Chapter 28 Magnetic Force

What Produces a Magnetic Field?

- No magnetic charges (*magnetic monopoles*) found yet.

- There are 2 ways to produce magnetic fields

  (1) use moving electrically charged particles, such as a current in a wire, to make an **electromagnet**.

  (2) by means of elementary particles such as electrons because these particles have an *intrinsic* magnetic field around them, which could form **permanent magnet**.

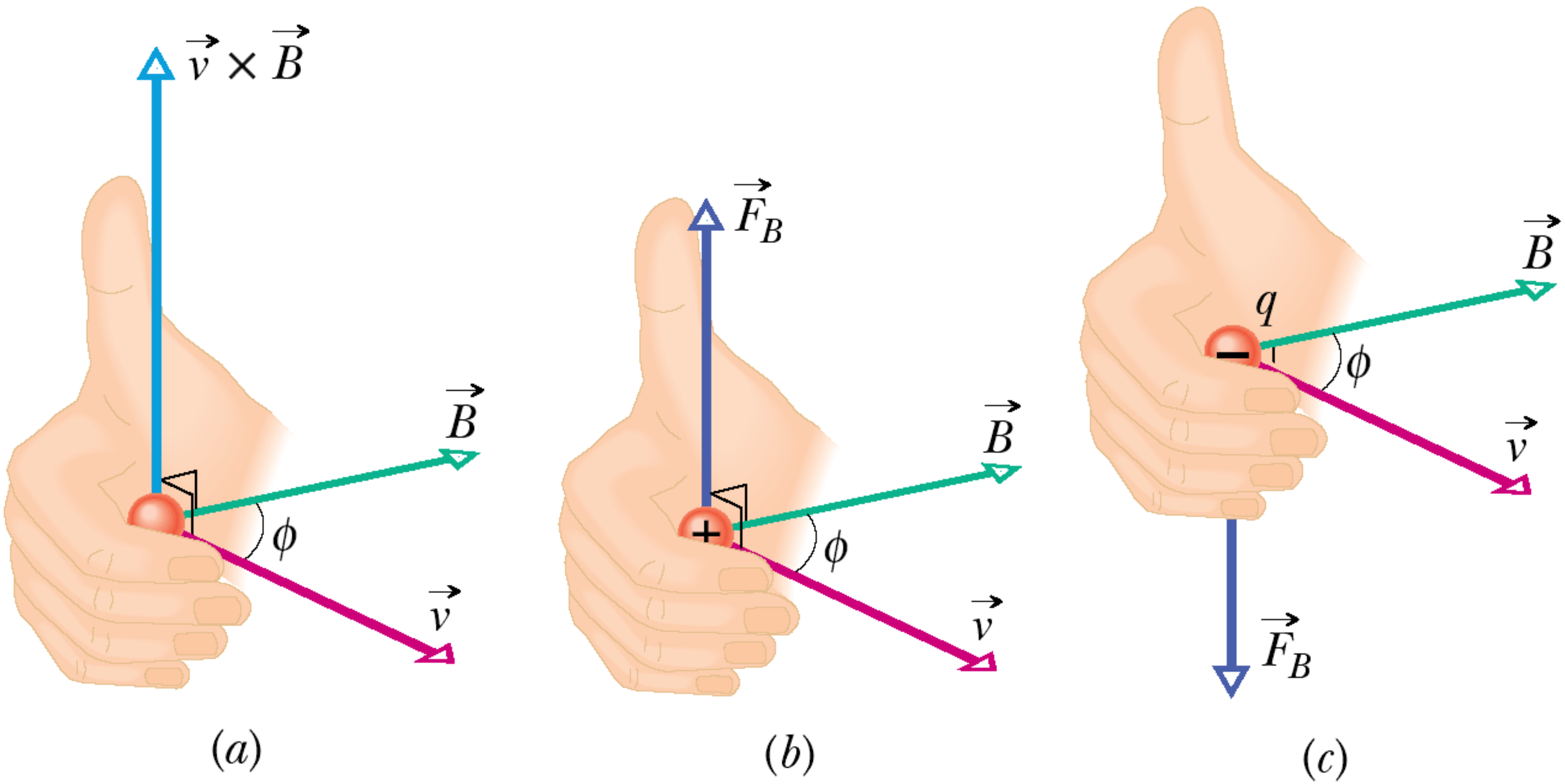
The Definition of Magnetic Field

- We define magnetic field in terms of the magnetic force exerted on a moving electrically charged test particle.

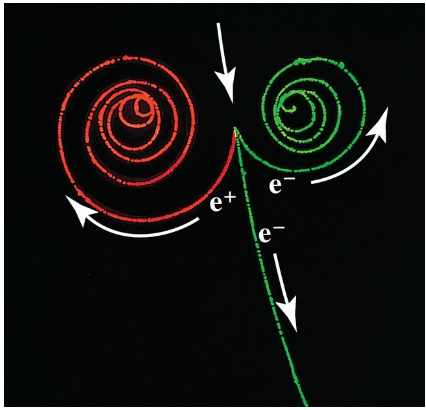
- We do this by firing a charged particle through the point at which the magnetic field is to be defined, using various directions and speeds for the particle and determining the magnetic force that acts on the particle at that point.

- The test suggests the following vector equation:

$$\vec{F}_B = q\,\vec{v} \times \vec{B} \quad \text{and} \quad |\vec{F}|_B \Rightarrow F_B = |q|\,v\,B\sin\phi \Leftarrow |q\,\vec{v} \times \vec{B}|$$



- The magnetic force on the particle is equal to the charge q times the cross product of its velocity and the magnetic field.



Finding the Magnetic Force on a Particle

- The magnetic force is equal to 0 if the charge is 0 or if the particle is stationary.

- The magnetic force is 0 if the velocity and the magnetic field are either parallel ($\phi=0$) or antiparallel ($\phi=\pi$), and the magnetic force is at its maximum when the velocity and the magnetic field are perpendicular to each other.

The force $\vec{F}_B$ acting on a charged particle moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ is *always* perpendicular to $\vec{v}$ and $\vec{B}$.

- $\vec{F}_B$ never has a component parallel to $\vec{v}$. This means that $\vec{F}_B$ cannot change the particle's speed v (and thus it cannot change the particle's kinetic energy). The force can change only the direction of $\vec{v}$ (and thus the direction of travel); only in this sense $\vec{F}_B$ can accelerate the particle.

- The SI unit for the magnetic field is the newton per coulomb-meter per second, or the **tesla** (T):

$$1\ \text{tesla} = 1\ \text{T} = 1\ \frac{\text{newton}}{(\text{coulomb})(\text{meter}/\text{second})}$$

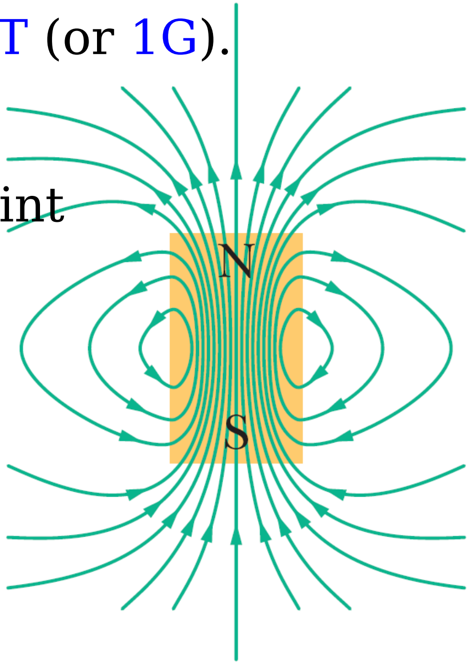
$$= 1\ \frac{\text{newton}}{(\text{coulomb}/\text{second})(\text{meter})} = 1\ \frac{\text{N}}{\text{A}\cdot\text{m}} = 10^4\ \text{gauss} = 10^4\ \text{G}$$

- Earth's magnetic field near the planet's surface is about $10^{-4}\ \text{T}$ (or 1G).

