Chapter 27 Circuits

- We restrict our discussion to circuits through which charge flows in one direction, which are called either *direct-current circuits* or *DC circuits*.

"Pumping" Charges

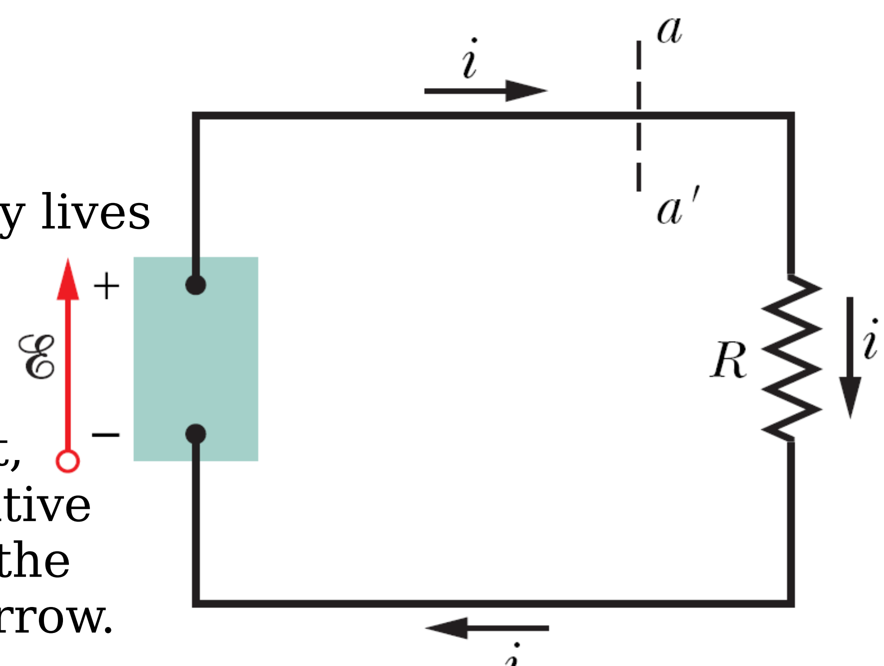
- An **emf device**: a device that does work on charge carriers and maintains a potential difference between a pair of terminals.

- The emf device will provide an **emf** $\mathscr{E}$, which means that it does work on charge carriers. An emf device is sometimes called *a seat of emf*.

- The term emf comes from the outdated phrase *electromotive force*.

- A common emf device is the *battery*.

- The emf device that most influences our daily lives is the *electric generator*.



Work, Energy, and Emf

- When an emf device is connected to a circuit, its internal chemistry causes a net flow of positive charge carriers from the negative terminal to the positive terminal, in the direction of the emf arrow.

- Within the emf device, positive charge carriers move from a region of low electric potential and thus low electric potential energy to a region of higher electric potential and higher electric potential energy.

- This motion is the opposite of what the electric field between the terminals would cause the charge carriers to do.

- There must be some source of energy within the device, enabling it to do work on the charges by forcing them to move as they do, eg, chemical forces, mechanical forces, or temperature differences.

- We define the emf of the emf device in terms of this work:

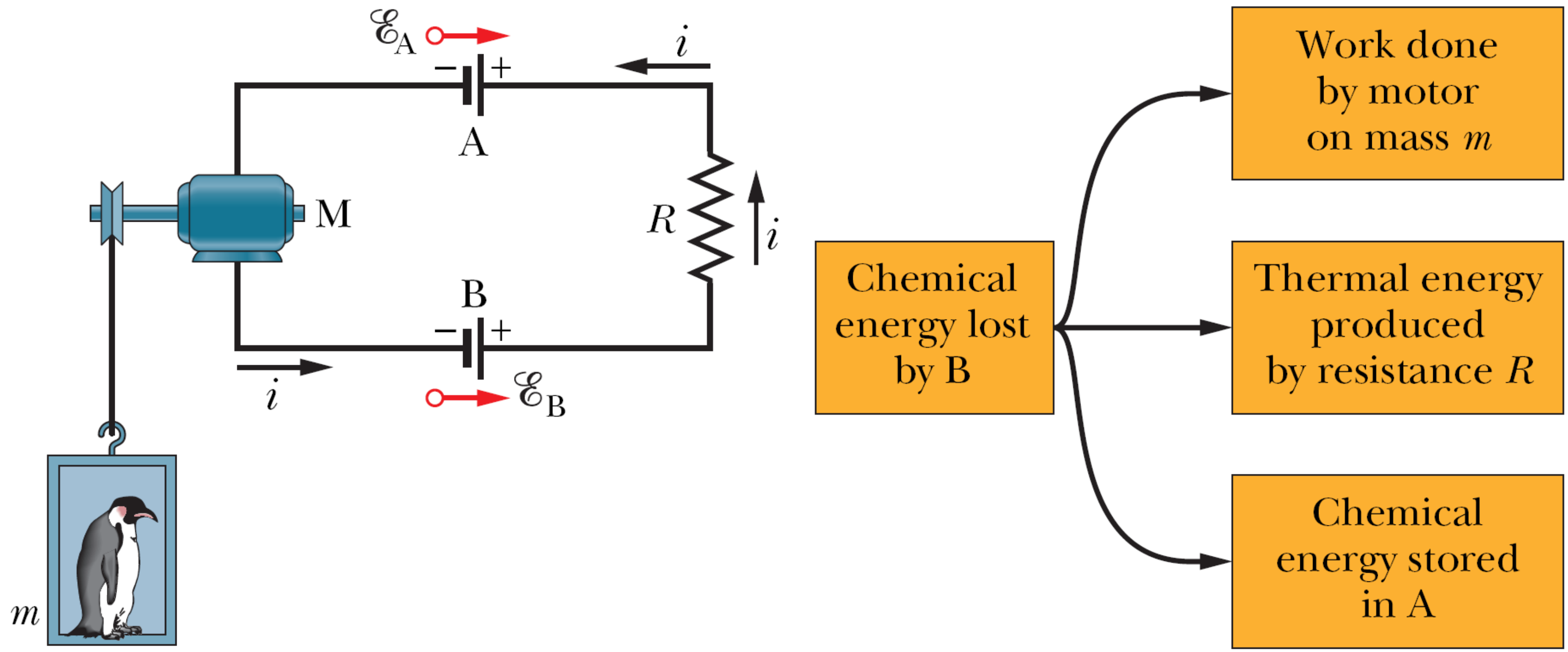
$$\mathscr{E} = \frac{\mathrm{d}\,W}{\mathrm{d}\,q} \quad \text{definition of } \mathscr{E}$$

the emf is the work per unit charge that the device does in moving charge from its low-potential terminal to its high-potential terminal.

- The SI unit for emf is the joule per coulomb, *volt*.

- An **ideal emf device** is one that lacks any internal resistance to the internal movement of charge from terminal to terminal. The potential difference between the terminals of an ideal emf device is equal to the emf of the device.

