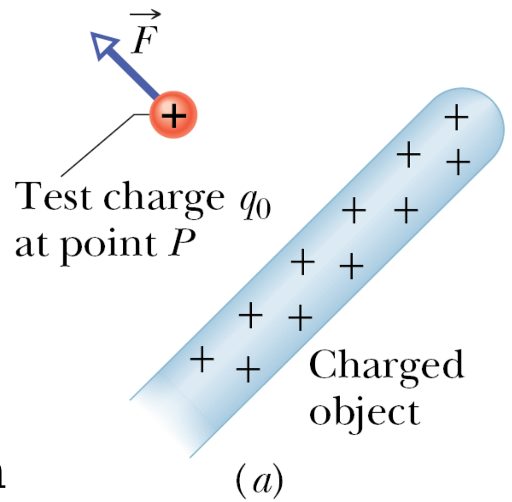
Chapter 22 Electric Fields

- A particle A sets up an **electric field** in the space surrounding itself. For another particle B at any given point in that space, particle B knows of the presence of particle A because it is affected by the electric field that particle A has already set up at that point.

The Electric Field

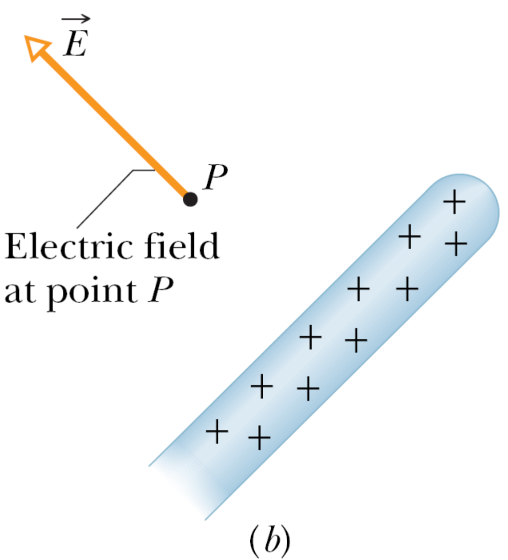


- *scalar fields*:

  temperature field: the resulting distribution of temperatures, one for each point in the space.

  pressure field: the distribution of air pressure values, one for each point in the atmosphere.

- The electric field is a *vector field*; it consists of a distribution of *vectors*, one for each point in the region around a charged object.



- define the electric field at some point near the charged object as follows: place a positive charge q_0, called a *test charge*, at the point. Then measure the electrostatic force acting on the test charge. Finally define the electric field at that point as

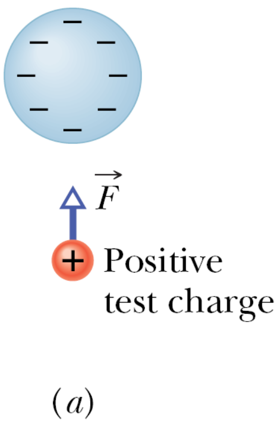
$$\boxed{\vec{E} = \frac{\vec{F}}{q_0} \qquad \text{electric field}}$$

- The SI unit for the electric field is the newton per coulomb (N/C).

- An electric field exists independently of the test charge (assuming the presence of the test charge does not affect the charge distribution on the charged object, and thus does not alter the electric field we are defining).

- 2 tasks:
(1) calculating the electric field produced by a given distribution of charge;
(2) calculating the force that a given field exerts on a charge placed in it.

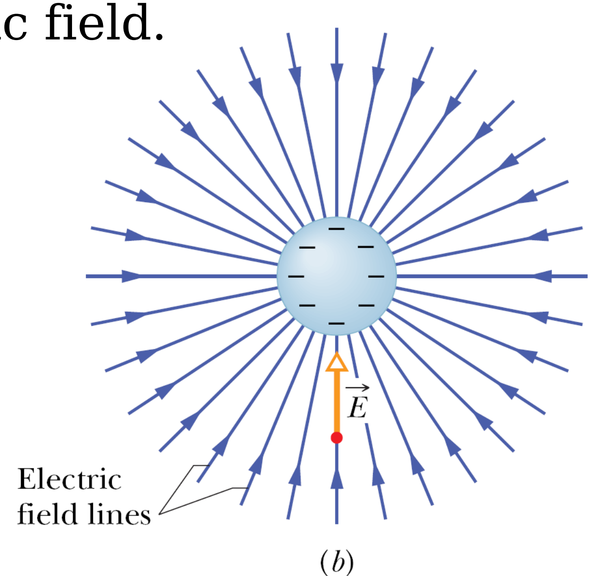
Electric Field Lines



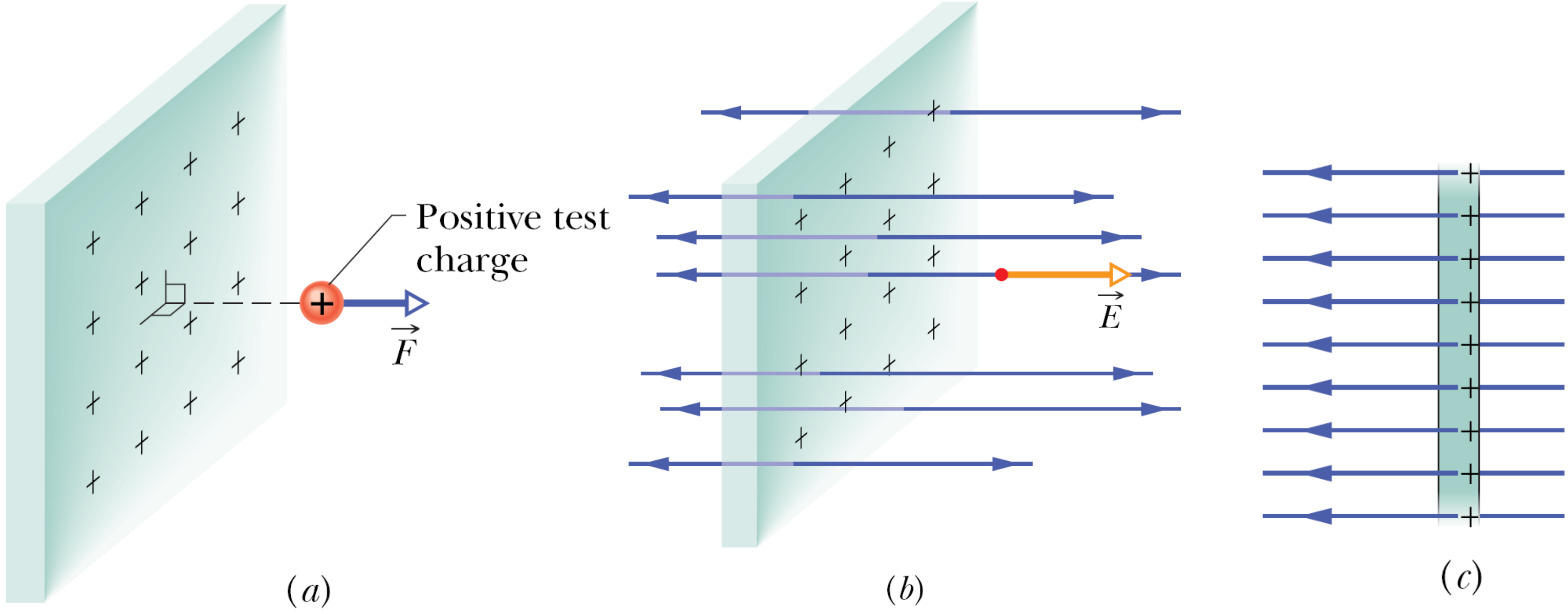
- **electric field lines:** to visualize patterns in electric fields.