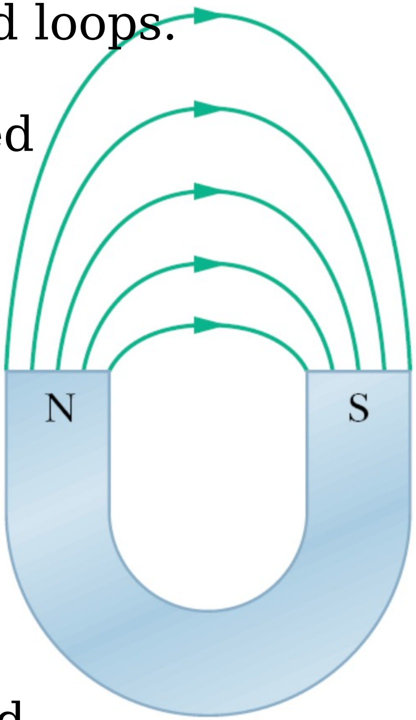
Magnetic Field Lines

- The direction of the tangent to a magnetic field line at any point gives the direction of the magnetic field at that point;

- The spacing of the lines represents the magnitude of the magnetic field – the magnetic field is stronger where the lines are closer together, and conversely.



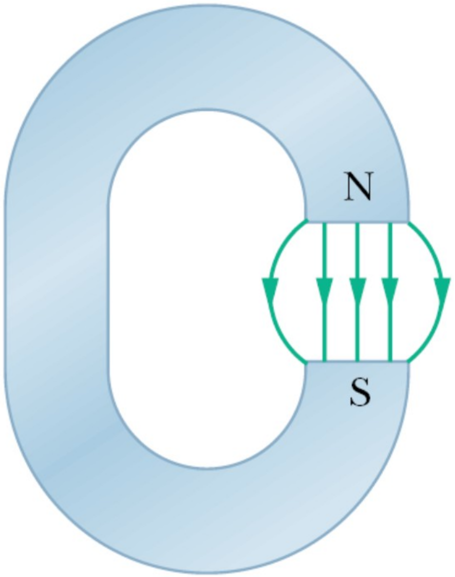
- The lines all pass through the magnet, and they all form closed loops.
- The end of a magnet from which the field lines emerge is called the *north pole* of the magnet; the other end, where field lines enter the magnet, is called the *south pole*.
- Because a magnet has 2 poles, it is said to be a **magnetic dipole.**



(*a*)

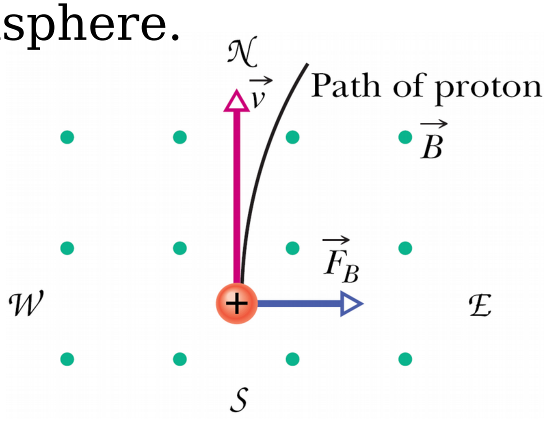
Opposite magnetic poles attract each other, and like magnetic poles repel each other.

- A compass turns because its north-pole end is attracted toward the Arctic region of Earth. Thus, the *south* pole of Earth's magnetic field must be located toward the Arctic. Logically, we then should call the pole there a south pole. However, because we call that direction north, we then say that Earth has a *geomagnetic north pole* in that direction. Correspondingly, the *geomagnetic south pole* is located in the southern hemisphere.



(*b*)

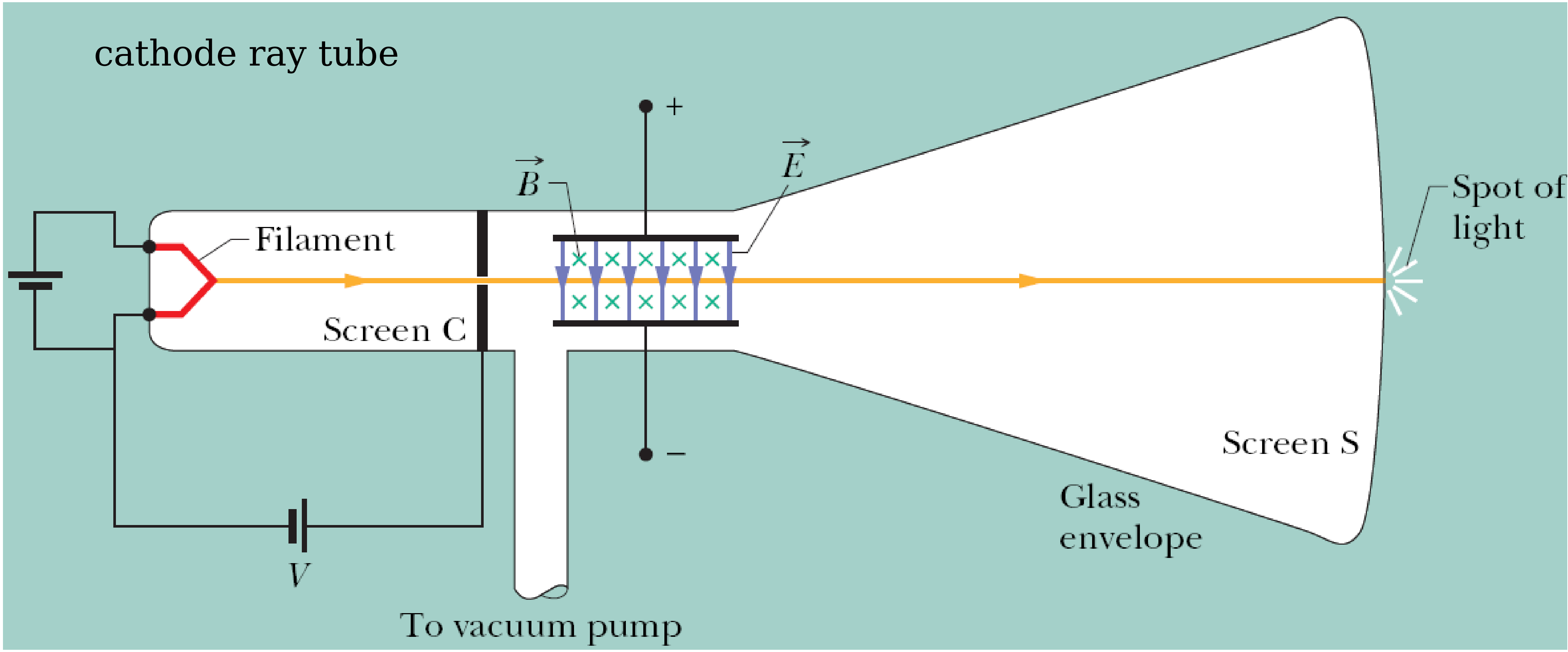
problem 28-1



Crossed Fields: Discovery of the Electron

- Both an electric field and a magnetic field can produce a force on a charged particle. When the 2 fields are perpendicular to each other, they are said to be *crossed fields*.

- J. J. Thomson in 1897 discovered the existence of electron by using a crossed field device.



- Electrons are forced up the slide by electric field and down the slide by magnetic field; that is, the forces are in opposition.

- Thomson's procedure was equivalent to the following series of steps:

(1) Set $E = 0$ and $B = 0$ and note the position of the spot on screen S due to the undeflected beam;

(2) Turn on the electric field and measure the resulting beam deflection;

(3) Maintaining the electric field, now turn on the magnetic field and adjust its value until the beam returns to the undeflected position.

- The deflection of the particle at the far end of the plates due to the electric field

$$y = \frac{1}{2} a_y t^2 = \frac{1}{2} \frac{q E}{m} \left(\frac{L}{v_x} \right)^2 = \frac{q E L^2}{2 m v^2}$$

- Because the direction of the deflection is set by the sign of the particle's charge, Thomson was able to show that the particles that were lighting up his screen were negatively charged.

- When the 2 fields are adjusted so that the 2 deflecting forces cancel (step 3), we have

$$q E = q v B \sin \frac{\pi}{2} = q v B \quad \Rightarrow \quad v = \frac{E}{B} \quad \Leftarrow$$

the crossed fields allow us to measure the speed of the charged particles

$$\Rightarrow \quad \frac{m}{q} = \frac{B^2 L^2}{2 y E}$$

- Thomson claimed that these particles are found in all matter, and they are lighter than the lightest known atom (hydrogen) by a factor of more than 1000. People consider this is the “discovery of the electron”.

Crossed Fields: The Hall Effect

- **Hall effect**: the drifting conduction electrons in a conductor can also be deflected by a magnetic field.

