Chapter 19 The Kinetic Theory of Gases

- 3 variables associated with a gas:

  (1) **volume**: a result of the freedom the atoms have to spread throughout the container
  (2) **pressure**: a result of the collisions of the atoms with the container's walls
  (3) **temperature**: related to the kinetic energy of the atoms

Avogadro's Number

One mole is the number of atoms in a 12g sample of carbon-12.

- **Avogadro's number** $N_A = 6.02 \times 10^{23} / \text{mole}$ Avogadro's number

- Avogadro suggested that all gases occupying the same volume under the same conditions of temperature and pressure contain the same number of atoms or molecules.

- The number of moles $n = \dfrac{N}{N_A}$

- the number of moles n in a sample from the mass M_{sam} of the sample and either the *molar mass M* (the mass of 1 mole) or the molecular mass m (the mass of one molecule):

$$n = \frac{M_{\text{sam}}}{M} = \frac{M_{\text{sam}}}{m \, N_A} \qquad \Leftarrow \qquad M = m \, N_A$$

Ideal Gases

- At low enough densities, all real gases tend to obey the relation

$$\boxed{p\,V = n\,R\,T} \quad \text{ideal gas law}$$

R is the **gas constant** $R = 8.31 \; \text{J}/\text{mol} \cdot \text{K}$.

- We can rewrite the equation in terms of the **Boltzmann constant**,

$$k = \frac{R}{N_A} = \frac{8.31 \; \text{J}/\text{mol} \cdot \text{K}}{6.02 \times 10^{23} \; \text{mol}^{-1}} = 1.38 \times 10^{-23} \; \text{J}/\text{K}$$

$$\Rightarrow \quad n\,R = N\,k \quad \Rightarrow \quad \boxed{p\,V = N\,k\,T} \quad \text{ideal gas law}$$

- An ideal gas is defined as a gas which behavior follows the above equation.

- *All real* gases approach the ideal state at low enough densities and at high enough temperature.

Work Done by an Ideal Gas at Constant Temperature
- **isothermal expansion** (or **isothermal compression**): A process carried out from an initial volume V_i to a final volume V_f at *constant temperature*.
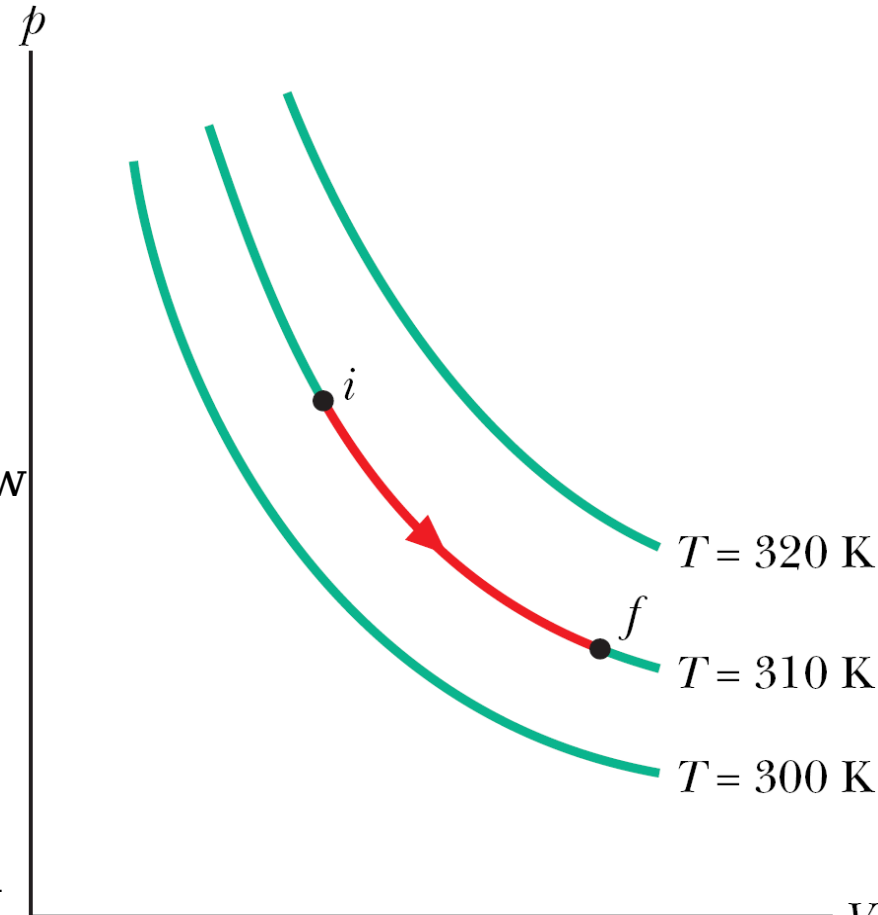
- On a *p-V* diagram, an *isotherm* is a curve that connects points that have the same temperature, ie, a graph of pressure versus volume for a gas whose temperature T is held constant.

$$p = n R T \frac{1}{V} = (\text{a constant}) \frac{1}{V}$$

- the isotherms with different temperatures doesn't cross each other.

- the work done by an ideal gas during an isothermal process

$$W = \int_{V_i}^{V_f} p \, d V = \int_{V_i}^{V_f} \frac{n R T}{V} d V \qquad \Leftarrow \qquad p V = n R T$$

- For an isothermal process: $T = \text{const} \quad \Rightarrow \quad W = n R T \int_{V_i}^{V_f} \frac{d V}{V} = n R T \ln V \Big|_{V_i}^{V_f}$

$$\Rightarrow \quad \boxed{W = n R T \ln \frac{V_f}{V_i}} \quad \text{ideal gass, isothermal process}$$

- For $V_f > V_i$ (expansion), the work W done by an ideal gas is positive;
  For $V_f < V_i$ (compression), the work W done by an ideal gas is negative.

Work Done at Constant Volume and at Constant Pressure

- If the volume of the gas is constant, then $W = 0$ constant-volume process

- If the volume changes while the pressure p of the gas is held constant, then

$$W = p(V_f - V_i) = p\,\Delta V \quad \text{constant-pressure process}$$

problem 19-1   problem 19-2

Pressure, Temperature, and RMS Speed

- n moles of an ideal gas are confined in a cubical box of volume V. The walls of the box are held at temperature T. We need to relate the macroscopic quantity - pressure p, to the microscopic quantity — the velocity of the molecule v.

- The molecules of gas in the box are moving in all directions and with various speeds, bouncing from the walls of the box.

- Ignore collisions of the molecules with one another and consider only elastic collisions with the walls.

