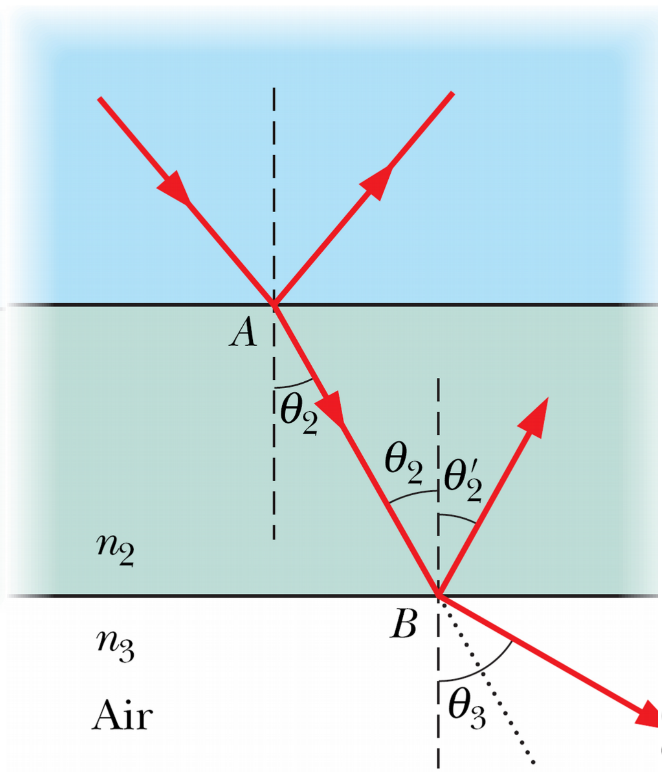
- Rainbows formed in this way involving one reflection of light inside each drop are called *primary rainbows*. A *secondary rainbow* involves 2 reflections inside a drop.

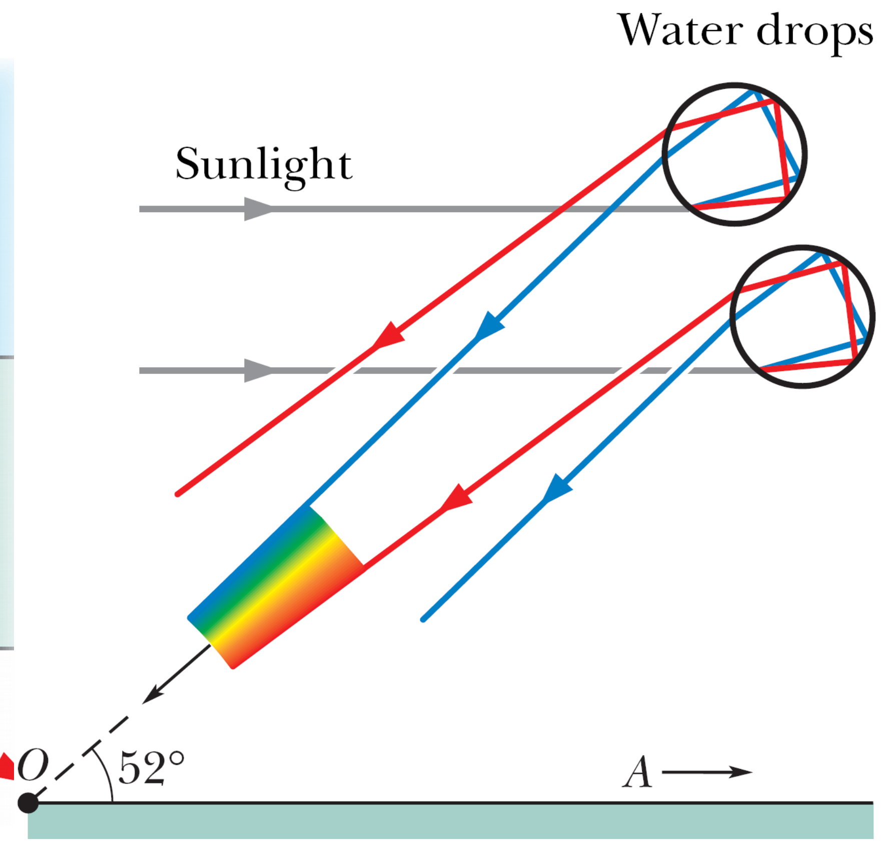
- Colors appear in the secondary rainbow at an angle of 52° from the direction of *A*. A secondary rainbow is wider and dimmer than a primary rainbow and thus is more difficult to see.

- The order of colors in a secondary rainbow is reversed from the order in a primary rainbow.