- The relation between the field lines and electric field vectors:
(1) At any point, the direction of the tangent to a curved field line gives the direction the electric field of at that point;
(2) the field lines are drawn so that the number of lines per unit area is proportional to the magnitude of the electric field.

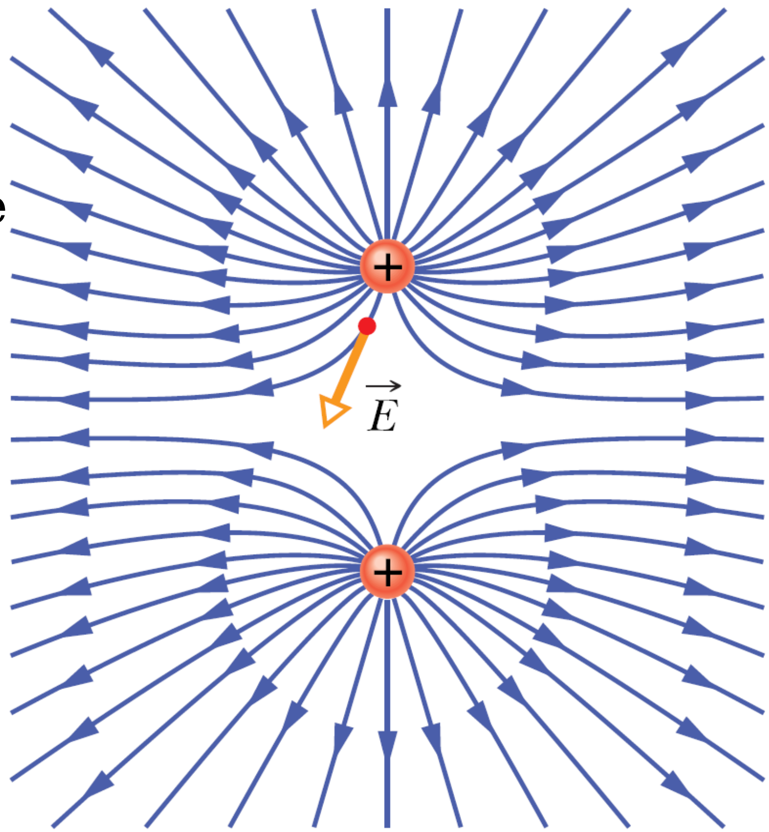
Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).

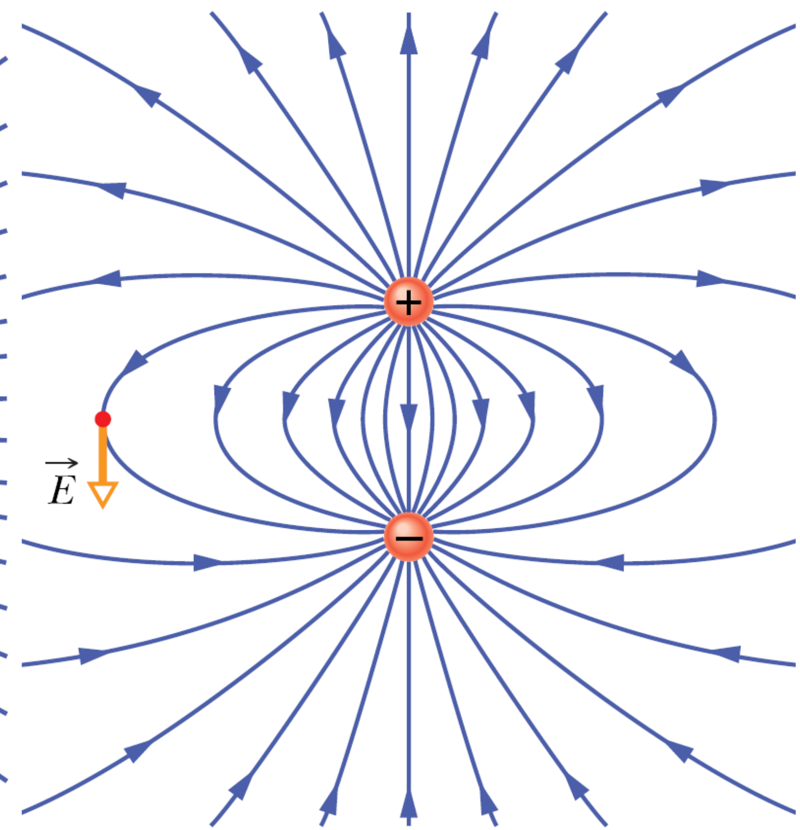


- an electric field, with the same magnitude and direction at every point, is a *uniform electric field*.

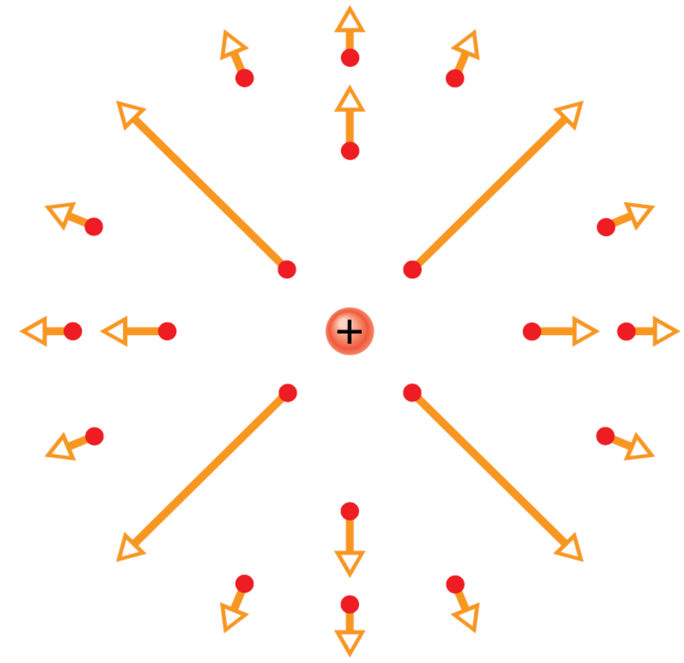


electric dipole: the pattern for 2 charges that are equal in magnitude but of opposite sign.





The Electric Field Due to a Point Charge



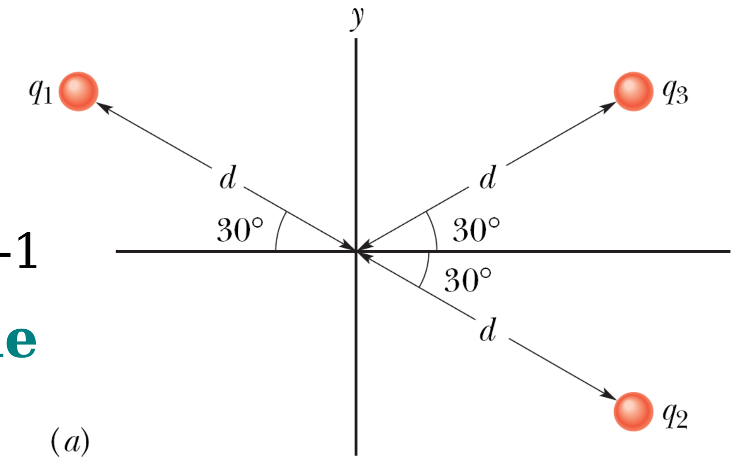
- For $\vec{F} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q\,q_0}{r^2}\,\hat{r}$

the electric field $\boxed{\vec{E} = \dfrac{\vec{F}}{q_0} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r^2}\,\hat{r} \quad \text{point charge}}$