- the change in the particle's momentum along the x axis is $\Delta p_x = (-m v_x) - m v_x = -2 m v_x$
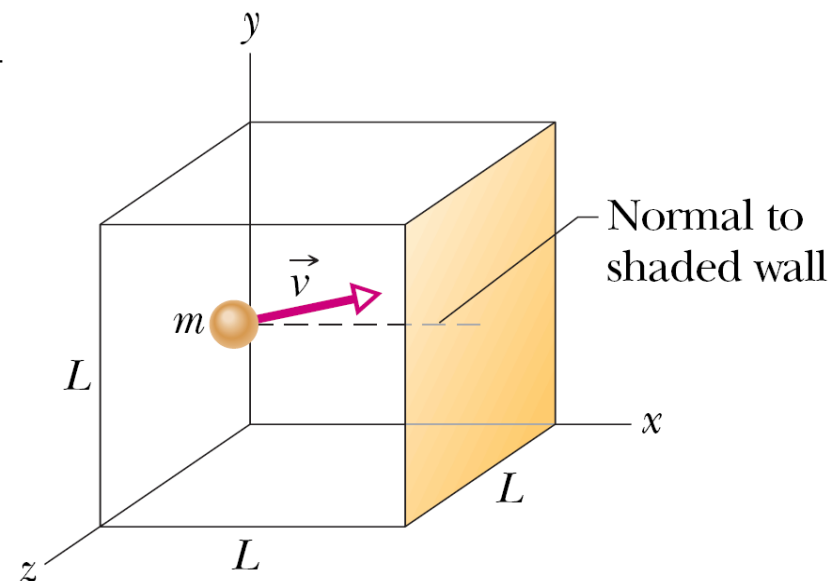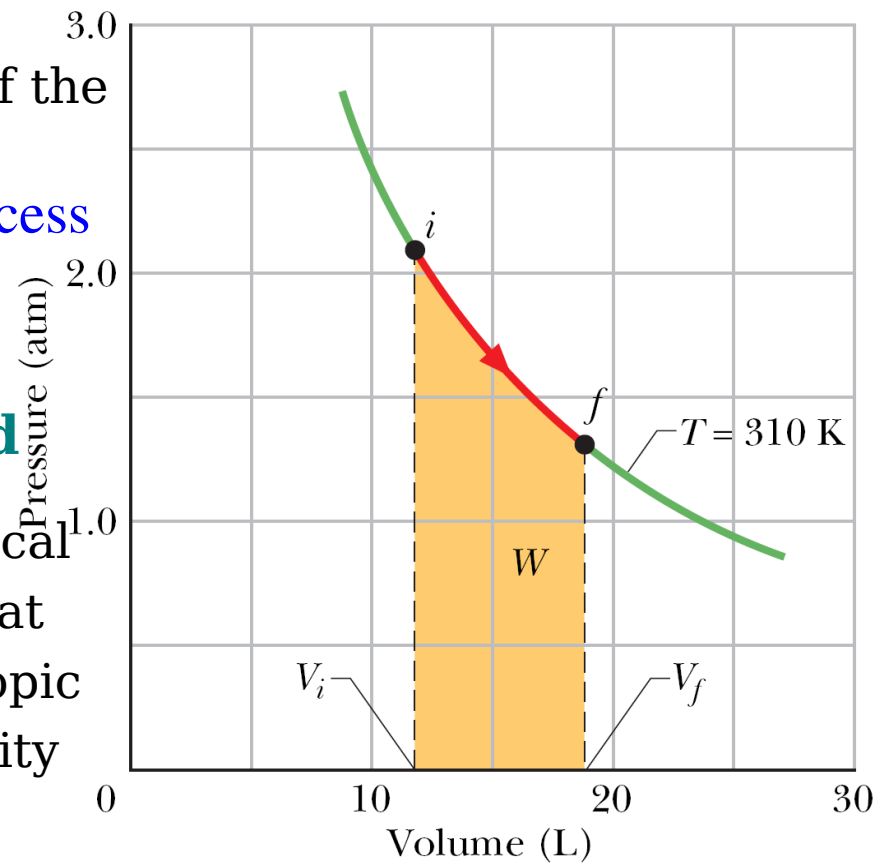
- the momentum Δp_x delivered to the wall by the molecule is $+2mv_x$.

- The molecule of will hit the wall repeatedly. The time Δt between collisions is the time the molecule takes to travel to the opposite wall and back again (a distance $2L$) at speed v_x

$$\Delta t = \frac{2L}{v_x}$$

- the average rate at which momentum is delivered to the wall by this single molecule is

$$\frac{\Delta p_x}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}$$

- From Newton's 2nd law, the rate at which momentum is delivered to the wall is the force acting on that wall.

- Dividing the magnitude of the total force F_x by the area of the wall $(=L^2)$ gives the pressure p on that wall.

- we can write this pressure: $p = \dfrac{F_x}{L^2} = \dfrac{mv_{x1}^2/L + \cdots + mv_{xN}^2/L}{L^2} = \dfrac{m}{L^3}\sum_{i=1}^{N} v_{xi}^2$

$$\left(v_x^2\right)_{avg} = \frac{1}{N}\sum_{i=1}^{N} v_{xi}^2, \quad N = n\,N_A \quad \text{and} \quad M = N_A\,m \quad \text{the molar mass}$$

$$\Rightarrow \quad p = \frac{m}{L^3}\sum_{i=1}^{N} v_{xi}^2 = \frac{m}{V}\,N\left(v_x^2\right)_{avg} = \frac{n\,m\,N_A}{V}\left(v_x^2\right)_{avg} = \frac{n\,M\left(v_x^2\right)_{avg}}{V}$$

- For any molecule, $v^2 = v_x^2 + v_y^2 + v_z^2$. Because there are many molecules and because they are all moving in random directions, the average values of the squares of their velocity components are equal, so that $v_x^2 = v^2/3$,

$$p = \frac{n\,M\,(v^2)_{avg}}{3\,V}$$

- **root-mean-square speed** $\quad v_{rms} \equiv \sqrt{(v^2)_{avg}} = \sqrt{\dfrac{1}{N} \sum_{i=1}^{N} v_i^2} \quad \Rightarrow \quad p = \dfrac{n\,M\,v_{rms}^2}{3\,V}$

- The equation tells how the pressure of the gas (a purely macroscopic quantity) depends on the speed of the molecules (a purely microscopic quantity).

- Using the ideal gas law ($p\,V = n\,R\,T$) gives $\quad \boxed{v_{rms} = \sqrt{\dfrac{3\,R\,T}{M}}}$

- The rms speed is only a kind of average speed; many molecules move much faster than this, and some much slower.

- The speed of sound in a gas is closely related to the rms speed of the molecules of that gas.

- If molecules move so fast, why does it take as long as a minute or so before you can smell perfume when someone opens a bottle across a room?

problem 19-3

Translational Kinetic Energy

● consider a single molecule of an ideal gas, Its average translational kinetic energy over the time is

$$K_{avg} = \left(\frac{1}{2} m v^2\right)_{avg} = \frac{1}{2} m \left(v^2\right)_{avg} = \frac{1}{2} m v_{rms}^2 = \frac{1}{2} m \frac{3\,R\,T}{M} = \frac{3\,R\,T}{2\,N_A}$$

$$k = \frac{R}{N_A} \quad \Rightarrow \quad \boxed{K_{avg} = \frac{3}{2} k\,T}$$

At a given temperature T, all ideal gas molecules — no matter what their mass — have the same *average* translational kinetic energy — namely, $3kT/2$. When we measure the temperature of a gas, we are also measuring the average translational kinetic energy of its molecules.