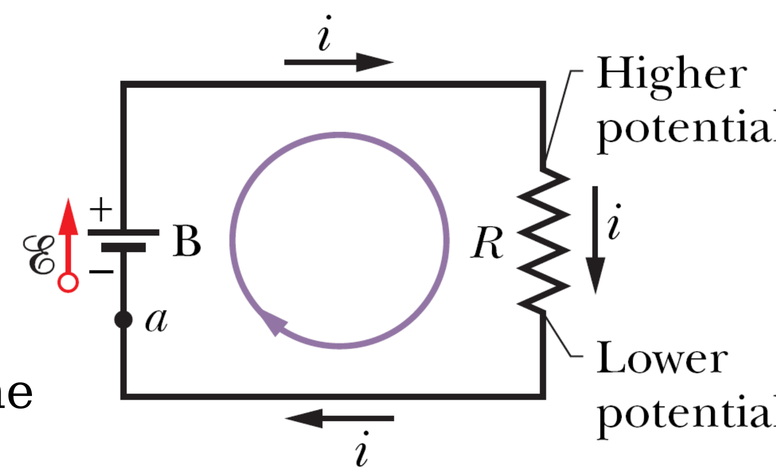
- A **real emf device** has internal resistance to the internal movement of charge. the potential difference between its terminals differs from its emf.



(a) (b)

Calculating the Current in a Single-Loop Circuit

- 2 ways to calculate the current in the simple *single-loop* circuit potential:
  a. energy conservation considerations,
  b. the concept of potential.



Energy Method

- The work done on a charge by a battery is

$$dW = \mathscr{E}\,dq = \mathscr{E}\,i\,dt$$

- From the principle of conservation of energy, the work done by the battery must equal the thermal energy that appears in the resistor: $\mathscr{E}\,i\,dt = i^2 R\,dt \Rightarrow \mathscr{E} = iR$

- The emf $\mathscr{E}$ is the energy per unit charge transferred to the moving charges by the battery. The quantity iR is the energy per unit charge transferred from the moving charges to thermal energy within the resistor.

- Solving for i, we find $i = \dfrac{\mathscr{E}}{R}$

Potential Method

Kirchhoff's loop rule (or *Kirchhoff's voltage law*):

LOOP RULE: The algebraic sum of the changes in potential encountered in a complete traversal of any loop of a circuit must be 0.

- Let us start at point a, whose potential is V_a, traverse a complete loop clockwise, and back at point a, the potential is again V_a, then

$$V_a + \mathscr{E} - i\,R = V_a \;\Rightarrow\; \mathscr{E} - i\,R = 0 \;\Rightarrow\; i = \frac{\mathscr{E}}{R}$$

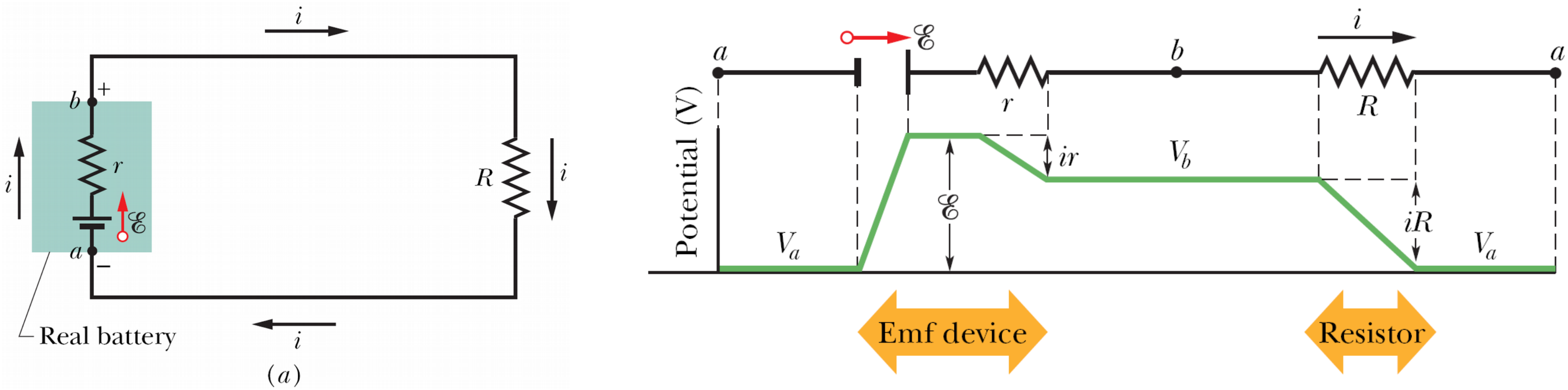
- If we apply the loop rule to a complete *counterclockwise* walk around the circuit, the rule gives us $-\mathscr{E} + iR = 0 \;\Rightarrow\; i = \dfrac{\mathscr{E}}{R}$ the same

- 2 rules for finding potential differences as we move around a loop:

RESISTANCE RULE: For a move through a resistance in the direction of the current, the change in potential is $-iR$; in the opposite direction it is $+iR$.

EMF RULE: For a move through an ideal emf device in the direction of the emf arrow, the change in potential is $+\mathscr{E}$; in the opposite direction it is $-\mathscr{E}$.

Other Single-Loop Circuits



(a) (b)

Internal Resistance

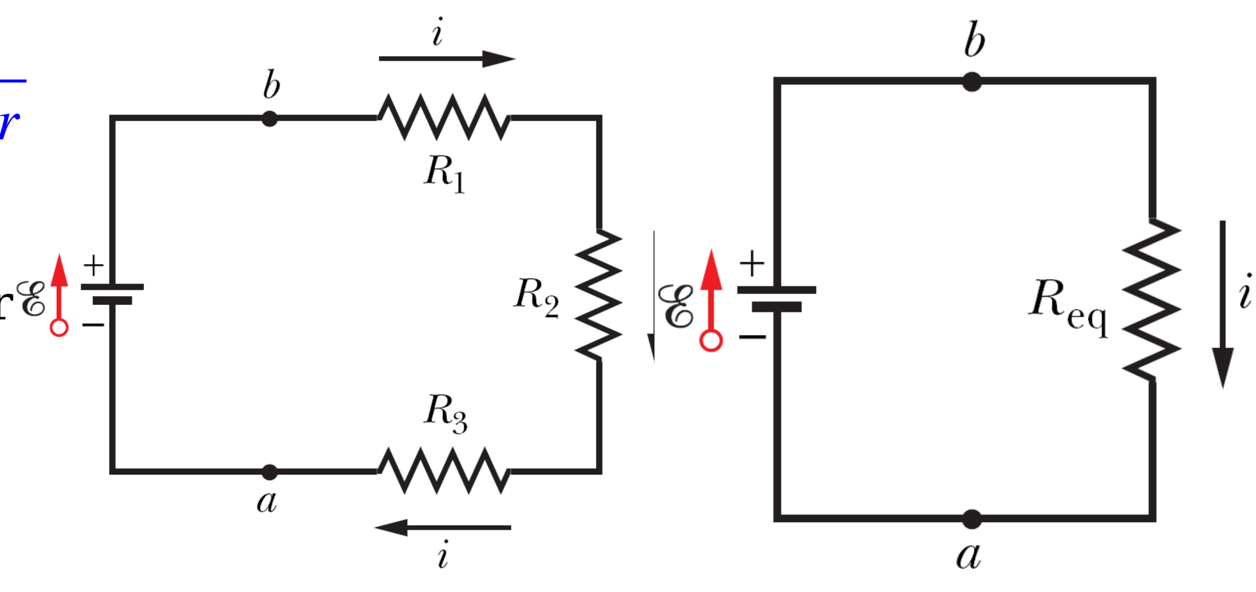
- The internal resistance of the battery is the electrical resistance of the conducting materials of the battery and thus is an unremovable feature of the battery.