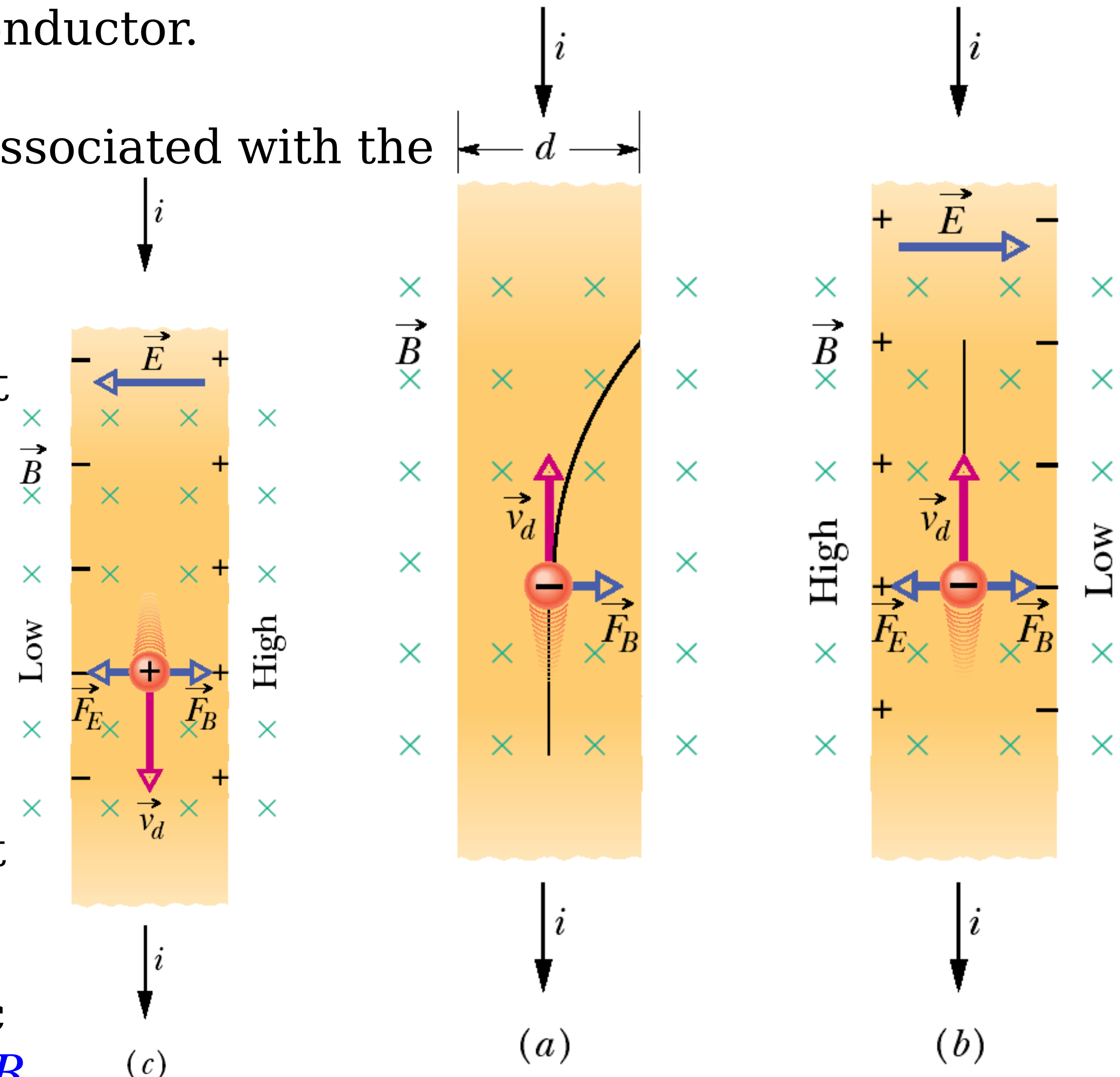
- The Hall effect allows us to find out whether the charge carriers in a conductor are positively or negatively charged. We can also measure the number of such carriers per unit volume of the conductor.

- A *Hall potential difference* V is associated with the electric field across strip width d,

$$V = E\,d$$

- If the charge carriers in current i are positively charged, then as these charge carriers move from top to bottom, they are pushed to the *right* edge and thus that the right edge is at higher potential. Because that this statement is contradicted by our voltmeter reading, the charge carriers must be negatively charged.

- When the electric and magnetic forces are in balance $e\,E = e\,v_d\,B$



$$\frac{E}{B} \Leftarrow v_d = \frac{J}{n\,e} = \frac{i}{n\,e\,A} \quad \Rightarrow \quad n = \frac{i}{e\,v_d\,A} = \frac{i\,B}{e\,\ell\,V} \quad \Leftarrow \quad \ell = \frac{A}{d}, \quad E = \frac{V}{d}$$

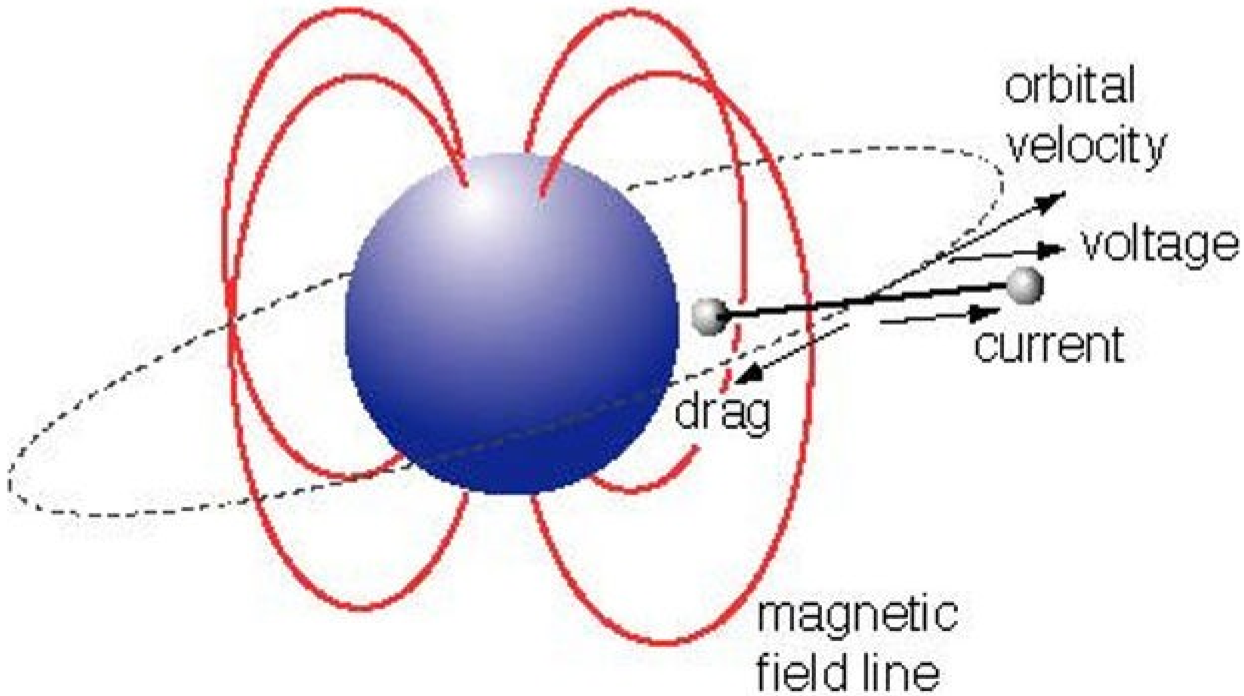
- We can also use the Hall effect to measure directly the drift speed of the charge carriers:
  (1) moved the metal strip mechanically through the magnetic field in a direction opposite that of the drift velocity of the charge carriers;
  (2) adjust the speed of the moving strip until the Hall potential difference vanishes;
  (3) At this condition, with no Hall effect, the velocity of the charge carriers *with respect to the laboratory frame* must be 0, so the velocity of the strip must be equal in magnitude but opposite the direction of the velocity of the negative charge carriers.