Mean Free Path

● the **mean free path** λ: is one useful parameter to describe this random motion of the molecules; it is the average distance traversed by a molecule between collisions.

● We expect (1) $\lambda \propto V/N$ (2) $\lambda \propto 1/d^2$, the cross section of a molecule determines its effective target area, thus

$$\boxed{\lambda = \frac{V}{\sqrt{2}\,\pi\,N\,d^2}} \quad \text{mean free path}$$

justification

- A collision will then take place if the centers of 2 molecules come within a distance d of each other.

- As a single molecule zigzags through the gas, it sweeps out a cylinder of cross-sectional area πd^2 between successive collisions.

- For a time interval Δt, it moves a distance $v_{avg}\Delta t$.

- The volume of the cylinder is $(\pi d^2)(v_{avg}\Delta t)$.

- The number of collisions that occur in time Δt is equal to the number of molecules that lie within this cylinder,

$$\lambda = \frac{\text{length of path during } \Delta t}{\text{number of collisions in } \Delta t} = \frac{v_{avg}\,\Delta t}{\pi\,d^2\,v_{rel}\,\Delta t\,N/V} = \frac{v_{avg}}{\pi\,d^2\,v_{rel}\,N/V}$$

v_{avg}: the mean speed of the molecules *relative to the container*;

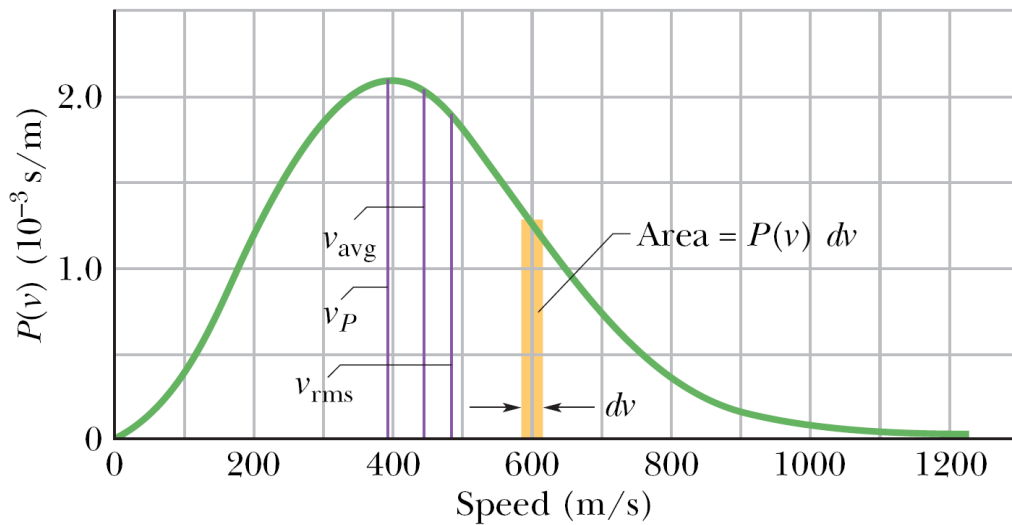v_{rel} : the mean speed of a single molecule *relative to the other molecules*.
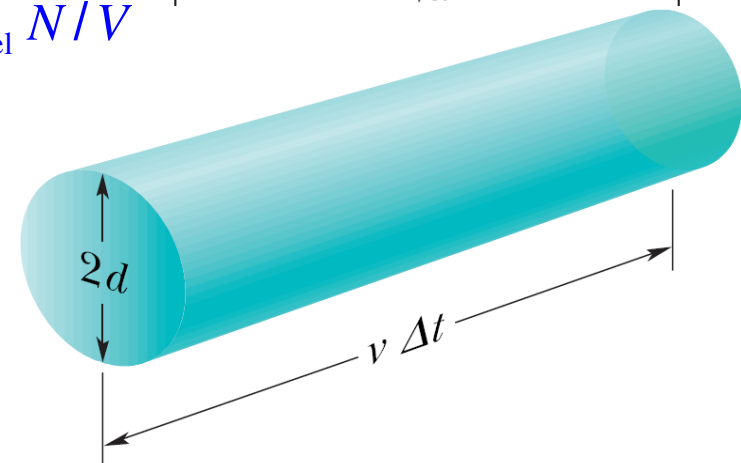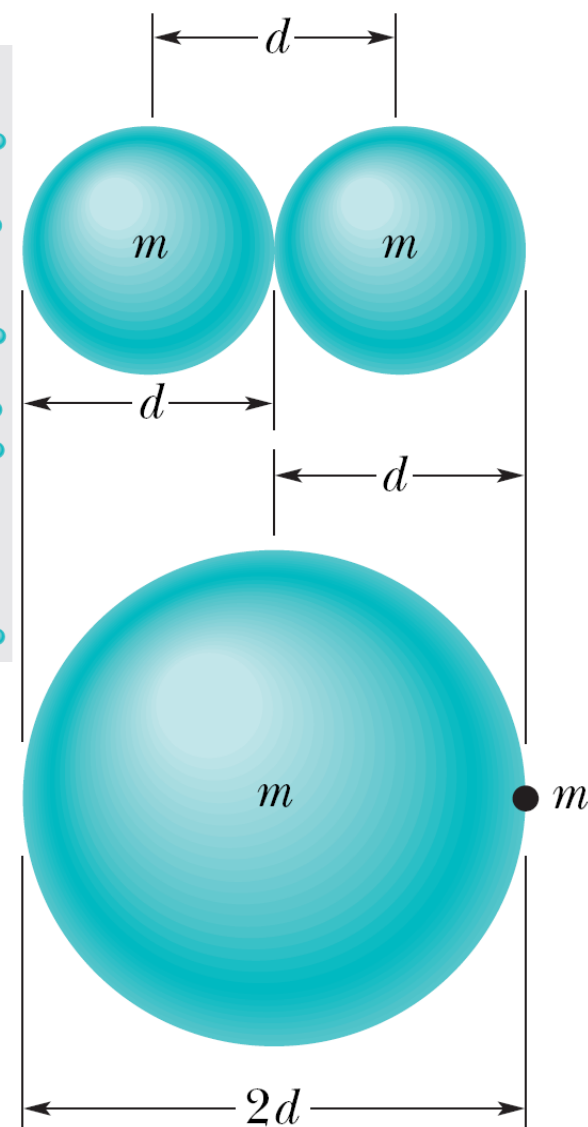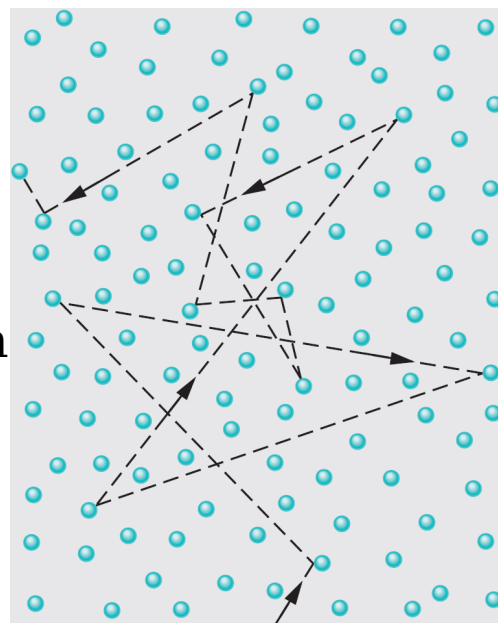
- A detailed calculation, taking into account the actual speed distribution of the molecules, gives

$$v_{rel} = \sqrt{2}\, v_{avg} \quad \Rightarrow \quad \lambda = \frac{V}{\sqrt{2}\,\pi\, N\, d^2}$$