- the net force from the n point charges acting on the test charge is $\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}$

therefore the net electric field at the position of the test charge

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n$$

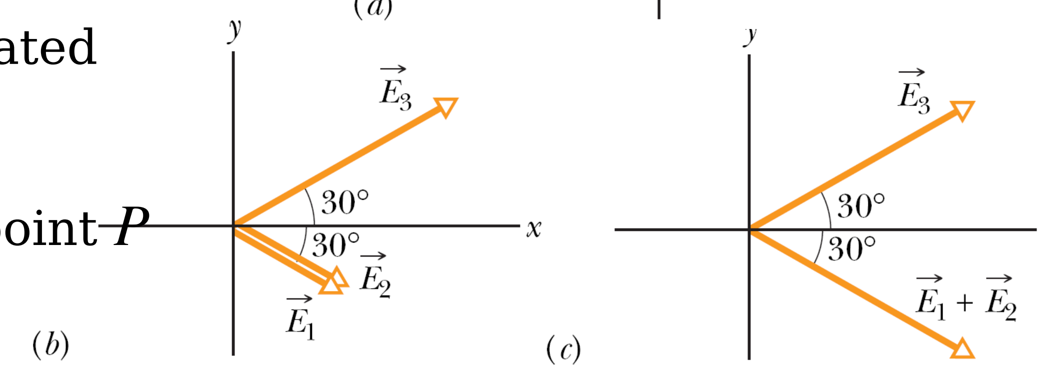


problem 22-1

The Electric Field Due to an Electric Dipole

- **electric dipole**: 2 charged particles of magnitude q but of opposite sign separated by a distance.

- From symmetry, the electric field at point P must lie along the dipole axis.



Applying the superposition principle for electric fields, the magnitude E of the electric field at P is

$$E = E_{+} + E_{-} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{+}^2} + \frac{1}{4\pi\varepsilon_0}\frac{-q}{r_{-}^2}$$

$$= \frac{q}{4\pi\varepsilon_0 (z - d/2)^2} - \frac{q}{4\pi\varepsilon_0 (z + d/2)^2}$$

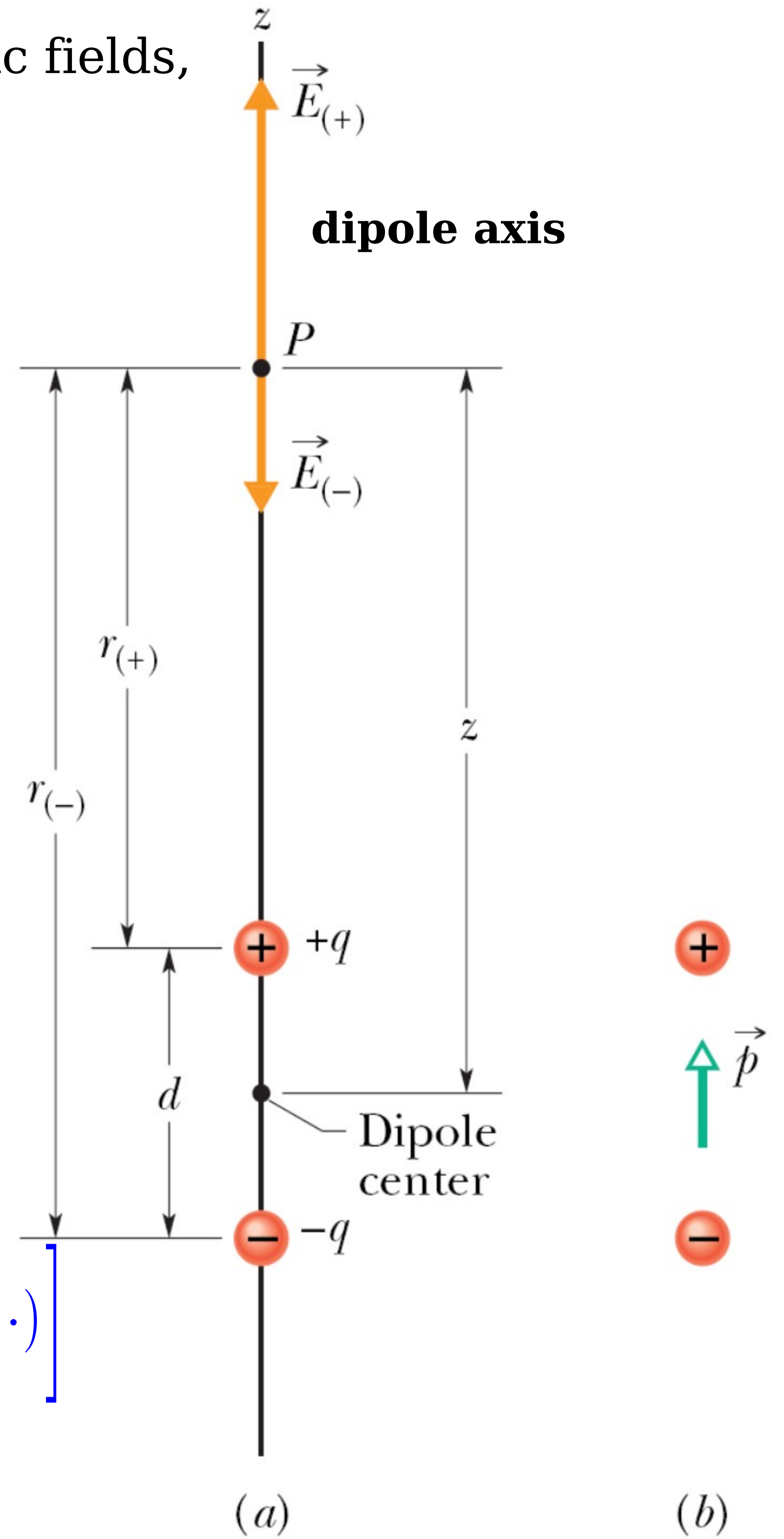
$$= \frac{q}{4\pi\varepsilon_0 z^2}\left[\left(1 - \frac{d}{2z}\right)^{-2} - \left(1 + \frac{d}{2z}\right)^{-2}\right]$$

Since $\left[\left(1 - \frac{d}{2z}\right)^{-2} - \left(1 + \frac{d}{2z}\right)^{-2}\right]$

$$= \left[\left(1 - \frac{d}{z} + \frac{d^2}{4z^2}\right)^{-1} - \left(1 + \frac{d}{z} + \frac{d^2}{4z^2}\right)^{-1}\right]$$

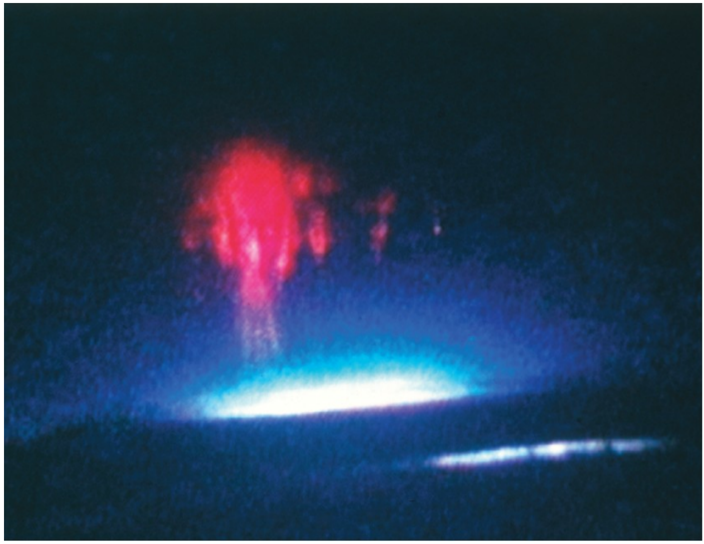
$$= \left[\left(1 + \frac{d}{z} + \cdots\right) - \left(1 - \frac{d}{z} + \cdots\right)\right]$$

$$\Rightarrow \quad E = \frac{q}{4\pi\varepsilon_0 z^2}\left[\left(1 + \frac{d}{z} + \cdots\right) - \left(1 - \frac{d}{z} + \cdots\right)\right]$$