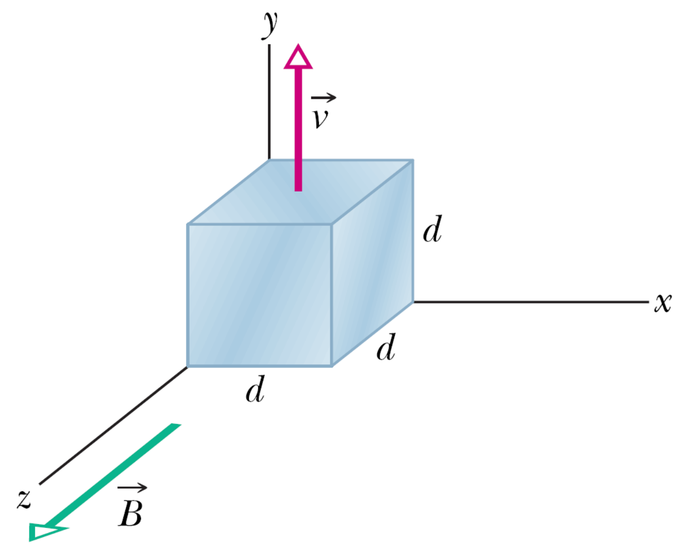
- When a conductor begins to move at speed v through a magnetic field, its conduction electrons do also. And an electric field $\vec{E}$ and potential difference V are quickly set up:

$$e\,E = e\,v\,B \quad \Rightarrow \quad V = v\,B\,d$$

- Such a motion-caused circuit potential difference can be of serious concern when a conductor in an orbiting satellite moves through Earth's magnetic field.

- If a conducting line (said to be an *electrodynamic tether*) dangles from the satellite, the potential produced along the line might be used to maneuver the satellite.





problem 28-2

A Circulating Charged Particle

- If a particle moves in a circle at constant speed, then the net force acting on the particle is constant in magnitude and points toward the center of the circle, perpendicular to the particle's velocity.

- A particle of charge magnitude $|q|$ and mass m moving perpendicular to a uniform magnetic field at speed v, then $F_B = |\vec{F}_B| = |q|\,v\,B$

- From Newton's 2nd law applied to uniform circular motion

$$F = m\,a = m\frac{v^2}{r} \Rightarrow |q|\,v\,B = \frac{m\,v^2}{r} \Leftarrow r = \frac{m\,v}{|q|\,B} \quad \text{radius}$$

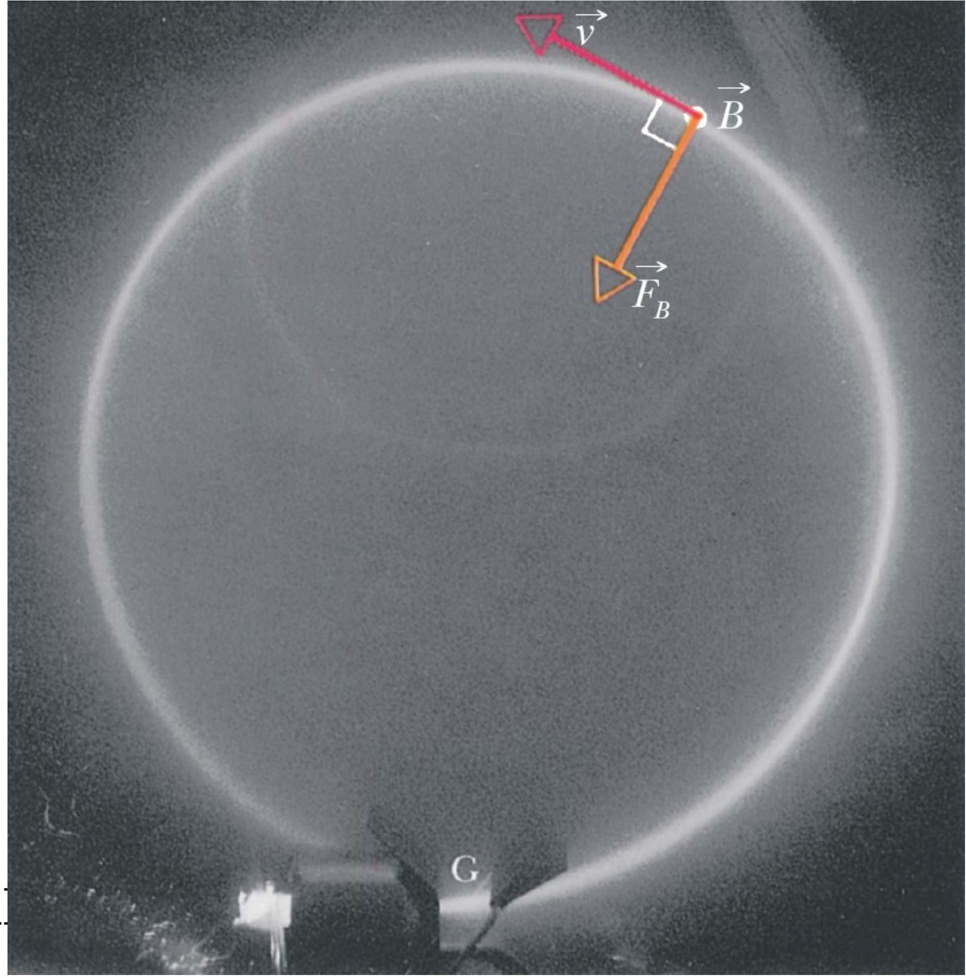
- The period T is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\frac{mv}{|q|B} = \frac{2\pi m}{|q|B} \quad \text{period}$$

The frequency and the angular frequency are

$$f = \frac{1}{T} = \frac{|q|B}{2\pi m} \qquad \text{frequency}$$

$$\omega = 2\pi f = \frac{|q|B}{m} \quad \text{angular frequency}$$



- The quantities T, f, and ω do not depend on the speed of the particle, fast particles move in large circles and slow ones in small circles, but all particles with the same charge-to-mass ratio $|q|/m$ take the same time to complete one round trip.

Helical Paths

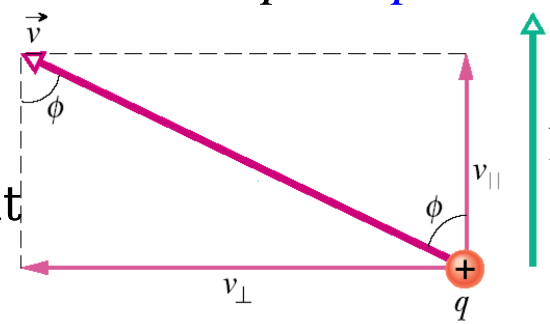
- If the velocity of a charged particle has a component parallel to the (uniform) magnetic field, the particle will move in a helical path about the direction of the field vector.

- the velocity vector can be resolved into 2 components, one parallel to the magnetic field and one perpendicular to it:

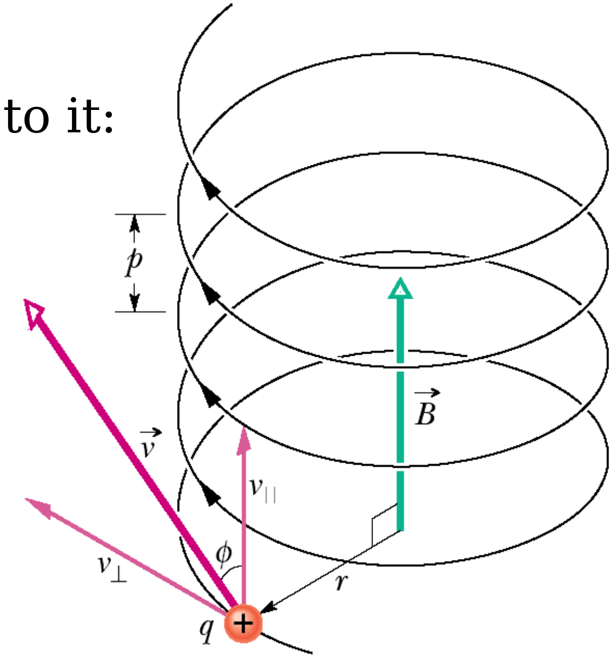
$$v_{\parallel} = v \cos\phi \quad \text{and} \quad v_{\perp} = v \sin\phi$$

- The parallel component determines the *pitch* p of the helix – the distance between adjacent turns.

- The perpendicular component determines the radius of the helix.