- Apply the loop rule clockwise beginning at point a,

$$\mathscr{E} - i\,r - i\,R = 0 \quad \Rightarrow \quad i = \frac{\mathscr{E}}{R + r}$$

Resistances in Series

- "in series" means that the resistances are wired one after another and that a potential difference V is applied across the 2 ends of the series.



(a) (b)

When a potential difference V is applied across resistances connected in series, the resistances have identical currents i. The sum of the potential differences across the resistances is equal to the applied potential difference V.

- Note that charge moving through the series resistances can move along only a single route.

Resistances connected in series can be replaced with an equivalent resistance R_{eq} that has the same current i and the same total potential difference V as the actual resistances.

- Starting at a and going clockwise around the circuit, we find

$$\mathscr{E} - iR_1 - iR_2 - iR_3 = 0 \quad \Rightarrow \quad i = \frac{\mathscr{E}}{R_1 + R_2 + R_3}$$

- With the 3 resistances replaced with a single equivalent resistance R_{eq},

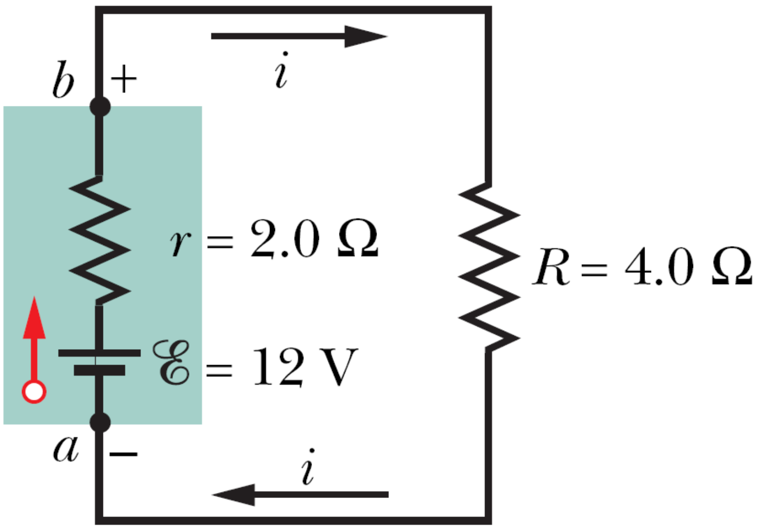
$$\mathscr{E} - iR_{eq} = 0 \quad \Rightarrow \quad i = \frac{\mathscr{E}}{R_{eq}} \quad \Rightarrow \quad R_{eq} = R_1 + R_2 + R_3$$

- The extension to n resistances is

$$R_{eq} = \sum_{j=1}^{n} R_j \quad n \text{ resistances in series}$$

- When resistances are in series, their equivalent resistance is greater than any of the individual resistances.

Potential Difference Between Two Points



- To find out the potential difference, let's start at one point and move through another point while keeping track of the potential changes.

- For the potential difference from a to b,

$$V_a + \mathscr{E} - i\,r = V_b \quad \text{or} \quad V_b - V_a = \mathscr{E} - i\,r$$

- We need i to evaluate the expression: $i = \dfrac{\mathscr{E}}{R + r}$

$$\Rightarrow \quad V_b - V_a = \mathscr{E} - \frac{\mathscr{E}}{R+r}\,r = \frac{\mathscr{E}}{R+r}\,R$$

- Substitute the data in the figure: $V_b - V_a = \dfrac{12\text{ V}}{4.0\ \Omega + 2.0\ \Omega}\,4.0\ \Omega = 8.0\text{ V}$

- If we move from a to b counterclockwise, then

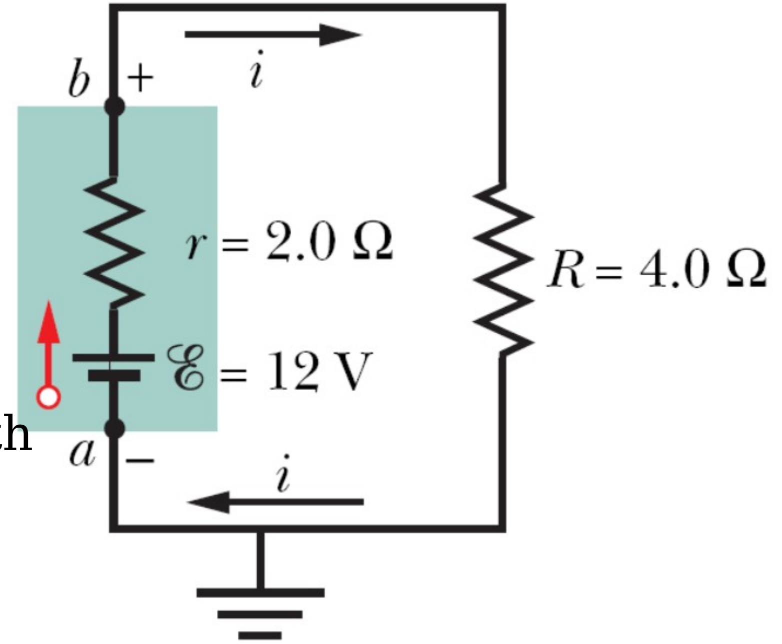
$$V_a + i\,R = V_b \quad \text{or} \quad V_b - V_a = i\,R \;\Rightarrow\; V_b - V_a = \frac{\mathscr{E}}{R+r}\,R \quad (\text{again})$$

To find the potential between any 2 points in a circuit, start at one point and traverse the circuit to the other point, following any path, and add algebraically the change in potential you encounter.

Potential Difference Across a Real Battery

- The potential difference V across a battery is $V = \mathscr{E} - i\,r$

- If the internal resistance r were 0, $V = \mathscr{E}$
- For the upper circuit, $V = 8.0$ V.
- The result depends on the value of the current.
- If the same battery were in a different circuit with different current, V would have some other value.

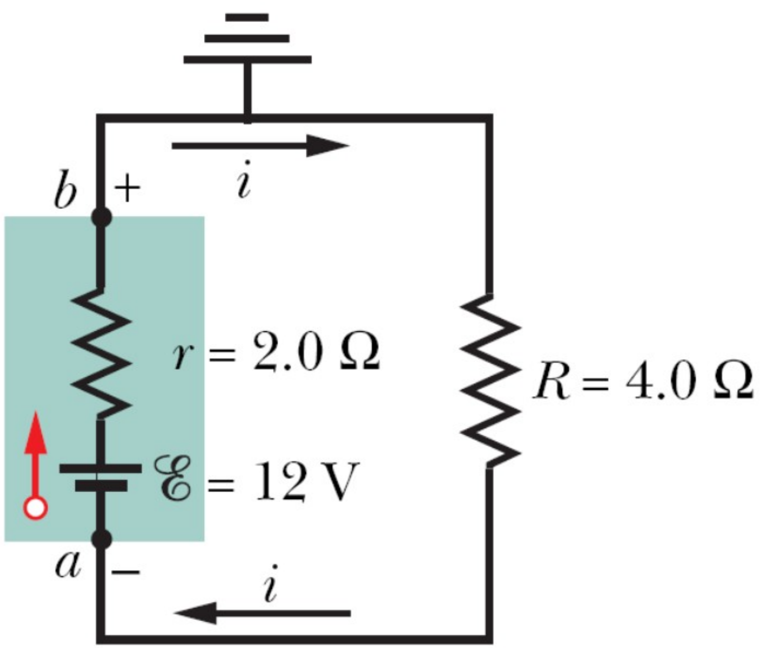


(a)

Grounding a Circuit

- *Grounding a circuit* usually means connecting the circuit to a conducting path to Earth's surface, as indicated by the symbol ⏚ .
- It means tat the potential is defined to be 0 at the grounding point in the circuit.
- In the upper figure, $V_a = 0,\ V_b = \ \ 8.0$V.