- usually interested in the electrical effect of a dipole only at distances that are large compared with the dimensions of the dipole, ie, $z \gg d$. In the approximation,

$$E \approx \frac{q}{4\pi\varepsilon_0 z^2}\frac{2d}{z} = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}$$



- define the **electric dipole moment** of the dipole (the unit is the coulomb-meter)

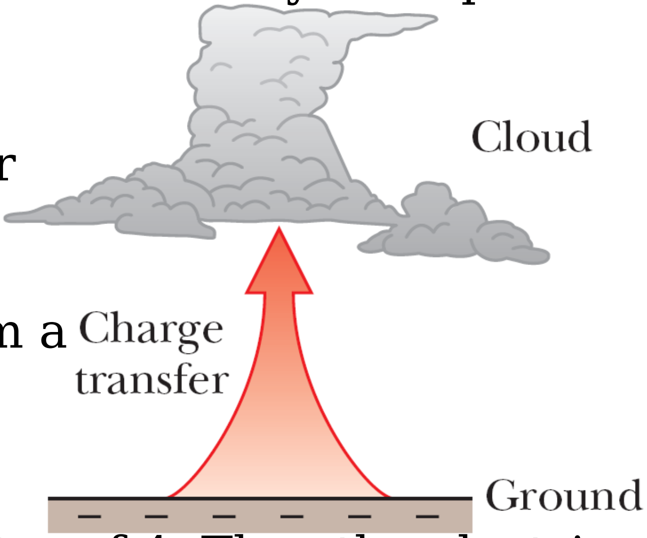
$$\vec{p} \equiv q\,\vec{d}$$

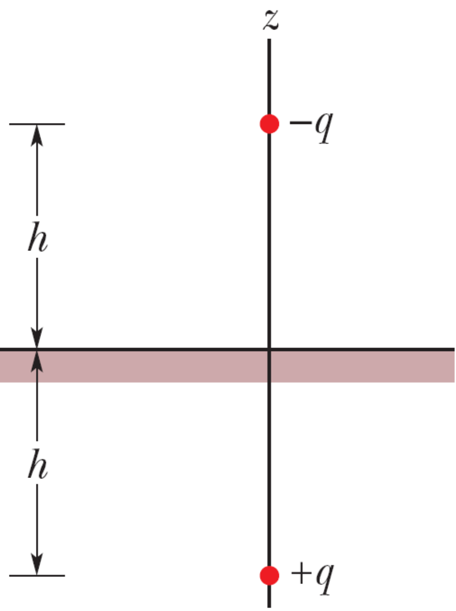
the direction is from the negative to the positive end of the dipole

$$\boxed{E \approx \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \qquad \text{electric dipole}}$$

- if we measure the electric field of a dipole only at distant points, we can never find q and d separately; instead, we can find only their product.

problem 22-2





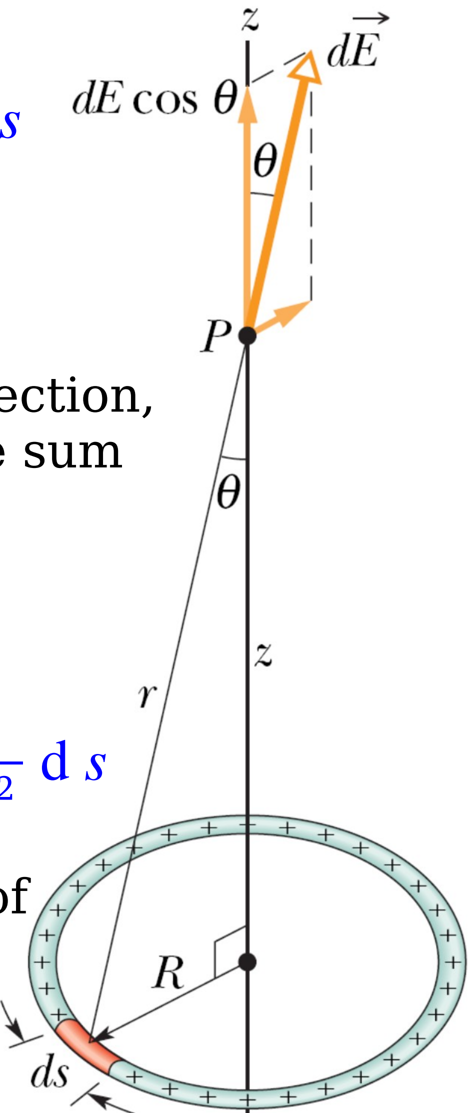
- E for a dipole varies as $1/r^3$ for all distant points, regardless of whether they lie on the dipole axis.

- Double the distance of a point from a dipole, the electric field at the point drops by a factor of 8. Double the distance from a single point charge, the electric field drops only by a factor of 4. Thus the electric field of a dipole decreases more rapidly with distance than does the electric field of a single charge.

The Electric Field Due to a Line of Charge

- a differential element has a charge of magnitude $dq = \lambda\, ds$ then the differential electric field is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2+R^2)}$$



- For any perpendicular component that points in a given direction, there is another one that points in the opposite direction. The sum of this pair of components, like the sum of all other pairs of oppositely directed components, is 0.

- The parallel component of the differential electric field

$$dE_{\parallel} = dE\cos\theta = dE\,\frac{z}{r} = dE\,\frac{z}{(z^2+R^2)^{1/2}} = \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\,ds$$

- Then the equation is integrated around the circumference of the ring to add the parallel components

$$E = E_{\parallel} = \int dE_{\parallel} = \int dE\cos\theta$$

$$= \frac{z\lambda}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}\int_0^{2\pi R} ds = \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2+R^2)^{3/2}}$$

- the total charge on the ring $q = \lambda(2\pi R)$

$$\Rightarrow \boxed{E = E_{\parallel} = \frac{qz}{4\pi\varepsilon_0(z^2+R^2)^{3/2}} \quad \text{charged ring}}$$