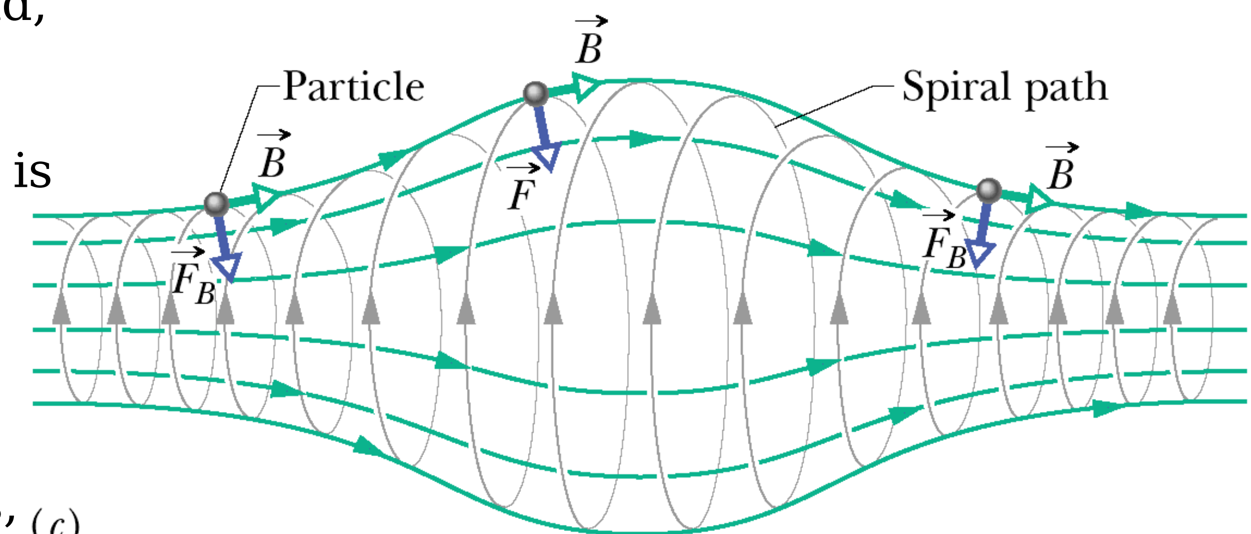
(a)



(b)

- In a nonuniform magnetic field, the more closely spaced field lines at the left and right sides indicate that the magnetic field is stronger there.

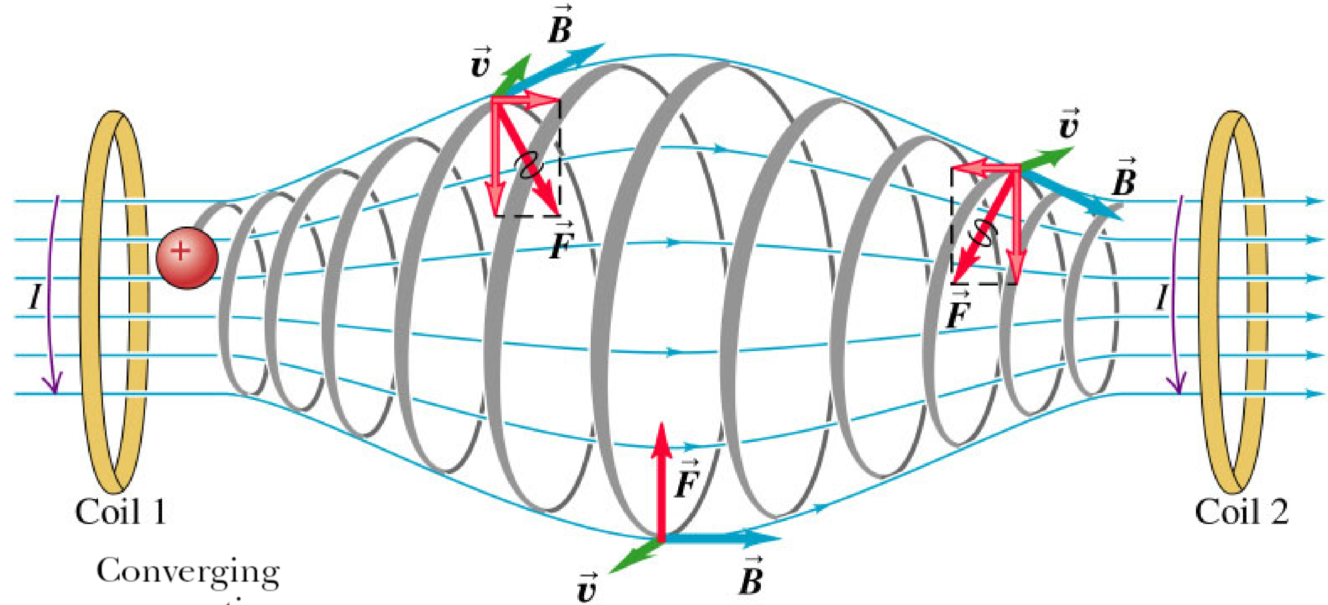
- When the field at an end is strong enough, the particle "reflects" from that end. If the particle reflects from both ends, it is said to be trapped in a *magnetic bottle*.



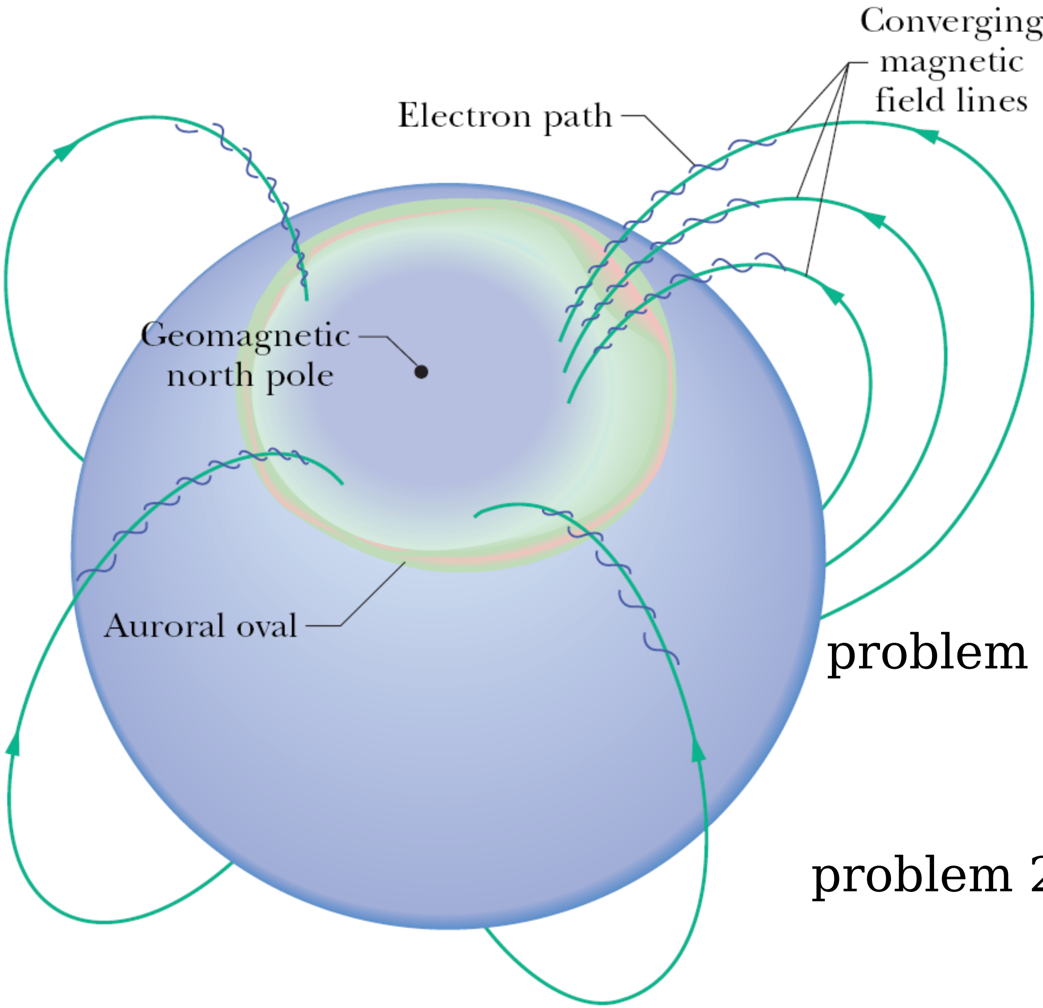
(c)

- This technique is used to confine very hot plasma with $T \sim 10^6$ K.