  In the lower figure, $V_b = 0,\ V_a = -8.0$V.



(b)

Power, Potential, and Emf

- When a battery does work on the charge carriers to establish a current, the battery transfers energy from its energy to the charge carriers.

- If the battery has an internal resistance r, it also transfers energy to internal thermal energy via resistive dissipation.

- The net rate P of energy transfer from battery to the charge carriers is

$$P = i\,V = i\,(\mathscr{E} - i\,r) = i\,\mathscr{E} - i^2\,r \qquad (a)$$

where $i^2 r$ is the rate P_r of energy transfer to thermal energy within the battery

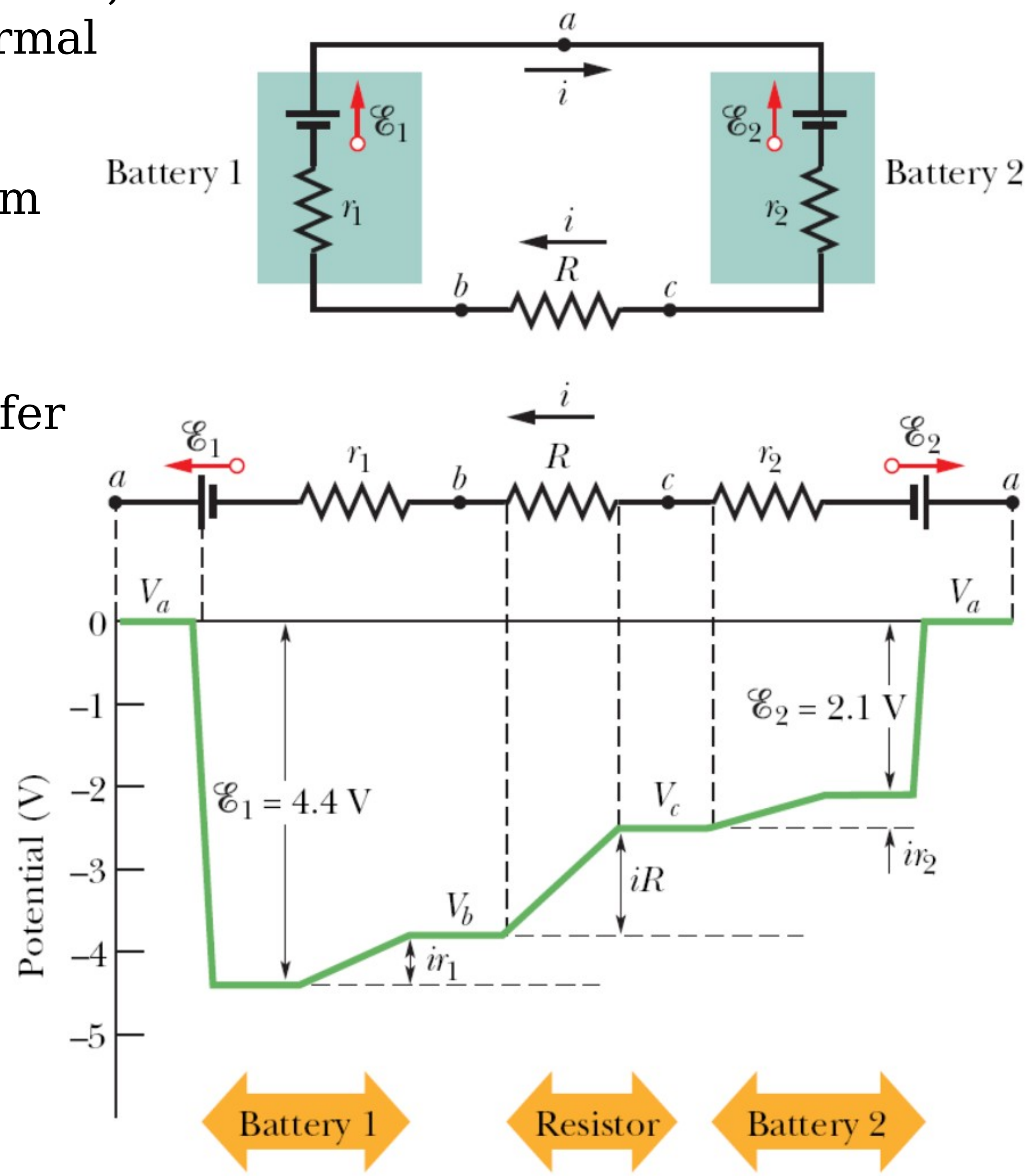
$$P_r = i^2\,r \quad \text{internal dissipation rate}$$

and $i\mathscr{E}$ is the rate P_{emf} at which the battery transfer its energy

$$P_{\text{emf}} = i\,\mathscr{E} \quad \text{power of emf device}$$

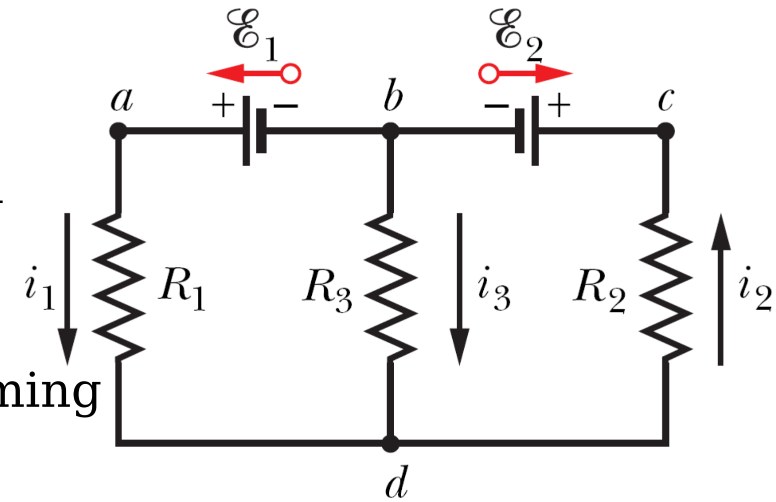
- If a battery is being *recharged*, with a "wrong way" current through it the energy transfer is *from* the charge carriers *to* the battery—both to the battery's chemical energy and the the energy dissipated in the internal resistance r.

(b)



problem 27-1

Multiloop Circuits



- There are 2 *junctions* in this circuit, at *b* and *d*, and there are 3 *branches* , *bad*, *bcd*, *bd*, connecting these junctions.

- Consider junction *d* for a moment: the total incoming current must equal the total outgoing current:

$$i_1 + i_3 = i_2$$

- *Kirchhoff's junction rule* (or *Kirchhoff's current law*):

JUNCTION RULE: The sum of the currents entering any junction must be equal to the sum of the currents leaving that junction.