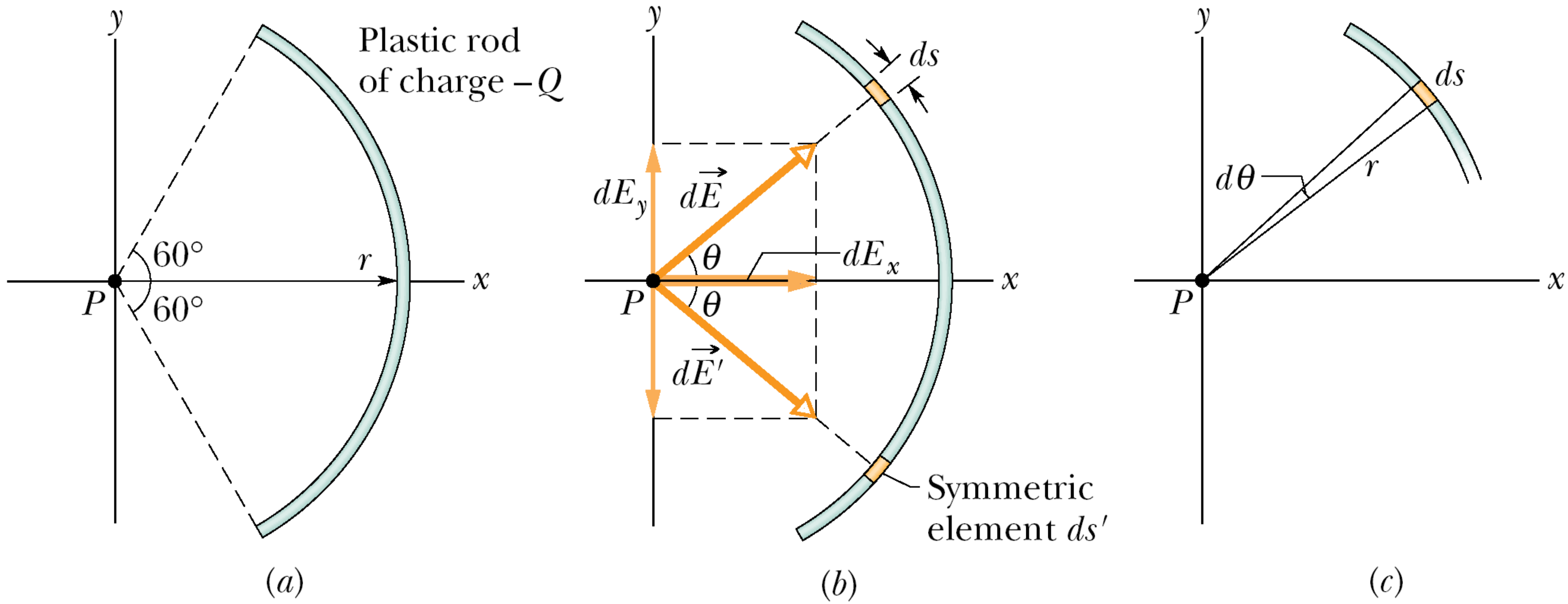
Some Measures of Electric charge

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

- If $z \gg R$, then $z^2 + R^2 \approx z^2$, then $E \approx \dfrac{q}{4\pi\varepsilon_0 z^2}$ charged ring at large distance
- This is a reasonable result because from a large distance, the ring looks like a point charge.

- If $z=0$, then $E=0$, ie, no parallel component at $z=0$, and the perpendicular force due to any element of the ring would be canceled by the force due to the element on the opposite side of the ring.

problem 22-3



The Electric Field Due to a Charged Disk

- the charge on the ring is $\mathrm{d}q = \sigma\, \mathrm{d}A = \sigma\,(2\pi r\, \mathrm{d}r)$

- the electric field at P due to the ring is

$$\mathrm{d}E = \frac{z\,\sigma\, 2\pi r\, \mathrm{d}r}{4\pi\varepsilon_0 (z^2+r^2)^{3/2}} = \frac{z\,\sigma}{4\varepsilon_0}\frac{2r\,\mathrm{d}r}{(z^2+r^2)^{3/2}}$$

$$\Rightarrow \quad E = \int \mathrm{d}E = \frac{z\,\sigma}{4\varepsilon_0}\int_0^R (z^2+r^2)^{-3/2}(2r)\,\mathrm{d}r$$

- define $X=(r^2+z^2)$, then

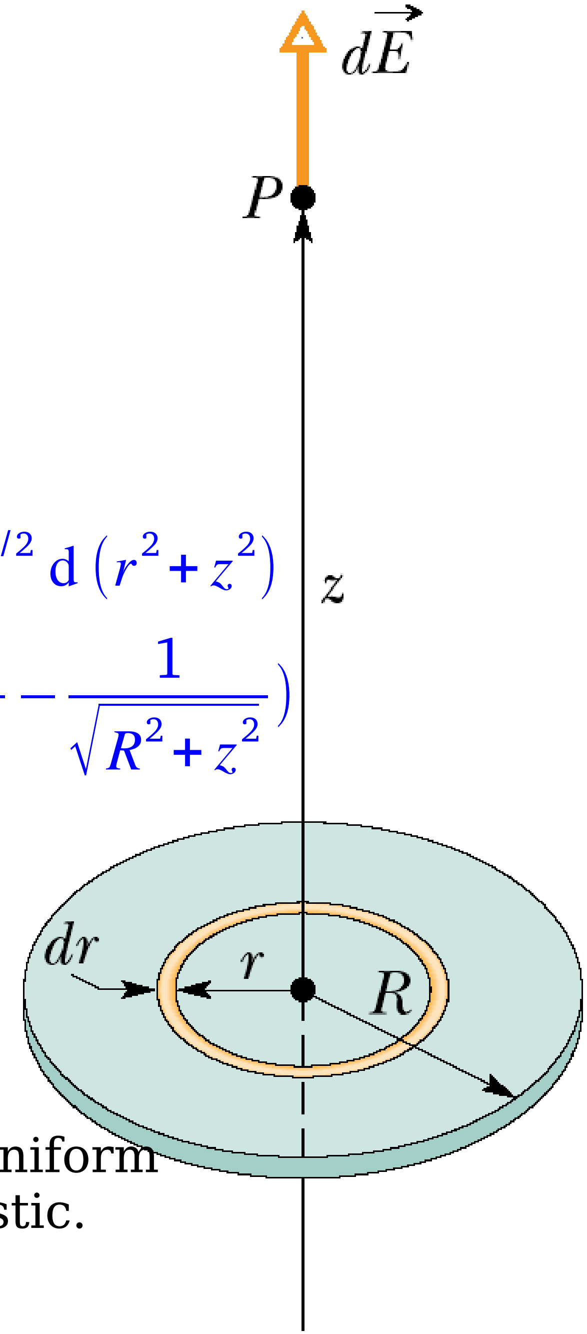
$$\int_0^R (r^2+z^2)^{-3/2}(2r)\,\mathrm{d}r = \int_0^R (r^2+z^2)^{-3/2}\,\mathrm{d}(r^2) = \int_0^R (r^2+z^2)^{-3/2}\,\mathrm{d}(r^2+z^2)$$

$$= \int_{z^2}^{R^2+z^2} X^{-3/2}\,\mathrm{d}X = -2\left[X^{-1/2}\right]_{z^2}^{R^2+z^2} = -2\left[(r^2+z^2)^{-1/2}\right]_0^R = 2\left(\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}}\right)$$

$$\Rightarrow \quad \boxed{E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right) \quad \text{charged disk}}$$

- If $R \to \infty$ and z is finite then $E \approx \dfrac{\sigma}{2\varepsilon_0}$ infinite sheet .

This is the electric field produced by an infinite sheet of uniform charge located on one side of a nonconductor such as plastic.