- The mean free path of air molecules: $0.1\,\mu m$ for H=0; 16cm for H=100km; 20km for H=300km.
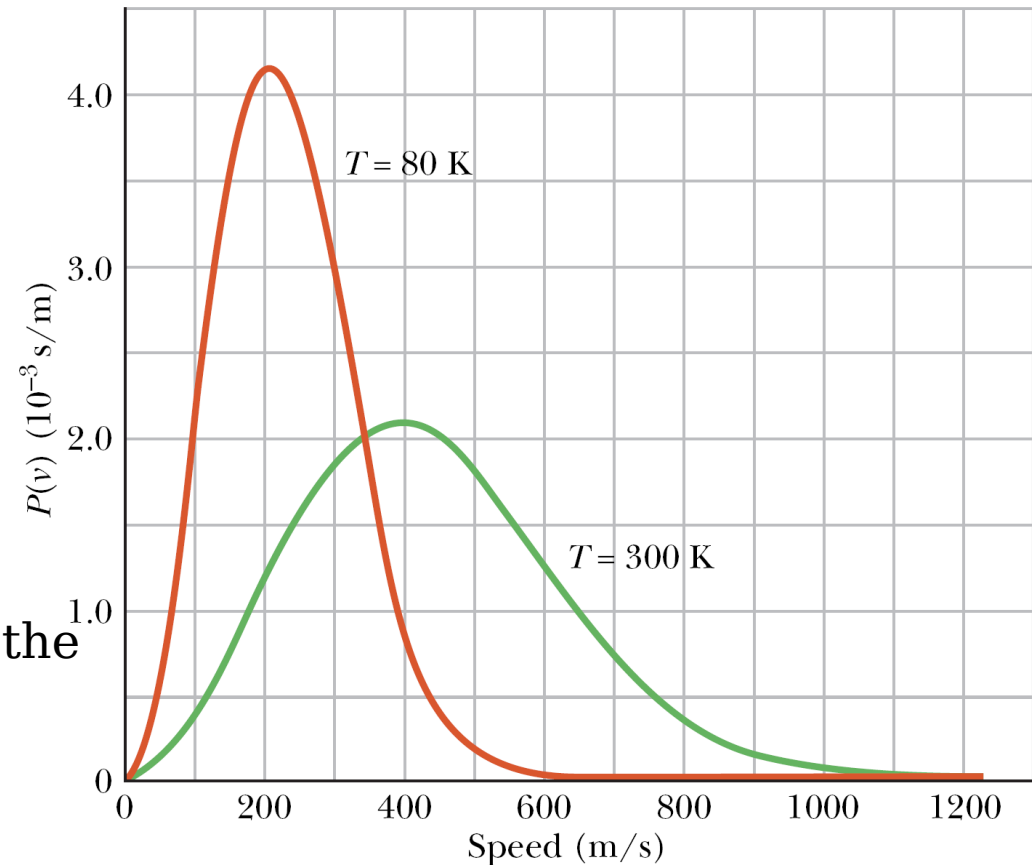
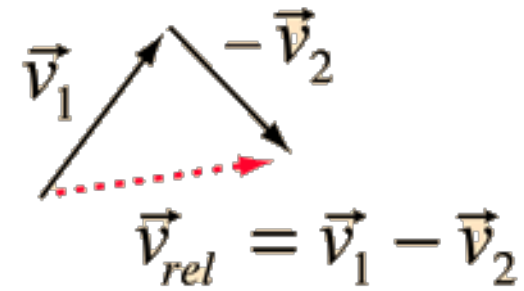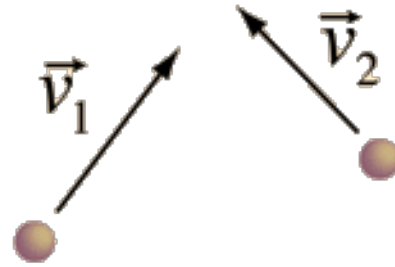problem 19-4

The Distribution of Molecular Speeds

- **Maxwell's speed distribution law**, the speed distribution of gas molecules, is

$$P(v) = \sqrt{\frac{2\,M^3}{\pi\, R^3\, T^3}}\; v^2\, e^{-M v^2 / 2 R T}$$

Random molecular velocities

$$v_{\text{rel}}^2 = \vec{v}_{\text{rel}}^2 = (\vec{v}_1 - \vec{v}_2) \cdot (\vec{v}_1 - \vec{v}_2) = \vec{v}_1 \cdot \vec{v}_1 - 2\, \vec{v}_1 \cdot \vec{v}_2 + \vec{v}_2 \cdot \vec{v}_2$$

$$\Rightarrow \quad \bar{v}_{\text{rel}} = \overline{\sqrt{\vec{v}_1 \cdot \vec{v}_1 - 2\, \vec{v}_1 \cdot \vec{v}_2 + \vec{v}_2 \cdot \vec{v}_2}}$$

$$= \sqrt{\overline{\vec{v}_1 \cdot \vec{v}_1} - 2\, \overline{\vec{v}_1 \cdot \vec{v}_2} + \overline{\vec{v}_2 \cdot \vec{v}_2}}$$

$$= \sqrt{\bar{v}_1^2 + \bar{v}_2^2}$$

$$= \sqrt{\bar{v}^2 + \bar{v}^2}$$

$$= \sqrt{2}\, \bar{v}$$

- The quantity $P(v)$ is a probability distribution function: For any speed v, the product $P(v)dv$ (a dimensionless quantity) is the fraction of molecules with speeds in the interval dv centered on speed v, $\int_0^\infty P(v)\,dv = 1$

- The fraction (frac) of molecules with speeds in an interval of v_1 to v_2 is

$$\text{frac} = \int_{v_1}^{v_2} P(v)\,dv$$

Average, RMS, and Most Probable Speeds

- find the **average speed** v_{avg} of the molecules in a gas with the distribution law

$$v_{avg} = \int_0^\infty v\,P(v)\,dv$$

- Substituting for $P(v)$ from the Maxwell's speed distribution law and find

$$\boxed{v_{avg} = \sqrt{\frac{8\,R\,T}{\pi\,M}}}\quad \text{average speed}$$

- Similarly, $\left(v^2\right)_{avg} = \int_0^\infty v^2\,P(v)\,dv = \dfrac{3\,R\,T}{M} \Rightarrow \boxed{v_{rms} = \sqrt{\left(v^2\right)_{avg}} = \sqrt{\frac{3\,R\,T}{M}}}\quad \text{rms speed}$