- Kirchhoff's junction rule is simply a statement of the conservation of charge for a steady flow of charge.

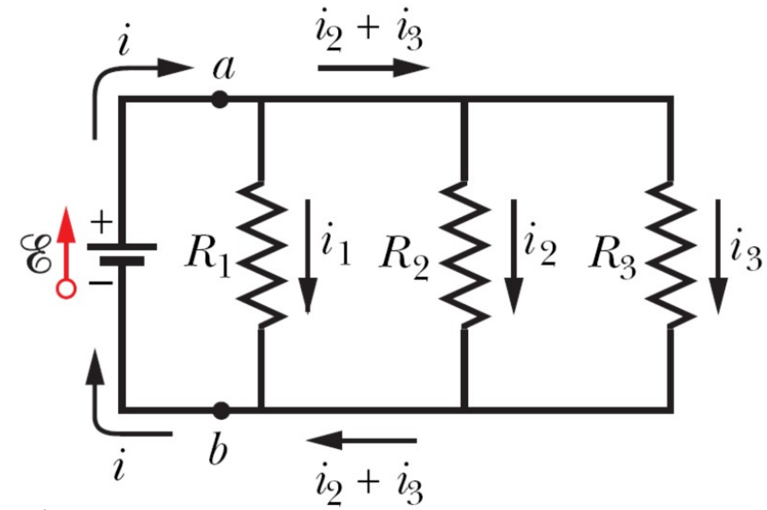
- The basic tools for solving complex circuits are the *loop rule* (based on the conservation of energy) and the *junction rule* (based on the conservation of charge).

- 3 unknowns, i_1, i_2, i_3, we need 2 more equations to solve them.

- Apply the loop rule to the left-hand loop *badb* and the right hand loop *bcdb*.

$$\Rightarrow \quad \mathscr{E}_1 - i_1 R_1 + i_3 R_3 = 0 \quad \text{for} \quad badb$$
$$-i_3 R_3 - i_2 R_2 - \mathscr{E}_2 = 0 \quad \text{for} \quad bcdb$$

now we have 3 equations for 3 unknowns.



(*a*)

- If we apply the loop rule to the big loop *badcb*,

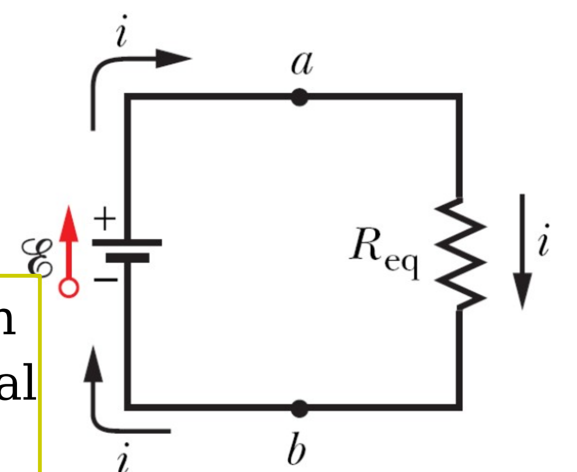
$$\Rightarrow \quad \mathscr{E}_1 - i_1 R_1 - i_2 R_2 - \mathscr{E}_2 = 0$$

but this equation is only the sum or the above equations.

Resistances in Parallel

When a potential difference V is applied across resistances connected *in parallel*, the resistances all have that same potential difference V.

Resistances connected in parallel can be replaced with an equivalent resistance R_{eq} that has the same potential difference V and the same *total* current i as the actual resistances.



(*b*)

- To get R_{eq}: $i_1 = \frac{V}{R_1}, \quad i_2 = \frac{V}{R_2}, \quad i_3 = \frac{V}{R_3} \quad \Rightarrow \quad i = i_1 + i_2 + i_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V}{R_{eq}}$

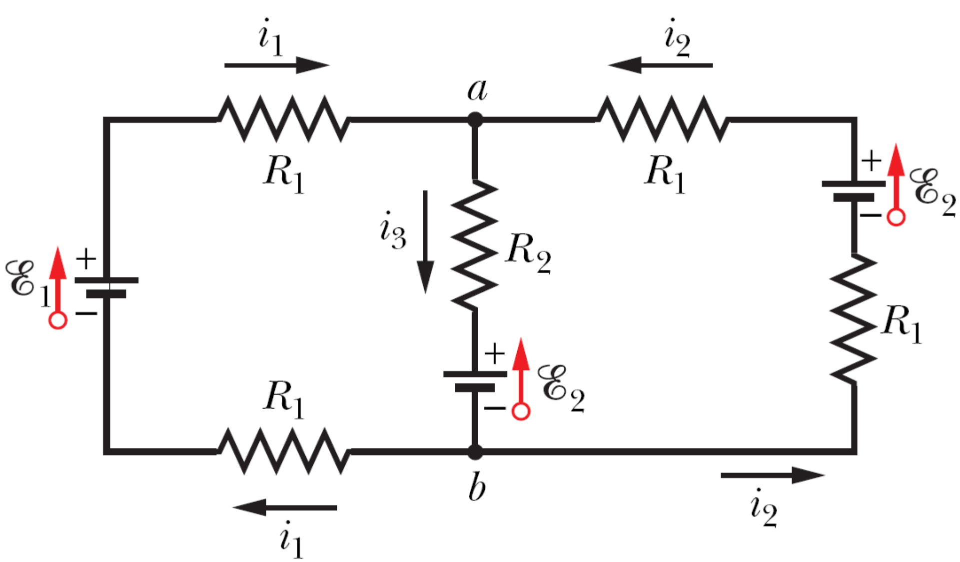
$$\Rightarrow \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad \Rightarrow \quad \text{extention} \quad \frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j} \quad n \ \text{resistances in parallel}$$

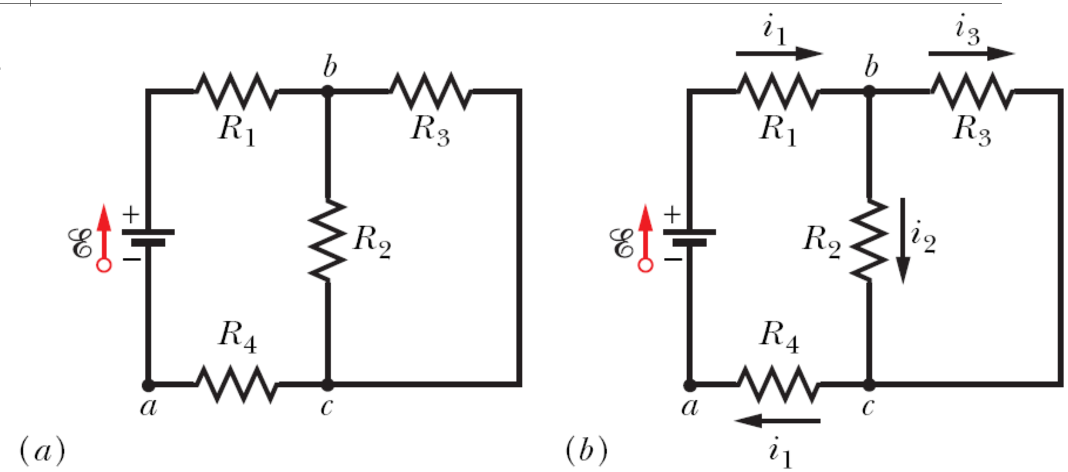
Series and Parallel Resistors and Capacitors