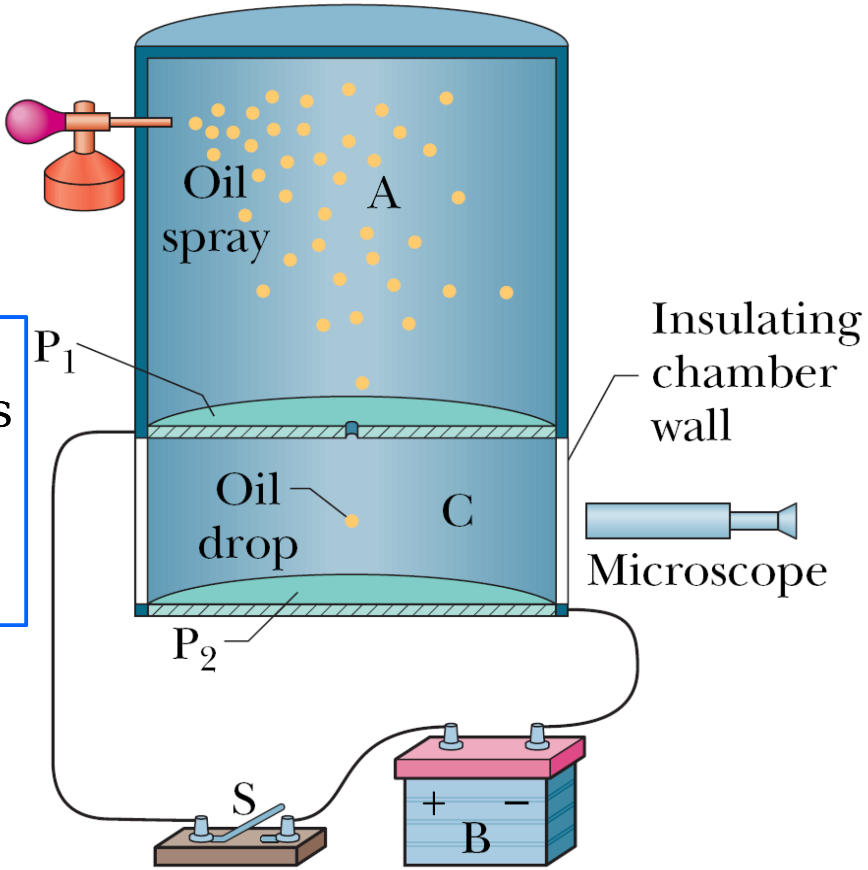


- If $z \to 0$ and R is finite, we will get the same result. This shows that at points very close to the disk, the electric field set up by the disk is the same as if the disk were infinite in extent.

A Point Charge in an Electric Field

- The electrostatic force acting on a charged particle by an external electric field is

$$\boxed{\vec{F} = q\,\vec{E}}$$



The electrostatic force acting on a charged particle located in an external electric field has the direction of the electric field if the charge of the particle is positive and has the opposite direction if the charge is negative.

Measuring the Elementary Charge

- By timing the motion of oil drops with the switch opened and with it closed and thus determining the effect of the charge q,

Millikan discovered that the values of q were always given by

$q = n\,e\,,\quad \text{for}\;\; n = \pm 0, \pm 1, \pm 2, \pm 3, \ldots,$ e is the elementary charge, 1.60×10^{-19} C.

- Millikan's experiment is a convincing proof that charge is *quantized*.

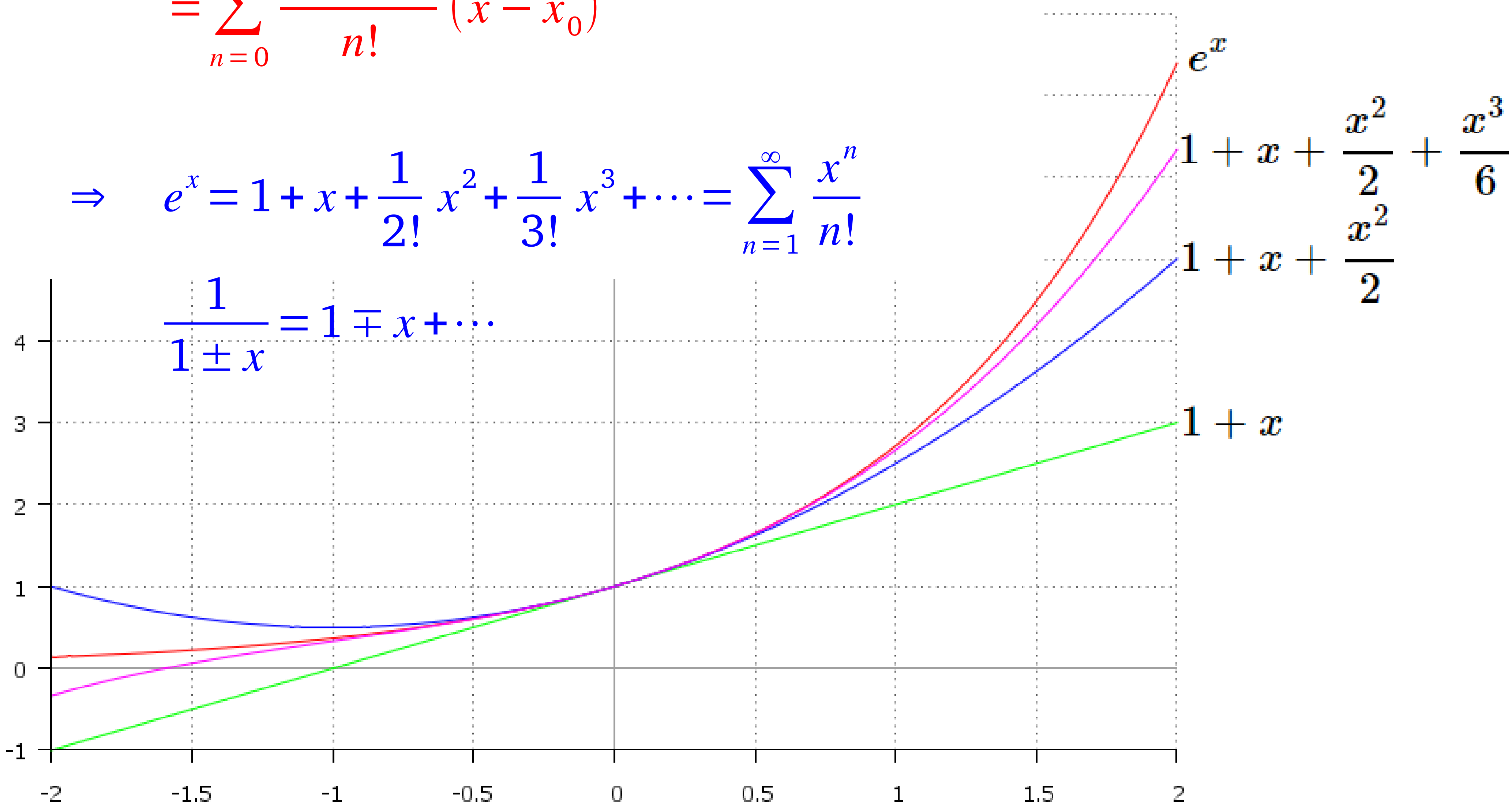
Taylor Expansion (Taylor Series)

$$f(x) = f(x_0) + f'(x_0)(x - x_0) + \frac{f''}{2!}(x_0)(x - x_0)^2 + \frac{f'''}{3!}(x_0)(x - x_0)^3 + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n$$

$$\Rightarrow \quad e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots = \sum_{n=1}^{\infty} \frac{x^n}{n!}$$