- The **most probable speed** v_P is the speed at which $P(v)$ is maximum, therefore solve it to find

$$\frac{d\,P}{d\,v} = 0 \quad \Rightarrow \quad \boxed{v_P = \sqrt{\frac{2\,R\,T}{M}}}\quad \text{most probable speed}$$

Gaussian Integral $\displaystyle\int_{-\infty}^{\infty} e^{-x^2}\, \mathrm{d}\,x = 2\int_{0}^{\infty} e^{-x^2}\, \mathrm{d}\,x = \sqrt{\pi}$

Proof:

$$\left(\int_{-\infty}^{\infty} e^{-x^2}\, \mathrm{d}\,x\right)^2 = \int_{-\infty}^{\infty} e^{-x^2}\, \mathrm{d}\,x \int_{-\infty}^{\infty} e^{-y^2}\, \mathrm{d}\,y = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)}\, \mathrm{d}\,x\, \mathrm{d}\,y$$

$$= \int_{0}^{\infty} \int_{0}^{2\pi} e^{-r^2}\,(r\, \mathrm{d}\,\theta)(\mathrm{d}\,r) \quad\Leftarrow\quad \begin{array}{l} r^2 = x^2 + y^2 \\ \mathrm{d}\,x\, \mathrm{d}\,y = (r\, \mathrm{d}\,\theta)(\mathrm{d}\,r) = r\, \mathrm{d}\,r\, \mathrm{d}\,\theta \end{array}$$

$$= \int_{0}^{2\pi} \mathrm{d}\,\theta \int_{0}^{\infty} e^{-r^2} r\, \mathrm{d}\,r = 2\pi\cdot\frac{1}{2}\int_{0}^{\infty} e^{-q}\, \mathrm{d}\,q \quad\Leftarrow\quad q \equiv r^2$$

$$= -\pi\, e^{-q}\Big|_{0}^{\infty} = \pi$$

$$s \equiv \sqrt{\frac{2\,M^3}{\pi\,R^3\,T^3}}\,, \quad u^2 = \frac{2\,R\,T}{M} \quad \Rightarrow \quad P(\nu) = s\,\nu^2\,e^{-\nu^2/u^2}$$

$$\int_0^\infty P(\nu)\,\mathrm{d}\nu = s\int_0^\infty \nu^2\,e^{-\nu^2/u^2}\,\mathrm{d}\nu = \frac{s}{2}\int_0^\infty \nu\,e^{-\nu^2/u^2}\,\mathrm{d}\nu^2 = -\frac{s\,u^3}{2}\int_0^\infty \frac{\nu}{u}\,\mathrm{d}\,e^{-\nu^2/u^2}$$

$$= -\frac{s\,u^3}{2}\left[\frac{\nu}{u}\,e^{-\nu^2/u^2}\Big|_0^\infty - \int_0^\infty e^{-\nu^2/u^2}\,\mathrm{d}\frac{\nu}{u}\right] = \frac{s\,u^3}{2}\int_0^\infty e^{-\nu^2/u^2}\,\mathrm{d}\frac{\nu}{u} = \frac{s\,u^3}{2}\frac{\sqrt{\pi}}{2} = 1$$

$$\overline{v} = \int_0^\infty v\,P(\nu)\,\mathrm{d}\nu = s\int_0^\infty \nu^3\,e^{-\nu^2/u^2}\,\mathrm{d}\nu = \frac{s\,u^4}{2}\int_0^\infty q\,e^{-q}\,\mathrm{d}q \quad \Leftarrow \quad q \equiv \frac{\nu^2}{u^2}$$

$$= -\frac{s\,u^4}{2}\left[q\,e^{-q}\Big|_0^\infty - \int_0^\infty e^{-q}\,\mathrm{d}q\right] = -\frac{s\,u^4}{2}e^{-q}\Big|_0^\infty = \sqrt{\frac{8\,R\,T}{\pi\,M}}$$

$$\overline{v^2} = \int_0^\infty v^2\,P(\nu)\,\mathrm{d}\nu = s\int_0^\infty \nu^4\,e^{-\nu^2/u^2}\,\mathrm{d}\nu = \frac{s\,u^5}{2}\int_0^\infty x^3\,e^{-x^2}\,\mathrm{d}x^2 \quad \Leftarrow \quad x \equiv \frac{\nu}{u}$$

$$= -\frac{s\,u^5}{2}\left[x^3\,e^{-x^2}\Big|_0^\infty - \frac{3}{2}\int_0^\infty x\,e^{-x^2}\,\mathrm{d}x^2\right] = -\frac{3\,s\,u^5}{4}\int_0^\infty x\,\mathrm{d}\,e^{-x^2} = \frac{3\,s\,u^5}{4}\frac{2}{s\,u^3} = \frac{3\,R\,T}{M}$$

- some molecules will have speeds that are many times of v_P. These molecules form the *high-speed tail* of a distribution curve, useful for rain & sunshine.

problem 19-5, 19-6

The Molar Specific Heats of an Ideal Gas

Internal Energy E_{int}

- assume that our ideal gas is a *monatomic gas* and the internal energy E_{int} of the ideal gas is simply the sum of the translational kinetic energies of its atoms.

- The average translational kinetic energy of a single atom depends only on the gas temperature, ie, $K_{avg} = 3kT/2$. The internal energy of a sample of n moles of the gas is

$$E_{int} = (n\,N_A)\,K_{avg} = (n\,N_A)\left(\frac{3}{2}\,k\,T\right) = (n\,N_A)\left(\frac{3\,R}{2\,N_A}\,T\right)$$

$$\Rightarrow \quad \boxed{E_{int} = \frac{3}{2}\,n\,R\,T} \quad \text{monatomic ideal gas}$$

The internal energy E_{int} of an ideal gas is a function of the gas temperature only; it does not depend on any other variable.

- 2 types of the molar specific heat of an ideal gas:
  C_V: molar specific heat at constant volume;
  C_p: molar specific heat at constant pressure.