Series	Paprallel	Series	Parallel
Resistors		Capacitors	
$R_{eq} = \sum_{j=1}^{n} R_j$	$\frac{1}{R_{eq}} = \sum_{j=1}^{n} \frac{1}{R_j}$	$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$	$C_{eq} = \sum_{j=1}^{n} C_j$
Same current through all resistors	same potential difference across all resistors	Same charge on all capacitors	same potential difference across all capacitors

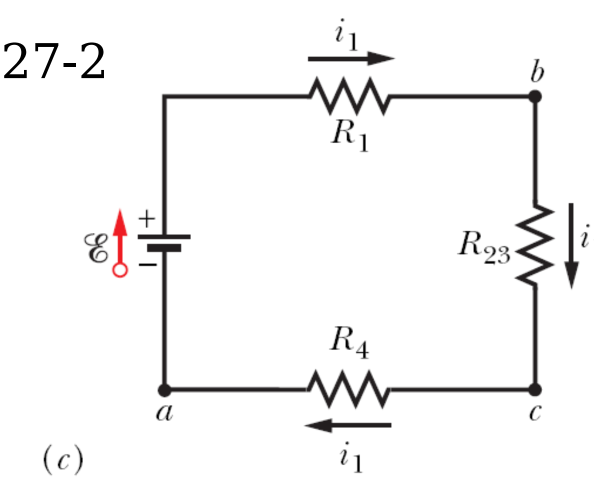
Note: when 2 or more resistances are connected in parallel, the equivalent resistance is smaller than any of the combining resistances.