$$\frac{1}{1 \pm x} = 1 \mp x + \cdots$$

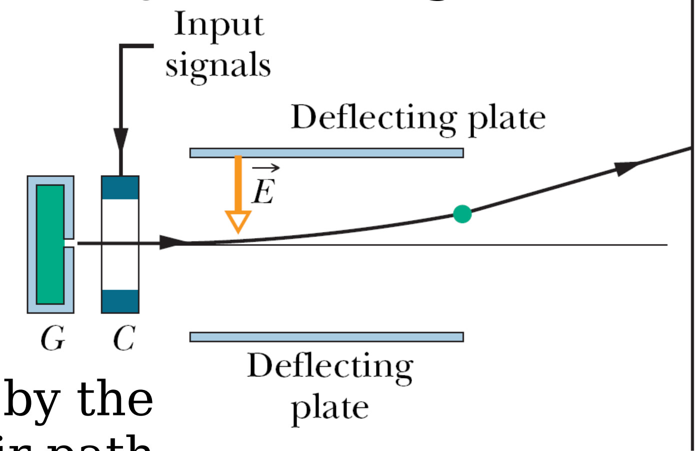


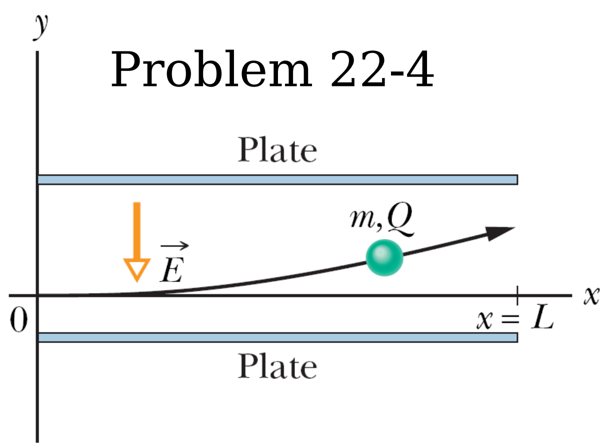
Electrical Breakdown and Sparking

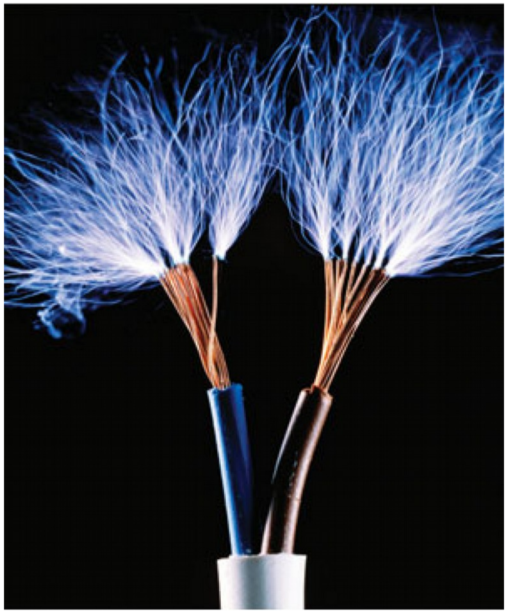
- If the magnitude of an electric field in air exceeds a certain critical value E_c, the air undergoes *electrical breakdown*, a process whereby the field removes electrons from the atoms in the air.

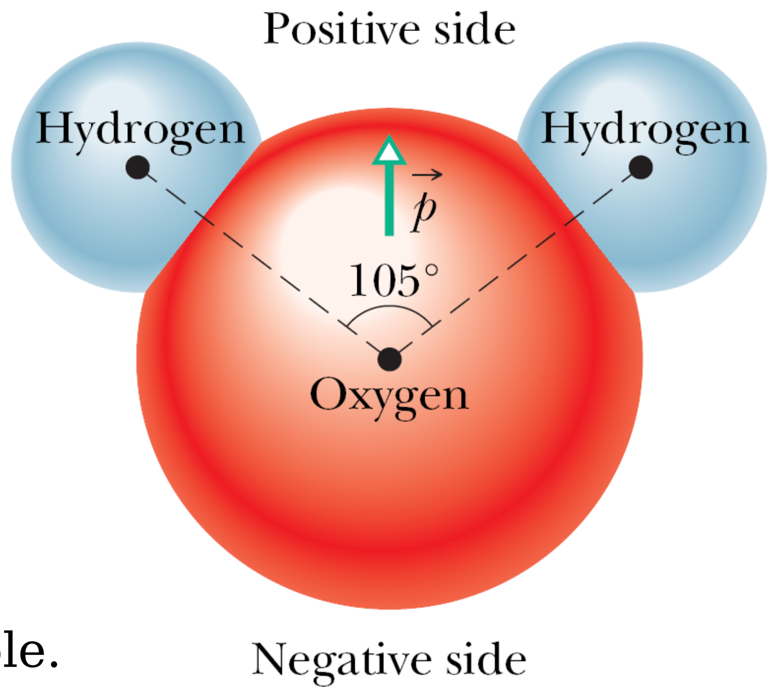
- The air then begins to conduct electric current because the freed electrons are propelled into motion by the field. As they move, they collide with any atoms in their path, causing those atoms to emit light — sparking.

Ink-Jet Printing









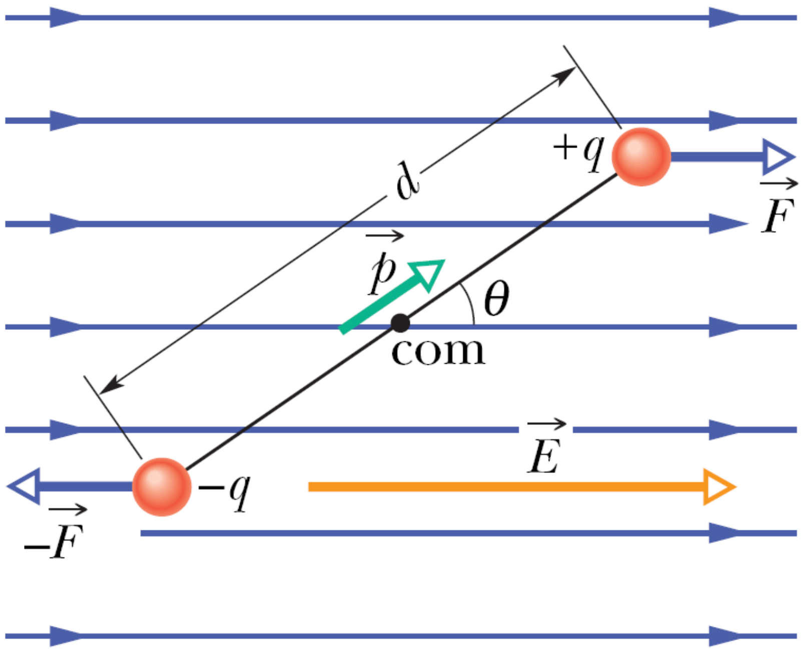
A Dipole in an Electric Field

- A molecule of water (H_2O) forms an electric dipole.

- In a water molecule, the 2 hydrogen atoms and the oxygen atom form an angle of about 105°. Then the 10 electrons of the molecule tend to remain closer to the oxygen nucleus than to the hydrogen nuclei.

- This makes the oxygen side of the molecule slightly more negative than the hydrogen side and creates an electric dipole moment that points along the symmetry axis of the molecule.

- Now consider such an abstract dipole in a uniform external electric field. Because the electric field is uniform, those forces act in opposite directions and with the same magnitude $F=qE$, the net force on the dipole from the field is 0 and the center of mass of the dipole does not move. But the forces do produce a net torque on the dipole about its center of mass.



(a)

- the net torque

$$\tau = -x F \sin\theta - F(d-x)\sin\theta = -F d \sin\theta$$
$$= -q E d \sin\theta = -p E \sin\theta$$

- generalize this equation to vector form as

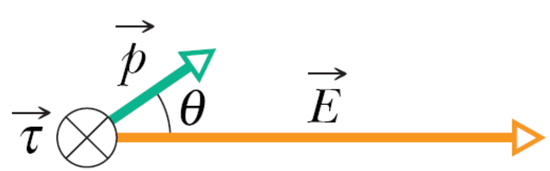
$$\boxed{\vec{\tau} = \vec{p} \times \vec{E} \qquad \text{torque on a dipole}}$$

- The torque acting on a dipole tends to rotate the dipole into the direction of the external field.