Molar Specific Heat at Constant Volume

● the heat Q is related to the temperature change ΔT by

$$\boxed{Q = n\, C_V\, \Delta T}\quad \text{constant volume}$$

the 1ˢᵗ law of thermodynamics gives

$$\Delta E_{\text{int}} = Q - W = n\, C_V\, \Delta T - W = n\, C_V\, \Delta T$$

$$\Rightarrow \quad C_V = \frac{\Delta E_{\text{int}}}{n\, \Delta T} = \frac{3\, n\, R\, \Delta T / 2}{n\, \Delta T}$$

$$\Rightarrow \quad \boxed{C_V = \frac{3}{2}\, R = 12.5 \text{ J/mol·K}}\quad \text{monatomic gas}$$
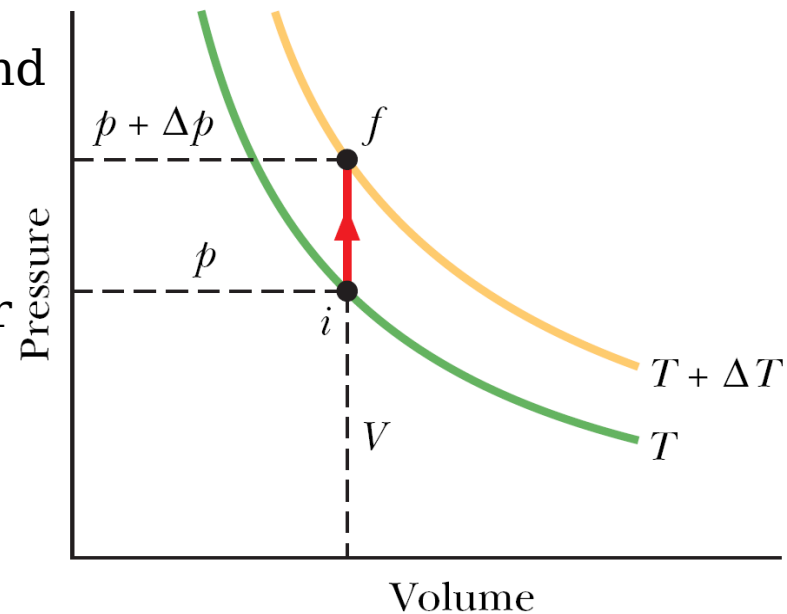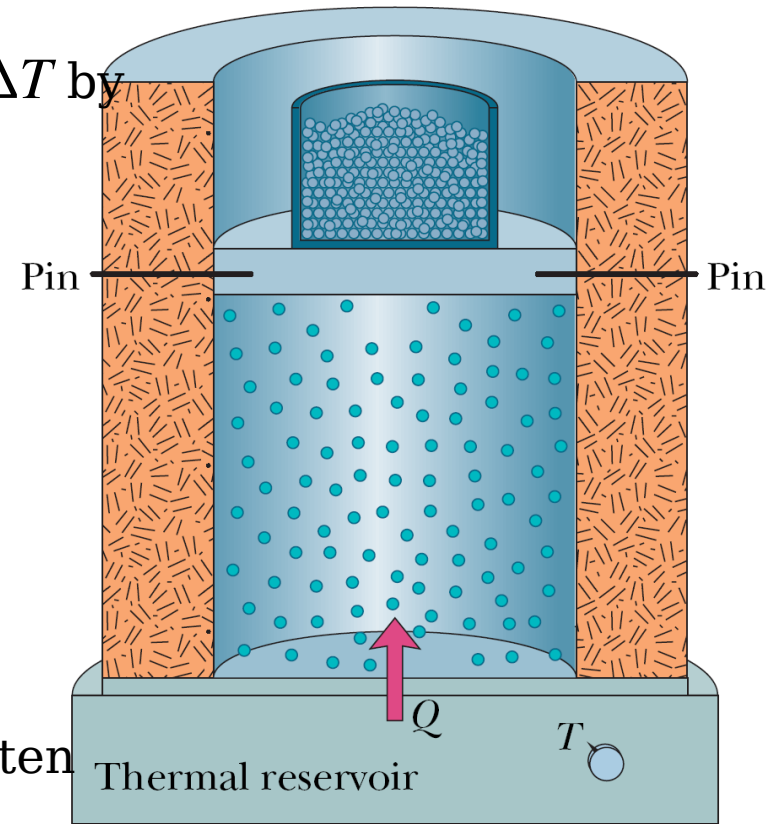
● the internal energy of any ideal gas can be rewritten

$$E_{\text{int}} = n\, C_V\, T \quad \text{any idea gas}$$

● This equation applies to monatomic, diatomic, and polyatomic ideal gases, provided the appropriate value of C_V is used.

● When an ideal gas that is confined to a container undergoes a temperature change ΔT, then the change in its internal energy as is

$$\boxed{\Delta E_{\text{int}} = n\, C_V\, \Delta T}\quad \text{ideal gas, any process}$$

Pin — Pin

Q

T

Thermal reservoir

Pressure

$p + \Delta p$ f

p

i

V

$T + \Delta T$

T

Volume

A change in the internal energy E_{int} of a confined ideal gas depends on the change in the gas temperature only; it does not depend on what type of process produces the change in the temperature.