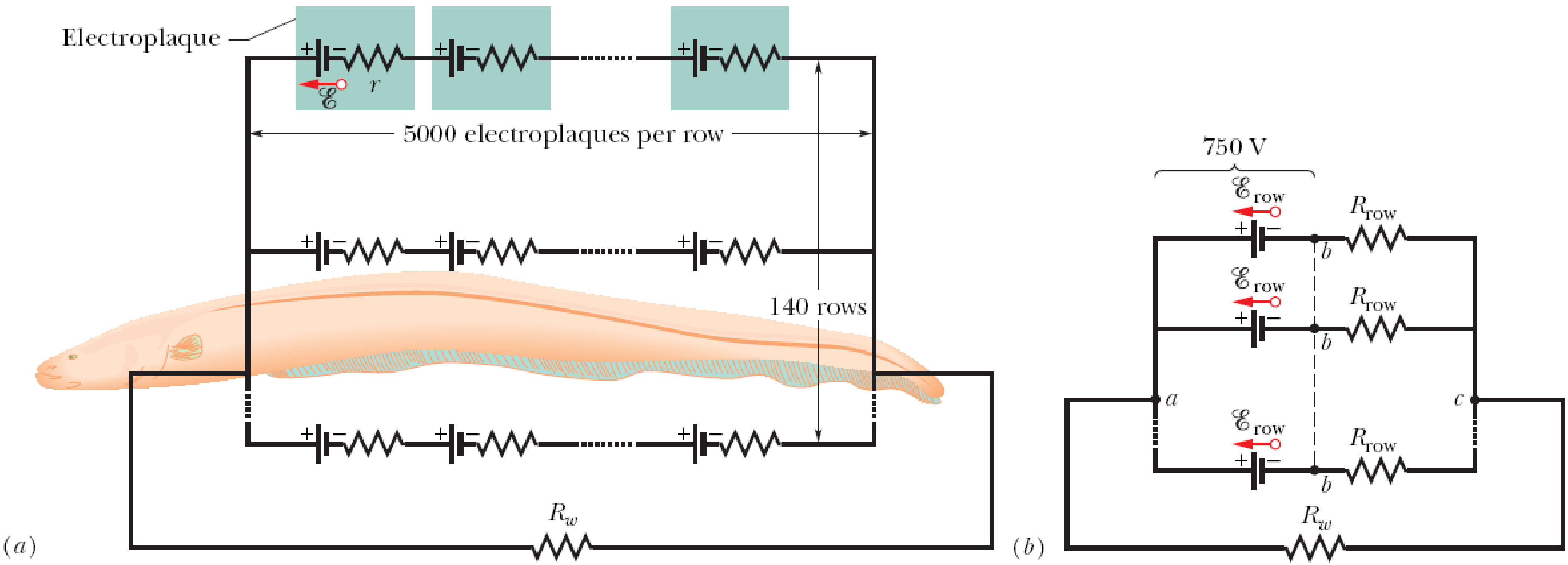
Problem 27-4





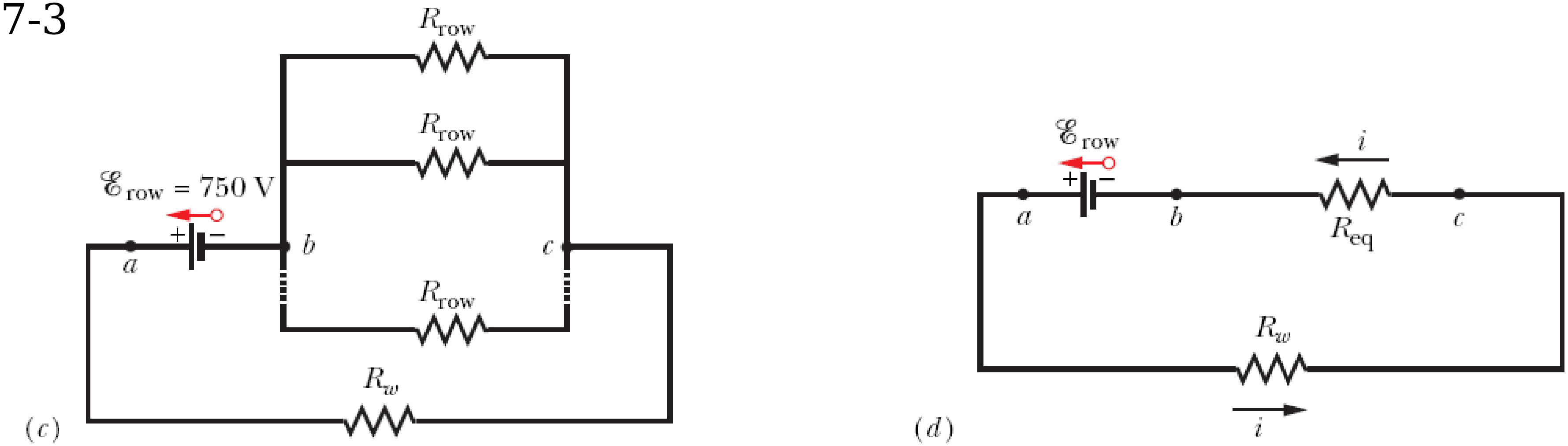
Problem 27-2





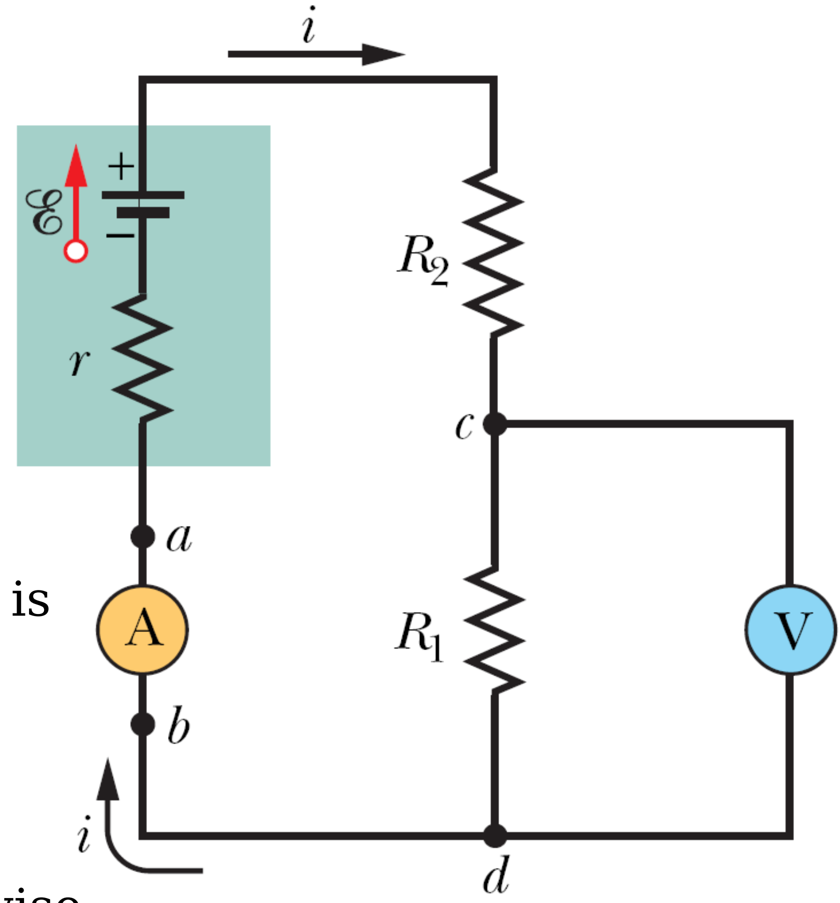
Electroplaque
5000 electroplaques per row
140 rows
R_w
(a)
750 V
$\mathscr{E}_{row}$
R_{row}
a
b
c
(b)

Problem 27-3



$\mathscr{E}_{row}$ = 750 V
R_{row}
R_w
a
b
c
(c)
R_{eq}
i
(d)

The Ammeter and the Voltmeter



- An instrument used to measure currents is called an *ammeter*.

- The resistance R_A of the ammeter should be very much smaller than other resistances in the circuit. Otherwise, the very presence of the meter will change the current to be measured.

- A meter used to measure potential differences is called a *voltmeter*.

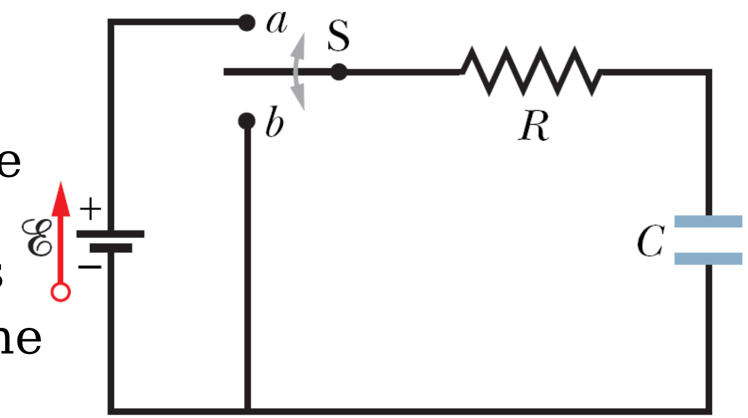
- The resistance R_V of a voltmeter be very much larger than the resistance of any circuit element across which the voltmeter is connected. Otherwise, the meter itself becomes an important circuit element and alters the potential difference that is to be measured.

- An *ohmmeter* is designed to measure the resistance of any element connected between its terminals.

- A versatile unit with an ammeter, a voltmeter, and an ohmmeter is called *multimeter*.

RC Circuits

Charging a Capacitor



- When the circuit for charging is complete, charge begins to flow between a capacitor plate and a battery terminal on each side of the capacitor. This current increases the charge q on the plates and the potential difference $V_C(=q/C)$ across the capacitor.

When the potential difference equals the potential difference across the battery, the current is 0.

- The *equilibrium* (final) *charge* on the fully charged capacitor is equal to $C\mathscr{E}$.

- Apply the loop rule to the circuit clockwise, and we find $\mathscr{E} - iR - V_C = \mathscr{E} - iR - \dfrac{q}{C} = 0$

- Since $i \equiv \dfrac{dq}{dt} \Rightarrow R\dfrac{dq}{dt} + \dfrac{q}{C} = \mathscr{E}$ charging equation

- Solve this equation: $R\dfrac{dq}{dt} + \dfrac{q}{C} = \mathscr{E} \Rightarrow \dfrac{dq}{dt} = \dfrac{\mathscr{E}}{R} - \dfrac{q}{RC} = \dfrac{1}{RC}(C\mathscr{E} - q)$

$$\frac{dq}{C\mathscr{E} - q} = \frac{dt}{RC} \Rightarrow \frac{d(C\mathscr{E} - q)}{C\mathscr{E} - q} = -\frac{dt}{RC} \Rightarrow \ln\left|C\mathscr{E} - q\right|_{q_0}^{q} = -\frac{1}{RC}\, t\Big|_0^t$$

assume $q = 0$ at $t = 0$: $\ln\dfrac{|C\mathscr{E} - q|}{C\mathscr{E}} = -\dfrac{t}{RC} \Rightarrow C\mathscr{E} - q = C\mathscr{E}\, e^{-\frac{t}{RC}}$

Thus $q = C\mathscr{E}\left(1 - e^{-\frac{t}{RC}}\right)$ charging a capacitor

- At $t = 0$ the term $e^{-t/RC}$ is unity; so $q = 0$. As t goes to ∞, the term $e^{-t/RC}$ goes to 0; so $q = C\mathscr{E}$, the proper value for the full (equilibrium) charge on the capacitor.

- The derivative of $q(t)$ is the current $i(t)$ charging the capacitor:

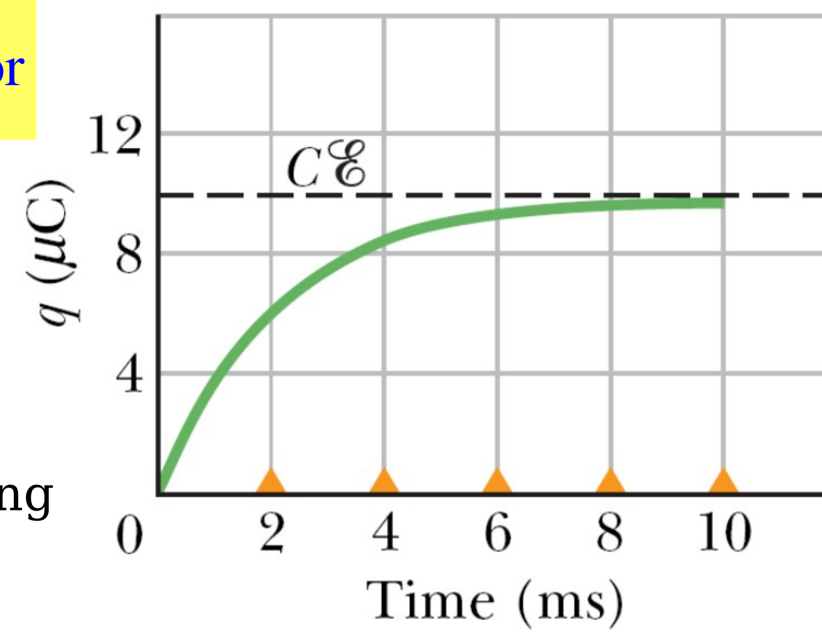
$$i = \frac{dq}{dt} = \frac{\mathscr{E}}{R} e^{-\frac{t}{RC}} \quad \text{charging a capacitor}$$

- The current has the initial value $\mathscr{E}/R$ and it decreases to 0 as the capacitor becomes fully charged.