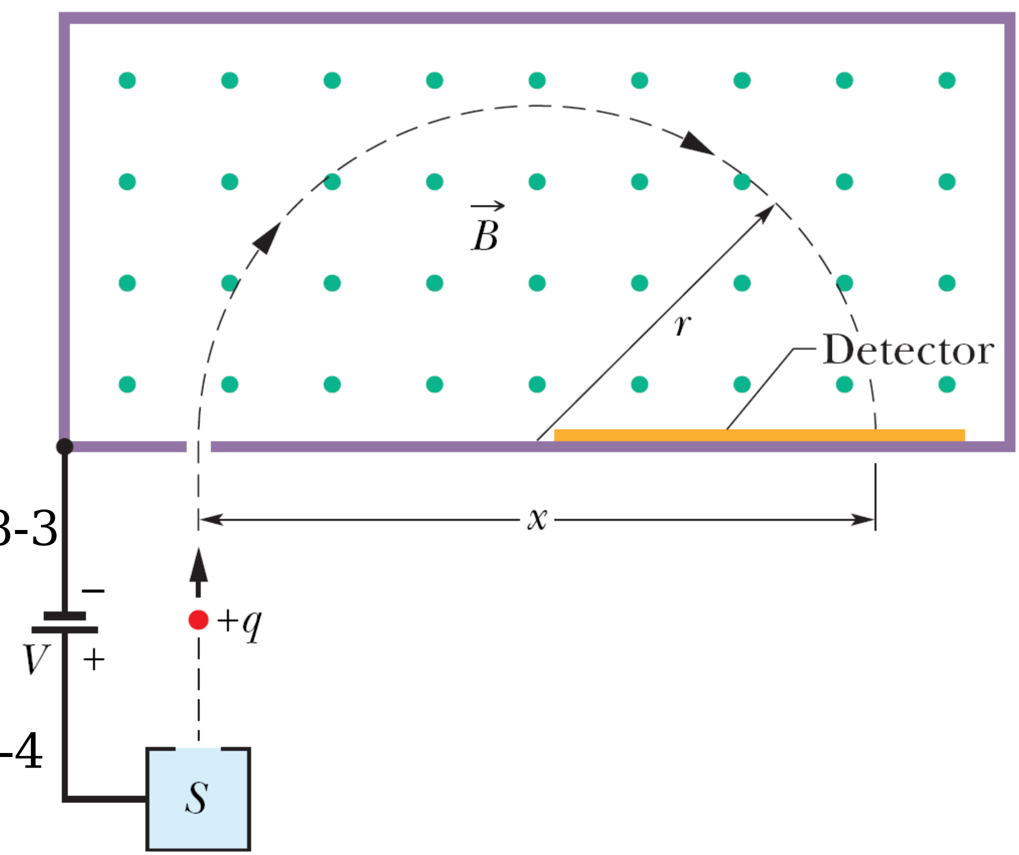
- *Van Allen radiation belts*
  ~ 100km high
  green light ⇐ oxygen
  pink light ⇐ nitrogen



- Aurora (long & thin) extend in arcs above Earth and can occur in a region called the *auroral oval*.



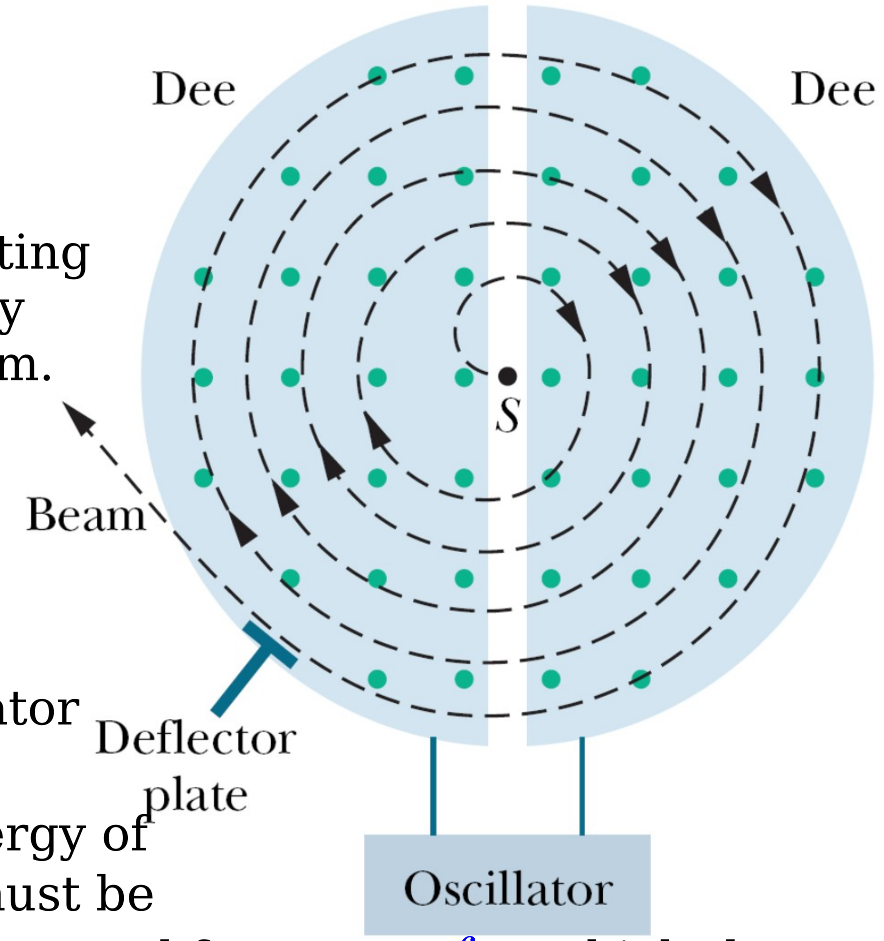
problem 28-3



problem 28-4

Cyclotrons and Synchrotrons

- *Accelerators* that employ a magnetic field to repeatedly bring particles back to an accelerating region, where they gain more and more energy until they finally emerge as a high-energy beam.



The Cyclotron

- The key to the operation of the cyclotron is

$$f = f_{OSC} \quad \text{resonance condition}$$

f_{osc}: the fixed frequency of the electrical oscillator

- This *resonance condition* says that, if the energy of the circulating proton is to increase, energy must be fed to it at a frequency f_{osc} that is equal to the natural frequency f at which the proton circulates in the magnetic field, $|q|B = 2\pi m f_{OSC}$

- Electrically neutral particles, such as neutrons, cannot be accelerated by a cyclotron.

The Proton Synchrotron

- At proton energies above 50 MeV, the conventional cyclotron begins to fail because the assumption that the frequency of revolution of a charged particle circulating in a magnetic field is independent of the particle's speed is true only for speeds that are much less than the speed of light – the relativistic effect.

- Another problem is the cost of building a cyclotron for high-energy protons.

- The solution is the *proton synchrotron*. The magnetic field and the oscillator frequency are made to vary with time during the accelerating cycle:

  (1) the frequency of the circulating protons remains in step with the oscillator at all times,
  (2) the protons follow a circular – not a spiral – path.

- The proton synchrotron at the Fermi National Accelerator Laboratory (Fermilab) in Illinois has a circumference of 6.3 km and can produce protons with energies of about 1 TeV ($=10^{12}$ eV).