● Although the values of heat Q and work W associated with these 3 paths differ, the values of ΔE_{int} associated with them are identical because they all involve the same temperature change ΔT.
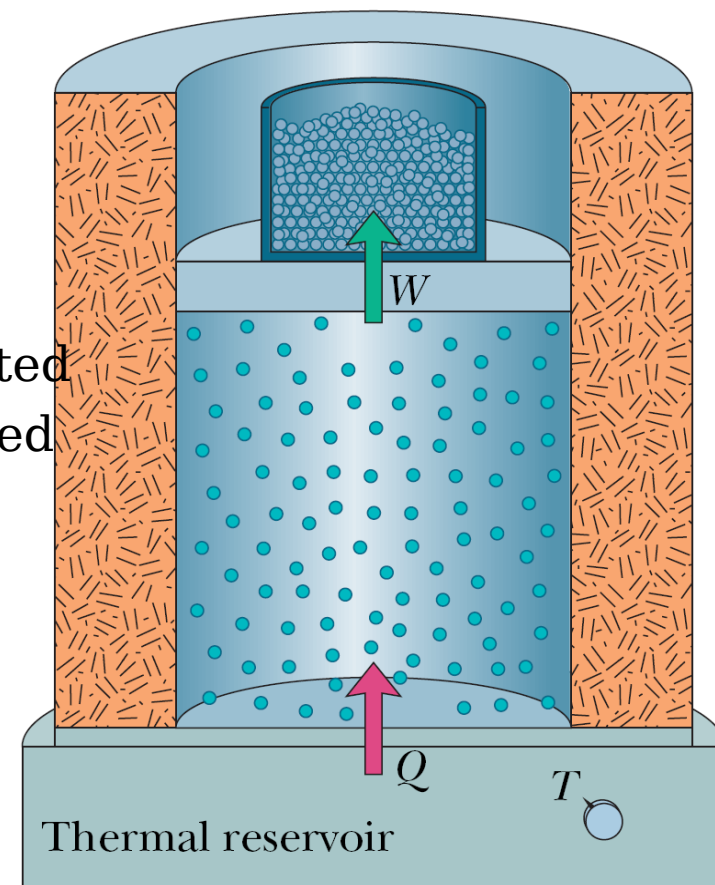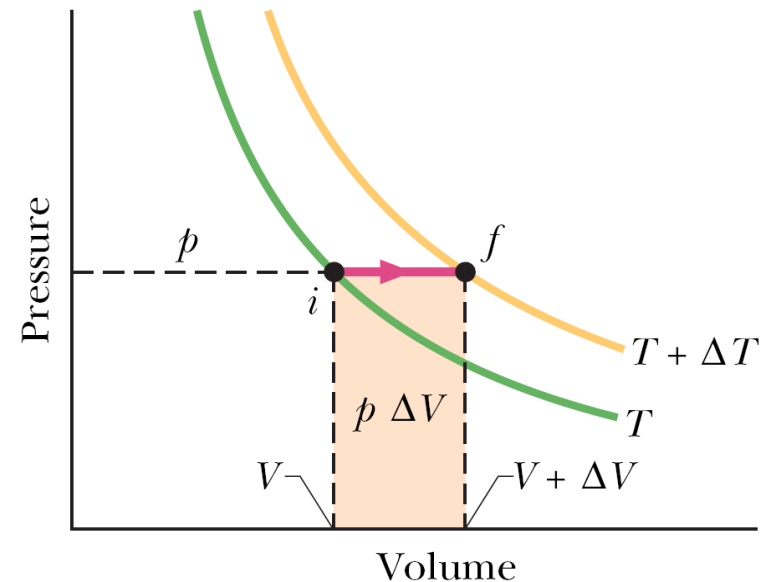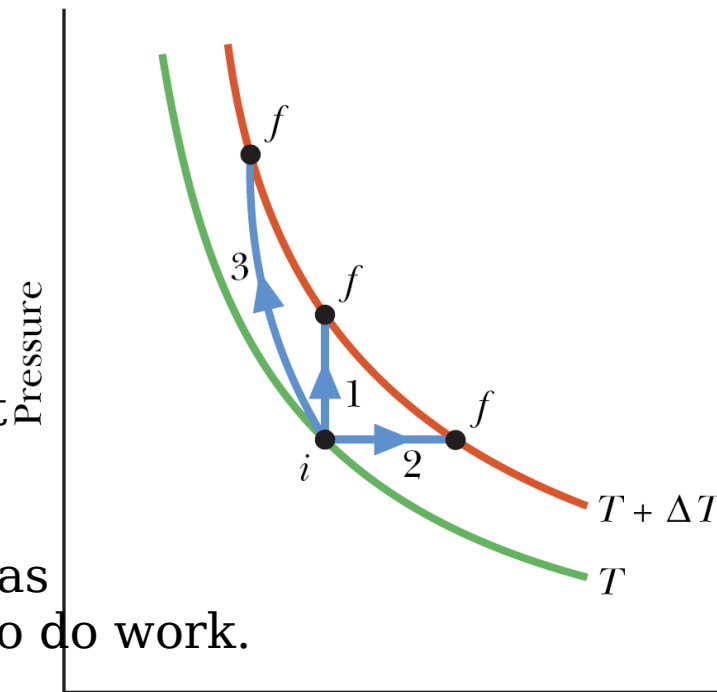
Molar Specific Heat at Constant Pressure

● Q is related to the temperature change ΔT by

$$Q = n\, C_p\, \Delta T$$

   constant pressure

● This C_p is greater than the molar specific heat at constant volume C_V, because energy must now be supplied not only to raise the temperature of the gas but also for the gas to do work.

Thermal reservoir

- From the 1st law of thermodynamics

$$n\, C_V\, \Delta T \Leftarrow \Delta E_{\text{int}} = Q - W = n\, C_p\, \Delta T - p\, \Delta V = n\, C_p\, \Delta T - n\, R\, \Delta T$$

$$\Rightarrow \quad C_V = C_p - R \quad \Rightarrow \quad \boxed{C_p = C_V + R}$$
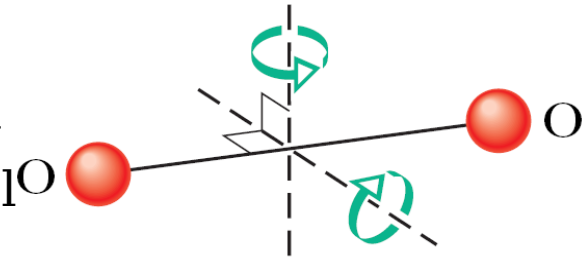
(a) **He**

problem 19-7

Degrees of Freedom and Molar Specific Heats

- to explain the discrepancy by considering the possibility that molecules with more than one atom can store internal energy in forms other than translational kinetic energy.

(b) $\mathbf{O_2}$

- assume that all 3 types (monatomic, diatomic, polyatomic) of molecules can have translational motions, and rotational motions.

- assume that the diatomic and polyatomic molecules can have oscillatory motions.

- the theorem of the **equipartition of energy**:

Every kind of molecule has a certain number f, *degrees of freedom*, which are independent ways in which the molecule can store energy. Each such degree of freedom has associated with it - on average - an energy of $kT/2$ per molecule (or $RT/2$ per mole).
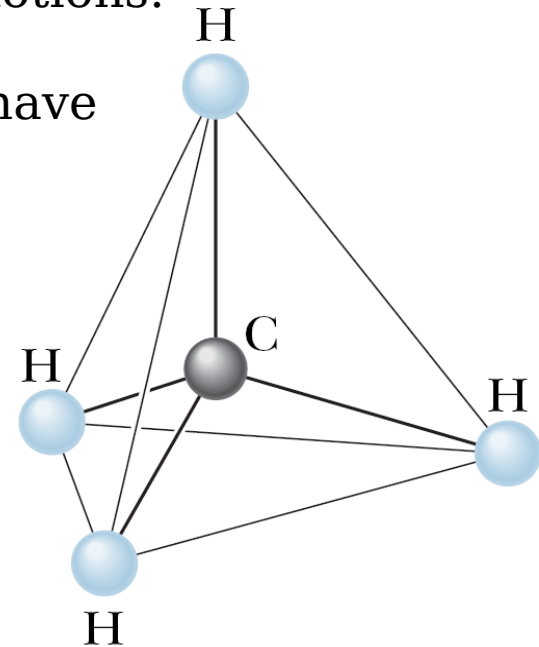
(c) $\mathbf{CH_4}$

- For the translational motion, the molecules have velocity components along xyz axes. Thus, gas molecules of all types have 3 degrees of translational freedom and, on average, an associated energy of 3($kT/2$) per molecule.

- A monatomic gas molecule does not rotate and so has no rotational energy.

- A diatomic molecule can rotate only about axes perpendicular to the line connecting the atoms and have only 2 degrees of rotational freedom and a rotational energy of only 2($kT/2$) per molecule.

- Only the polyatomic molecules are able to rotate with an angular velocity component along each of the 3 axes, and have 3 degrees of rotational freedom, and an energy of 3($kT/2$)per molecule.

- Now the internal energy: $E_{int} = \dfrac{f}{2} n R T \implies C_V = \dfrac{f}{2} R = 4.16 \, f \text{ J/mol·K}$

where f is the number of degrees of freedom.
- f = 3 for monatomic gases, f = 5 for diatomic gases, f = 6 for polyatomic gases.