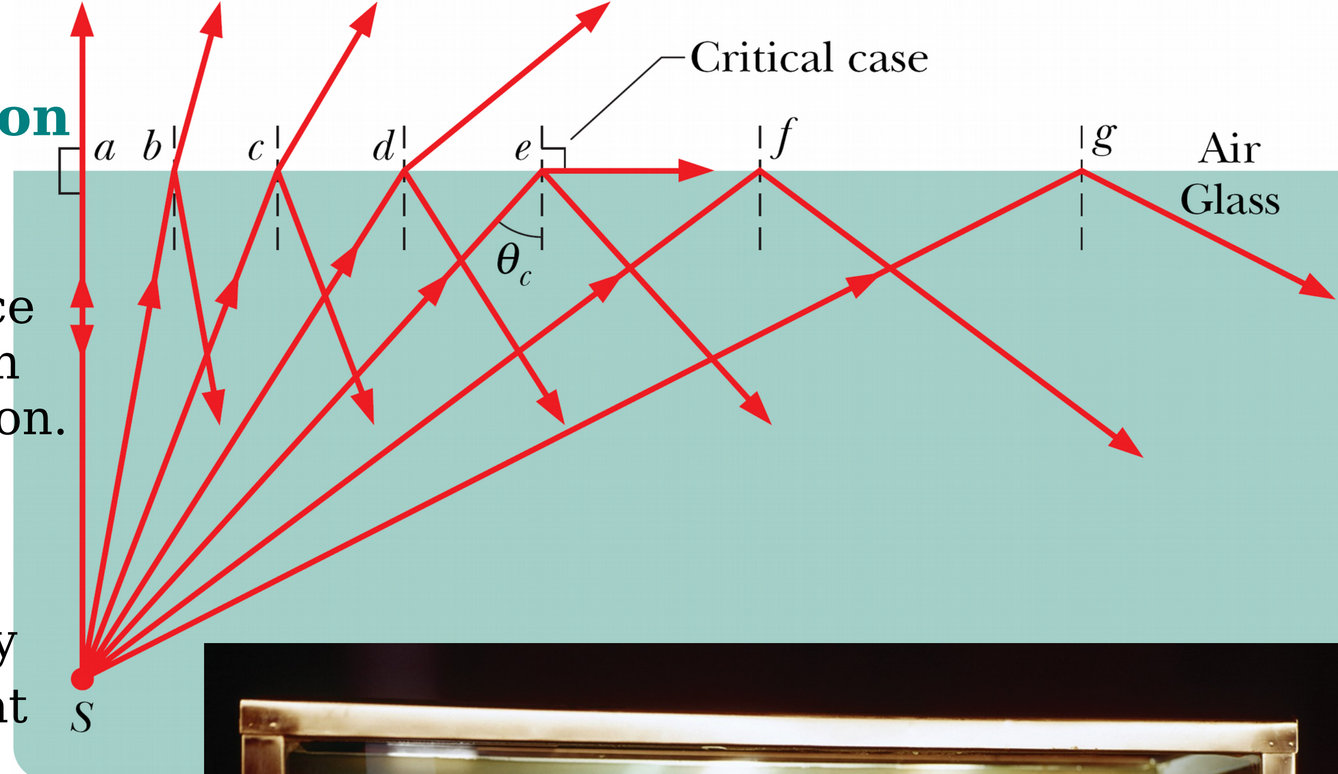


Problem 33.3





Total Internal Reflection



- For ray a, which is ⊥ to the interface, part of the light reflects at the interface and the rest travels through it with no change in direction.

- As the angle of incidence increases, the angle of refraction increases; for ray e it is 90°, which means that the refracted ray points directly along the interface.

- The angle of incidence giving this situation is the **critical angle** θ_c.

- For angles of incidence larger than θ_c, there is no refracted ray and all the light is reflected; this effect is called **total internal reflection**: all the light remains inside the glass.