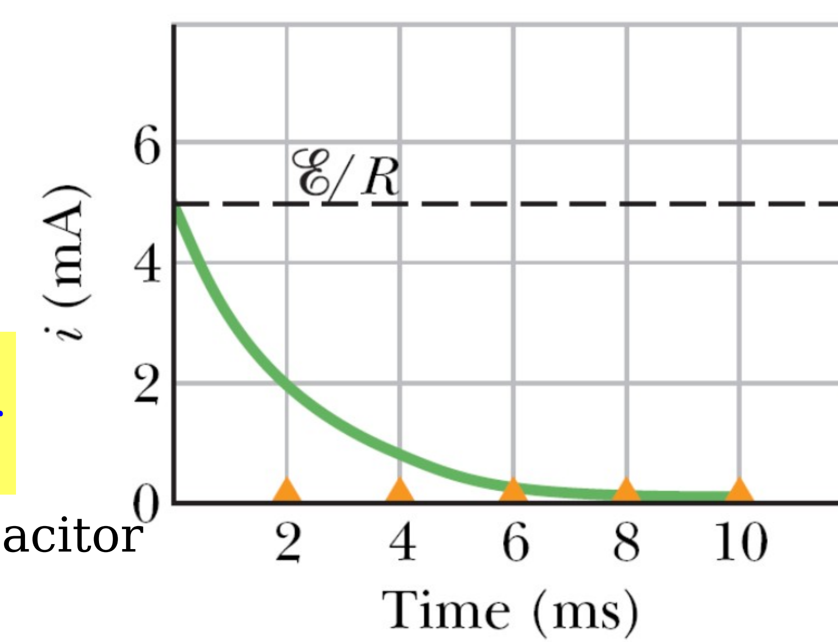
- The potential difference $V_C(t)$ across the capacitor during the charging process is

$$V_C = \frac{q}{C} = \mathscr{E}\left(1 - e^{-\frac{t}{RC}}\right) \quad \text{charging a capacitor}$$

- $V_C = 0$ at $t = 0$ and that $V_C = \mathscr{E}$ when the capacitor becomes fully charged as $t \to \infty$.



(a)



(b)

A capacitor that is being charged initially acts like ordinary connecting wire relative to the charging current. A long time later, it acts like a broken wire.

The Time Constant

- The product RC has the dimensions of time and is called the **capacitive time constant** of the circuit and is represented with the symbol τ:

$$\tau = RC \quad \text{time constant}$$

- At time $t = \tau$ ($= RC$), the charge on an initially uncharged capacitor has increased from 0 to

$$q = C\mathscr{E}\left(1 - e^{-1}\right) = 0.63\, C\mathscr{E}$$

during the first time constant τ the charge has increased from 0 to 63% of its final value $C\mathscr{E}$.

Discharging a Capacitor

- The differential equation for discharging is $$R\frac{dq}{dt} + \frac{q}{C} = 0 \quad \text{discharging equation}$$

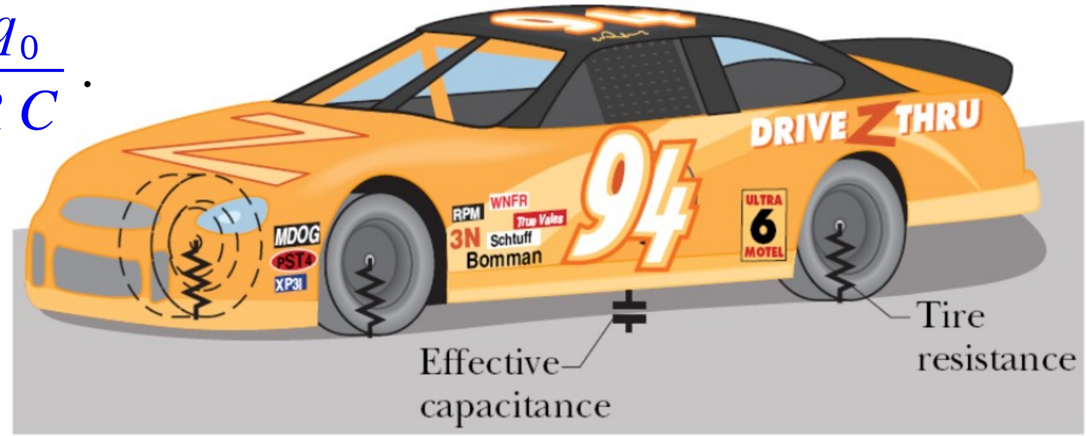
- The solution to this differential equation is $$q = q_0\, e^{-\frac{t}{RC}} \quad \text{discharging a capacitor}$$

where $q_0 (= CV_0)$ is the initial charge on the capacitor.

- q decreases exponentially with time, at a rate that is set by the capacitive time constant $\tau = RC$. At time $t = \tau$, the capacitor's charge has been reduced to $q_0 e^{-1}$, or about 37% of the initial value.

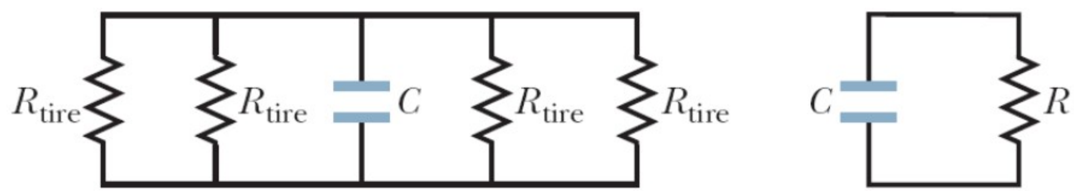
- The current $i(t)$: $i = \frac{dq}{dt} = -\frac{q_0}{RC} e^{-\frac{t}{RC}}$ discharging a capacitor

- The initial current $i_0 = \frac{q_0}{RC}$.



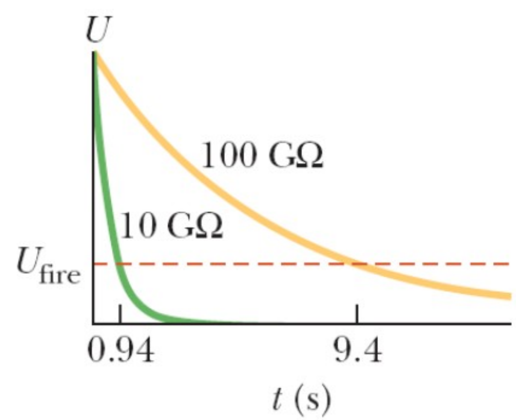
problem 27-5

(a)



(b) (c)

Selected problems: 8, 20, 44, 60, 66



(d)