Potential Energy of an Electric Dipole

- The dipole has its least potential energy when it is in its equilibrium orientation, which is when its dipole moment is lined up with the external field,

$$\vec{\tau} = \vec{p} \times \vec{E} = 0$$



(b)

- the dipole is like a pendulum, which has its least gravitational potential energy in its equilibrium orientation at its lowest point.

- choose the potential energy to be 0 when the angle is 90°, then the potential energy U at any angle is

$$U=-W=-\int_{\pi/2}^{\theta}\tau\, d\theta'=\int_{\pi/2}^{\theta} pE\sin\theta'\, d\theta'=pE\int_{\pi/2}^{\theta}\sin\theta'\, d\theta'=-pE\cos\theta$$

$$\Rightarrow \boxed{U=-\vec{p}\cdot\vec{E} \quad \text{potential energy of a dipole}}$$

- the potential energy of the dipole is least (U=-pE) when θ=0, which is when the dipole moment and the external field are in the same direction; the potential energy is greatest (U=pE) when θ=180°, which is when the dipole moment and the external field are in opposite directions.

- When a dipole rotates from an initial orientation to another orientation, the work W done on the dipole by the electric field is $W=-\Delta U=-(U_f-U_i)$

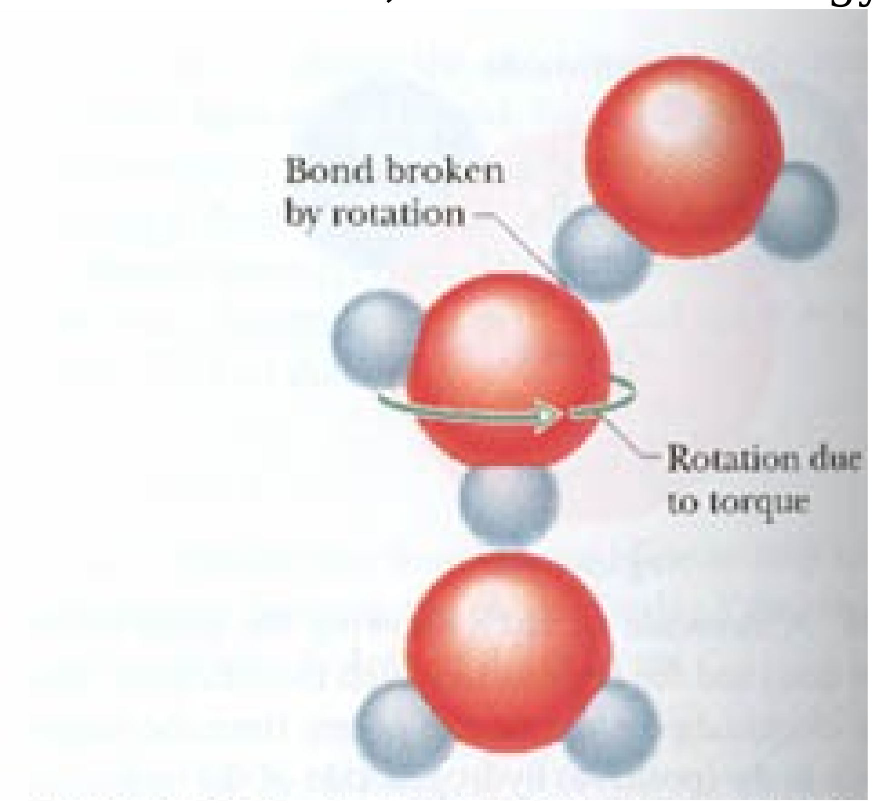
- If the change in orientation is caused by an applied torque, then the work W_a done on the dipole by the applied torque is the negative of a the work done on the dipole by the field $W_a=-W=U_f-U_i$

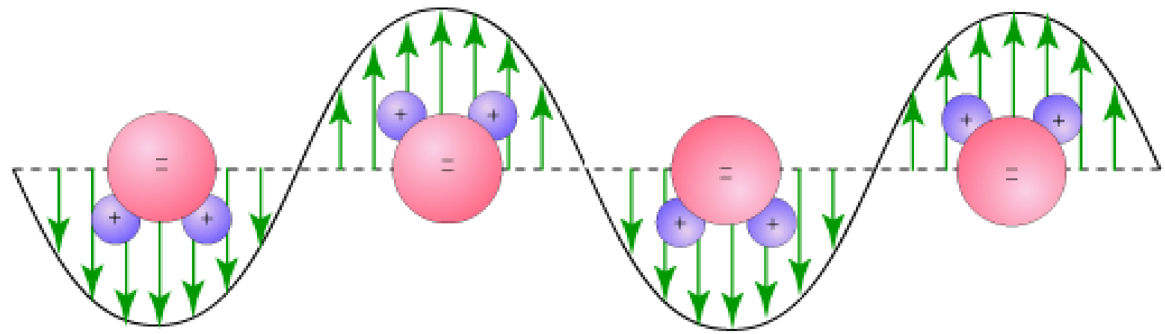
Microwave Cooking

- Food can be warmed and cooked in a microwave oven if the food contains water because water molecules are electric dipoles.

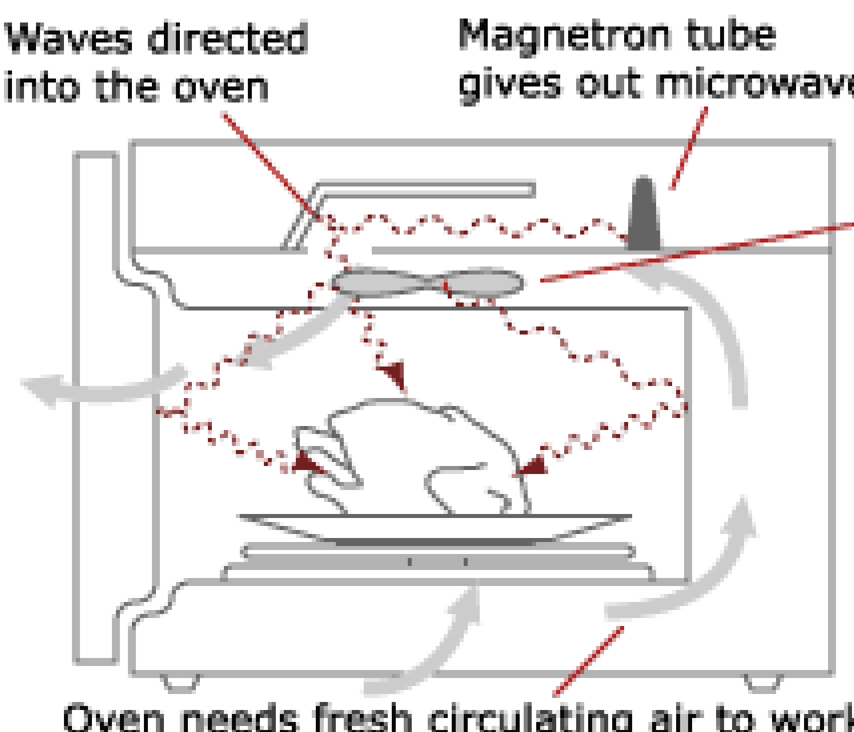
- When you turn on the oven, the microwave source sets up a rapidly oscillating electric field within within the food. Then the water molecules continuously flip-flop in an attempt to align with the electric field.

- Energy is transferred from the electric field to the thermal energy of the water. Soon, the thermal energy of the water is enough to cook the food.





Frequency ~ 2.5 GHz



- Avoid conducting container.

Problem 22-5

Selected problems: 10, 26, 50, 58