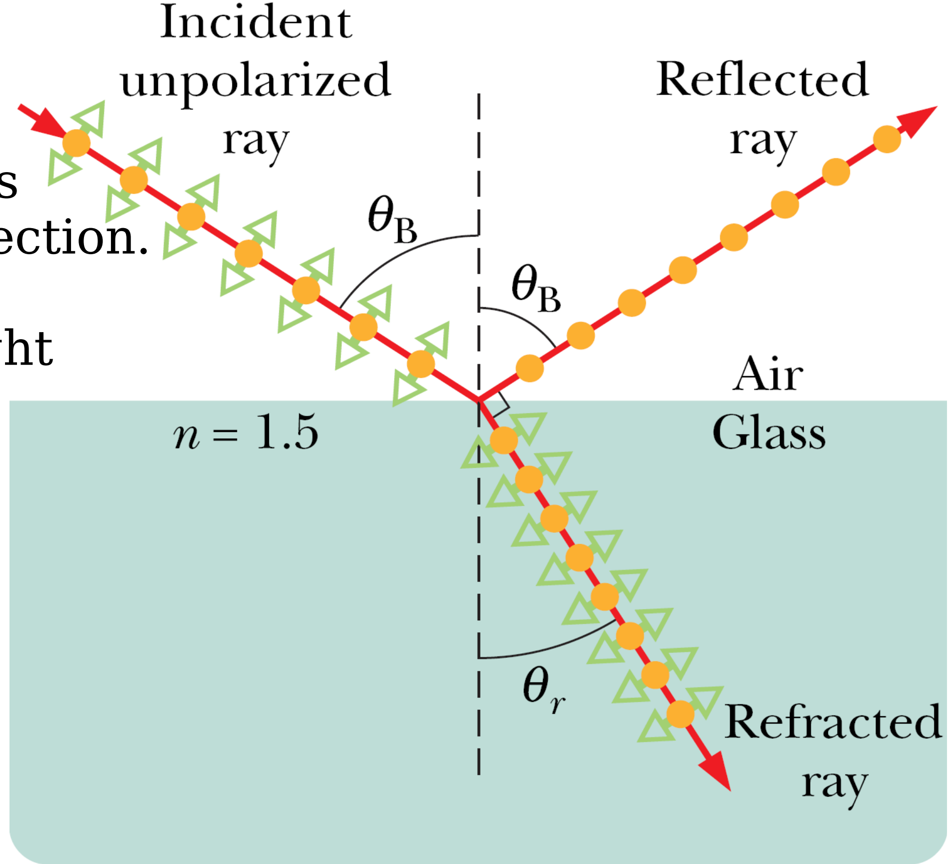
- To find θ_c, $n_1 \sin\theta_c = n_2 \sin\frac{\pi}{2} \Rightarrow$ $\boxed{\theta_c = \sin^{-1}\frac{n_2}{n_1} \quad \text{critical angle}}$

- $n_2 \leq n_1$ tells us that total internal reflection cannot occur when the incident light is in the medium of lower index of refraction.

- For 2 thin bundles of optical fibers, light introduced at the outer end of one bundle undergoes repeated total internal reflection within the fibers so that, even though the bundle provides a curved path, most of the light ends up exiting the other end. Some of the light reflected from the interior then comes back up the second bundle in a similar way.

- The physician can then perform a minimally invasive surgery with it..

Polarization by Reflection



- Any light that is reflected from a surface is either fully or partially polarized by the reflection.

- Resolve the electric field vectors of the light into 2 components: the *perpendicular components* are ⊥ to the plane of incidence; the *parallel components* are ∥ to the plane.

- Because the light is unpolarized, these 2 components are of equal magnitude.

- In general, the reflected light also has both components but with unequal magnitudes. This means that the reflected light is partially polarized — the electric fields along one direction have greater amplitudes than those along other directions.

- When the light is incident at a particular incident angle, ie, the *Brewster angle* θ_B, the reflected light has only perpendicular components.

- The reflected light is then fully polarized ⊥ to the plane of incidence. The parallel components of the incident light do not disappear but (along with perpendicular components) refract into the glass.

- Glass, water, and the other dielectric materials can partially and fully polarize light by reflection.

- When you intercept sunlight reflected from such a surface, you see a bright spot (the glare) on the surface where the reflection takes place. The reflected light is partially or fully polarized horizontally.

- To eliminate such glare from the surfaces, the lenses in polarizing sunglasses are mounted with their polarizing direction vertical.

Brewster's Law

- For light incident at the Brewster angle θ_c, we find experimentally that the reflected and refracted rays are $\perp$ to each other:

$$\theta_B + \theta_r = \frac{\pi}{2} \quad + \quad n_1 \sin\theta_B = n_2 \sin\theta_r \quad \Rightarrow \quad n_1 \sin\theta_B = n_2 \sin\left(\frac{\pi}{2} - \theta_B\right) = n_2 \cos\theta_B$$

$$\Rightarrow \quad \boxed{\theta_B = \tan^{-1}\frac{n_2}{n_1} \quad \text{Brewster angle}}$$

- If the incident and reflected rays travel *in air*, ie, $n_1 = 1$. Let $n_2 = n$,

$$\Rightarrow \quad \theta_B = \tan^{-1} n \quad \text{Brewster's law}$$