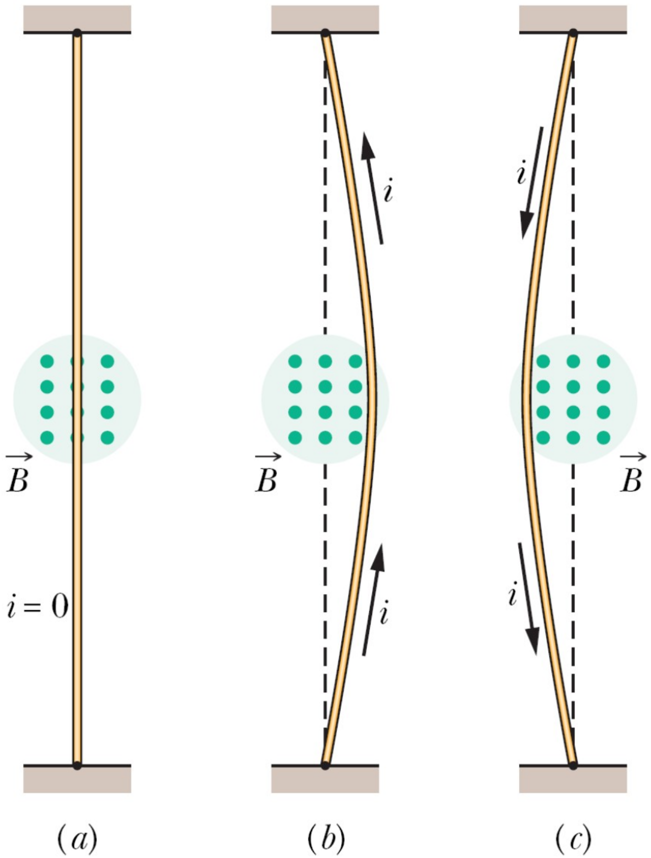
problem 28-5

Magnetic Force on a Current-Carrying Wire

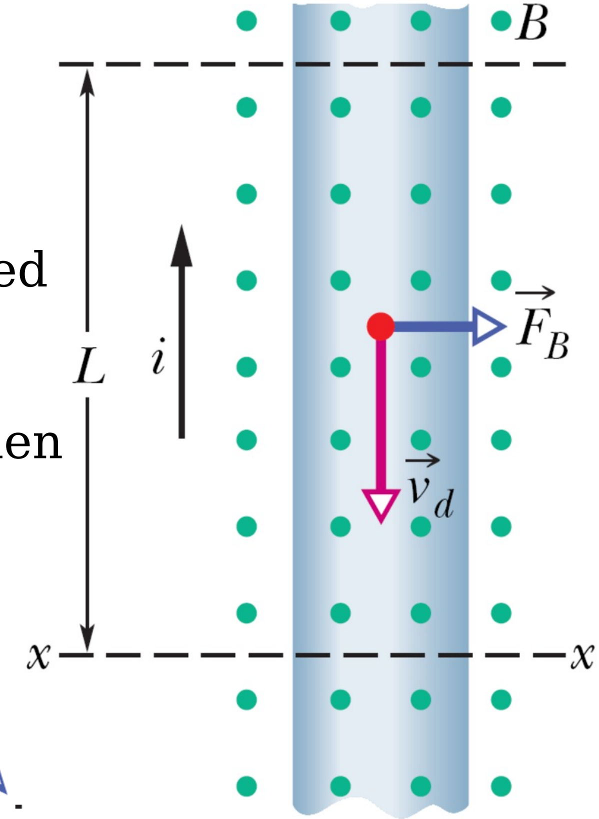
- A wire with a current upward in the magnetic field directed outward from the page deflects to the right; a wire with a current downward defects to the left.

- Consider a length L of the wire. All the conduction electrons in this section of wire will drift past plane *xx* in a time $t = L/v_d$. Then the charge passing through that plane in the time is,

$$q = i\,t = i\,\frac{L}{v_d}$$



Therefore $F_B = q\, v_d\, B \sin\phi = \dfrac{i\,L}{v_d}\, v_d\, B \sin\dfrac{\pi}{2} = i\,L\,B$



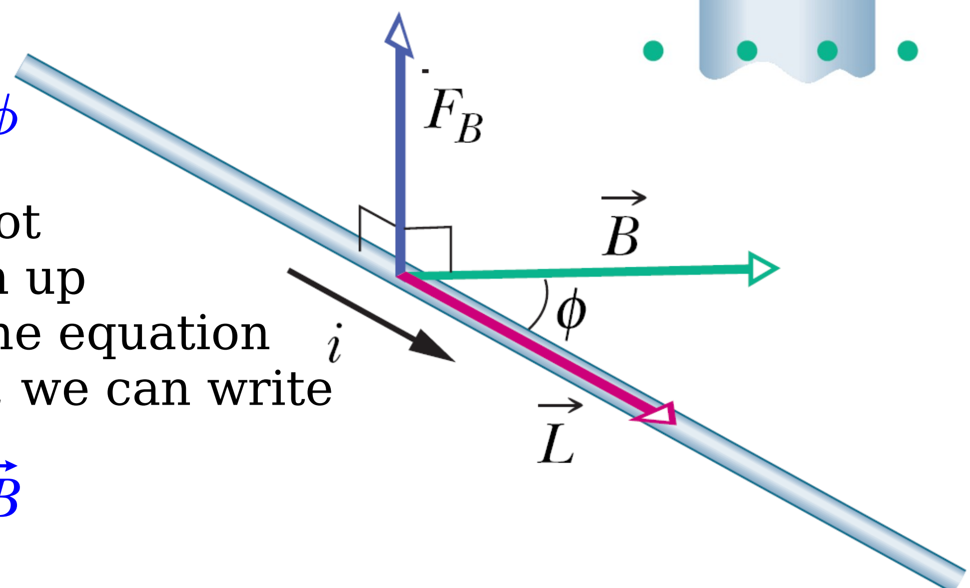
- This equation gives the magnetic force that acts on a length L of straight wire carrying a current i and immersed in a magnetic field that is perpendicular to the wire.

- If the magnetic field is not perpendicular to the wire, then

$$\vec{F}_B = i\,\vec{L} \times \vec{B} \quad \text{force on a current}$$

where $\vec{L}$ is a length vector that has magnitude L and is directed along the wire segment in the direction of the (conventional) current.

- The force magnitude is $F_B = i\,L\,B \sin\phi$



- If a wire is not straight or the field is not uniform, we can imagine the wire broken up into small straight segments and apply the equation to each segment. In the differential limit, we can write

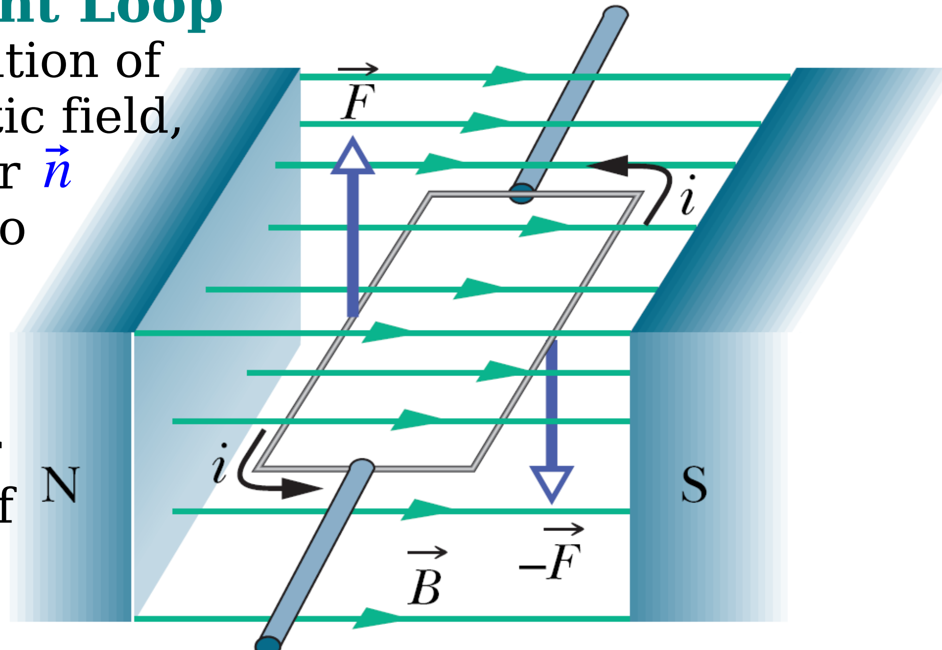
$$\vec{F}_B = \int d\vec{F}_B = \int i\, d\vec{L} \times \vec{B}$$

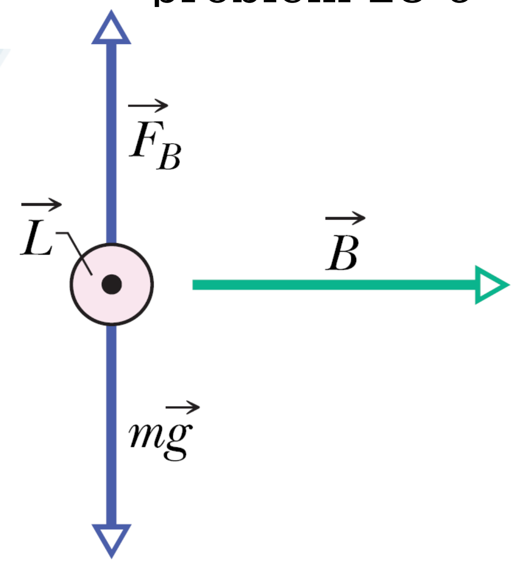
- There is no such thing as an isolated current-carrying wire segment of length dL. There must always be a way to introduce the current into the segment at one end and take it out at the other end.

Torque on a Current Loop

problem 28-6

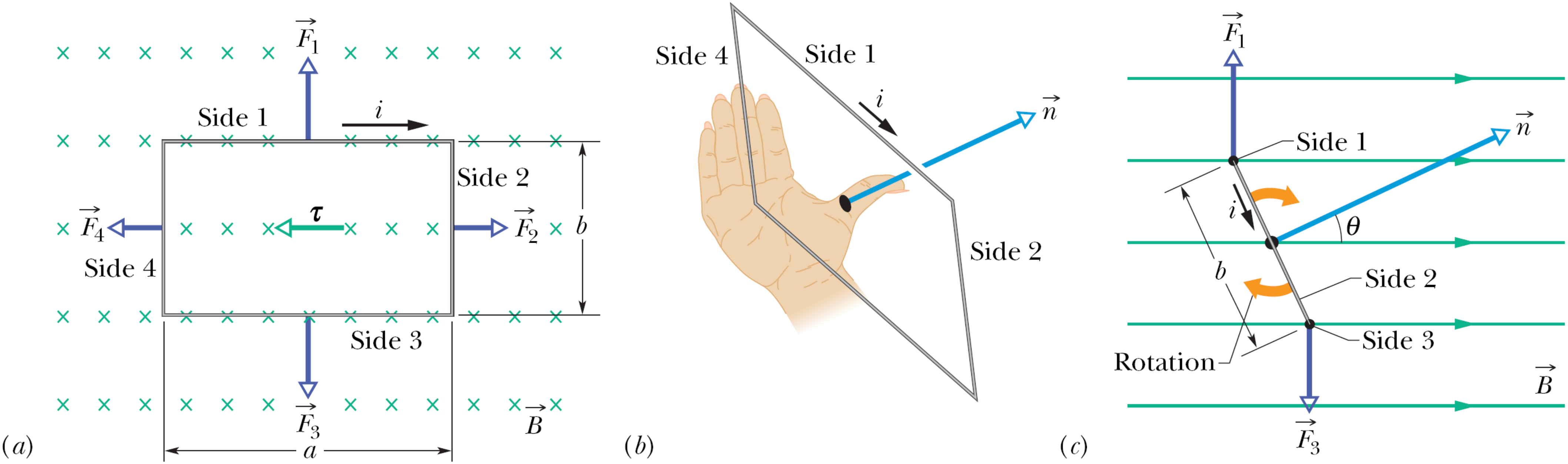
- To define the orientation of the loop in the magnetic field, we use a normal vector $\vec{n}$ that is perpendicular to the plane of the loop that rotates it.





- A right-hand rule for finding the direction of $\vec{n}$— point or curl the fingers of your right hand in the direction of the current at any point on the loop. Your extended thumb then points in the direction of the normal vector $\vec{n}$.

- The net force on the loop is the vector sum of the forces acting on its 4 sides.



- The magnitude of the force acting on side 2 $F_2 = i\, b\, B \sin\left(\frac{\pi}{2} - \theta\right) = i\, b\, B \cos\theta$
- The force acting on side 4 has the same magnitude as the force on side 2 but the opposite direction. Thus, they cancel out exactly. Their net force is 0 and, because their common line of action is through the center of the loop, their net torque is also 0.
- Since $\vec{L} \perp \vec{B}$, so the forces on side 1 and 3 have the common magnitude iaB.
- The magnitude of the torque due to forces on side 1 and 3 is

$$\tau' = i\, a\, B \frac{b}{2} \sin\theta + i\, a\, B \frac{b}{2} \sin\theta = i\, a\, b\, B \sin\theta = i\, A\, B \sin\theta \quad \Leftarrow \quad A = a\, b$$

- If we replace the single loop of current with a *coil* of N *turns*, then the total torque on the coil then has magnitude

$$\tau = N\, \tau' = (N\, i\, A)\, B \sin\theta \quad \Rightarrow \quad \vec{\tau} = I\, \vec{A} \times \vec{B} \quad \Leftarrow \quad I \equiv N\, i$$

- A current-carrying flat coil placed in a magnetic field will tend to rotate so that $\vec{n}$ has the same direction as the field.
- In a motor, the current in the coil is reversed as $\vec{n}$ begins to line up with the field direction, so that a torque continues to rotate the coil.

The Magnetic Dipole Moment

- Since the coil behaves like a bar magnet placed in the magnet field, a current-carrying coil is said to be a *magnetic dipole*.

- Assign a magnetic dipole moment to the coil $\vec{\mu} = I\,\vec{A} = N\,i\,\vec{A}$ magnetic moment

- The unit of $\vec{\mu}$ is ampere-square meter ($\mathrm{A \cdot m^2}$).

- The torque now can be expressed as $\vec{\tau} = \vec{\mu} \times \vec{B}$

and its magnitude is $\tau = \mu\,B\sin\theta$

- It is very similar to the torque exerted by an *electric field* on an *electric* dipole.

$$\vec{\tau} = \vec{p} \times \vec{E}$$

- A magnetic dipole in an external magnetic field has a **magnetic potential energy** that depends on the dipole's orientation in the field

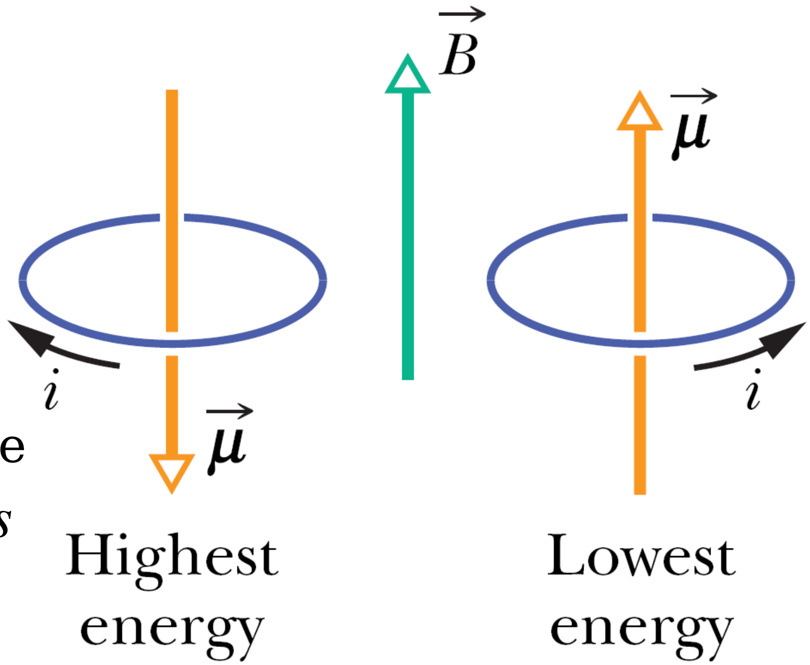
$$U(\theta) = -\vec{\mu} \cdot \vec{B}$$

- A magnetic dipole has its lowest energy ($= -\mu B\cos 0 = -\mu B$) when its dipole moment is lined up with the magnetic field. It has its highest energy ($= -\mu B\cos\pi = +\mu B$) when its dipole moment is directed opposite the field.

- When a magnetic dipole rotates from an initial orientation to another orientation, the work W done on the dipole by the magnetic field is

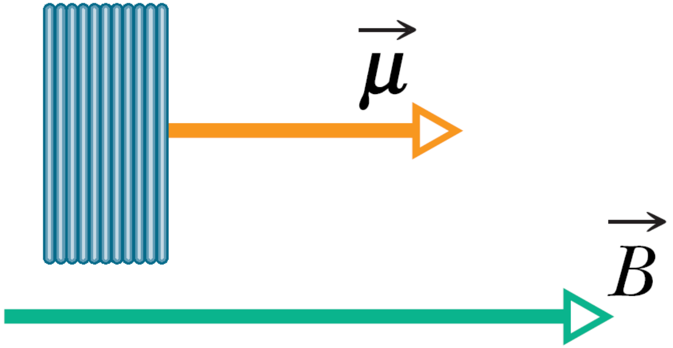
$$W = -\Delta U = -(U_f - U_i)$$

- If an applied torque acts on the dipole during the change in its orientation, then work W_a is done on the dipole by the applied torque. *If the dipole is stationary* before and after the change in its orientation, then work W_a is the negative of a the work done on the dipole by the field, $W_a = -W = U_f - U_i$



- Bar magnet, Earth, most subatomic particles (electron, the proton, and the neutron) have magnetic dipole moments. All these quantities can be viewed as current loops.

problem 28-7



Selected problems: 7, 22, 32, 58, 64