Degrees of Freedom for Various Molecules

|		Degree of Freedom			specific heat	
Molecule	Example	Translational	Rotational	Total (f)	C_V	C_p
monatomic	He	3	0	3	$\frac{3}{2} R$	$\frac{5}{2} R$
diatomic	O_2	3	2	5	$\frac{5}{2} R$	$\frac{7}{2} R$
polyatomic	CH_4	3	3	6	$3 R$	$4 R$

problem 19-8

A Hint of Quantum Theory

- improve the agreement of kinetic theory by including the oscillations of the atoms in a gas of diatomic or polyatomic molecules.

- experiment shows that such oscillations are "turned on" only when the gas molecules have relatively large energies.

- quantum theory shows that rotational motion of the molecules requires a certain minimum amount of energy; oscillatory motion of the molecules requires a certain (higher) minimum amount of energy.
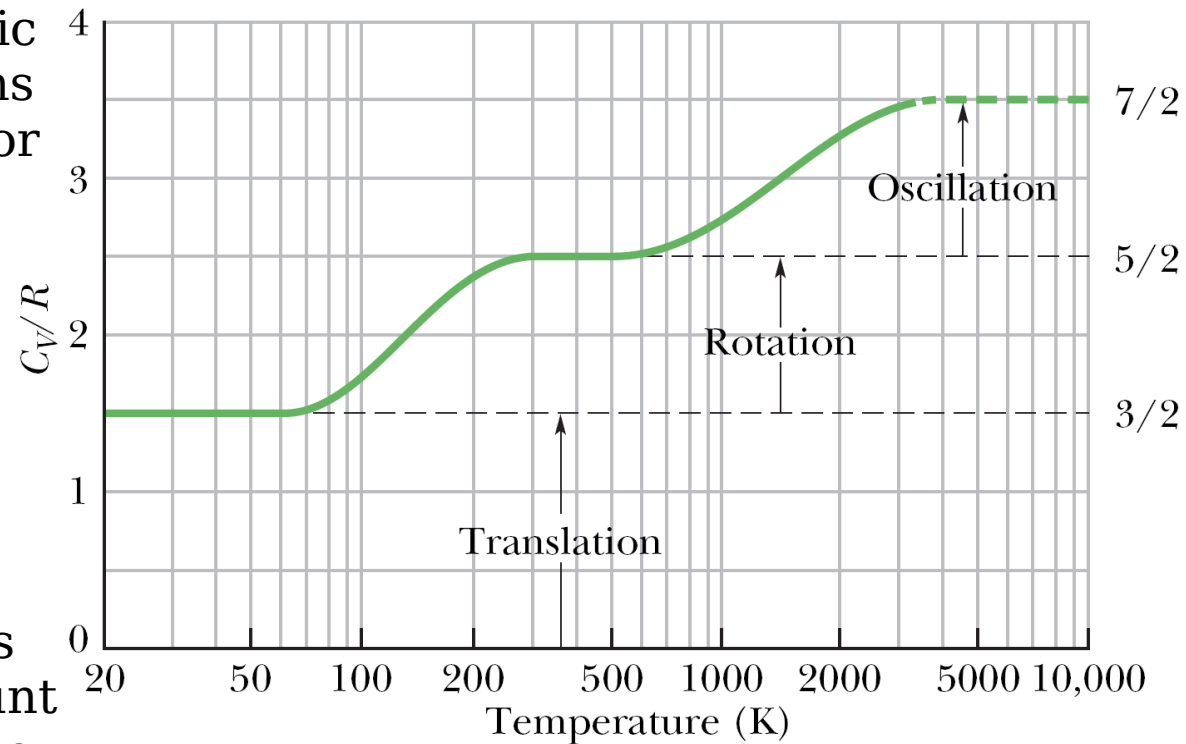

The Adiabatic Expansion of an Ideal Gas

- ensure that $Q = 0$ either by carrying out the process very quickly (as in sound waves) or by doing it (at any rate) in a well-insulated container.

- the relation between the pressure and the volume during an adiabatic process is

$$p\,V^{\gamma} = \text{a constant} \qquad \text{adiabatic process}$$

in which $\gamma = C_p/C_V$, the ratio of the molar specific heats for the gas.

Adiabat $(Q = 0)$

Isotherms:
700 K
500 K
300 K

i

f

Pressure

Volume

W

Insulation
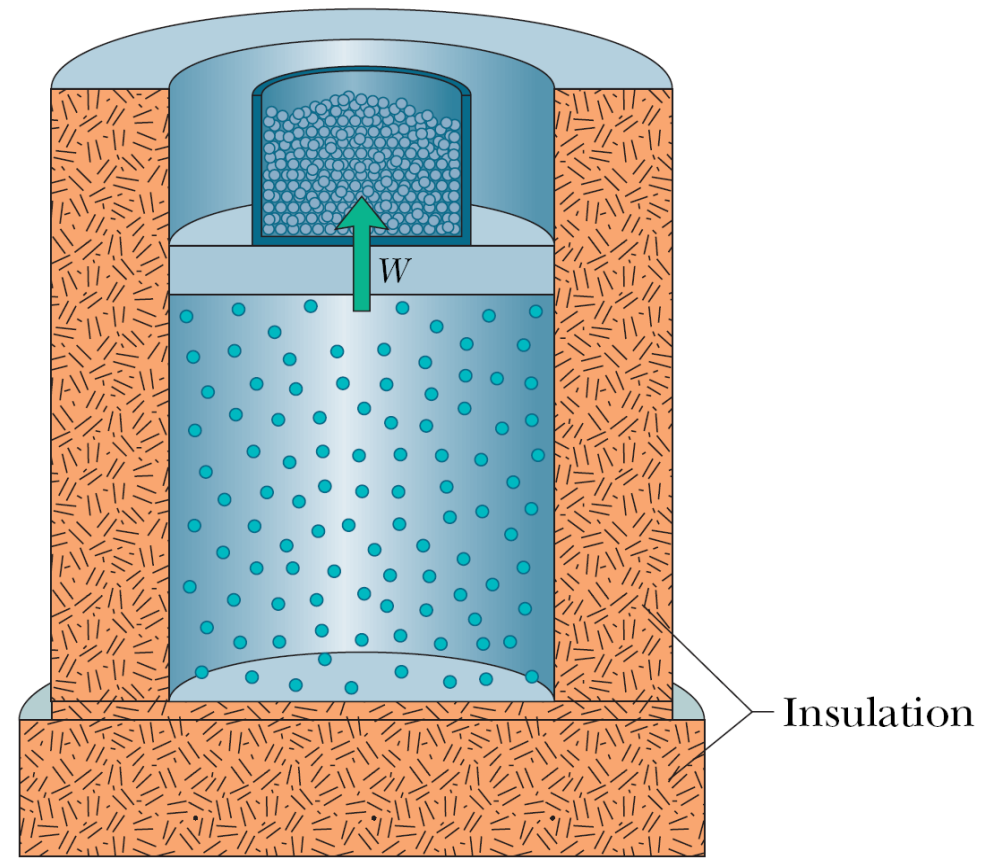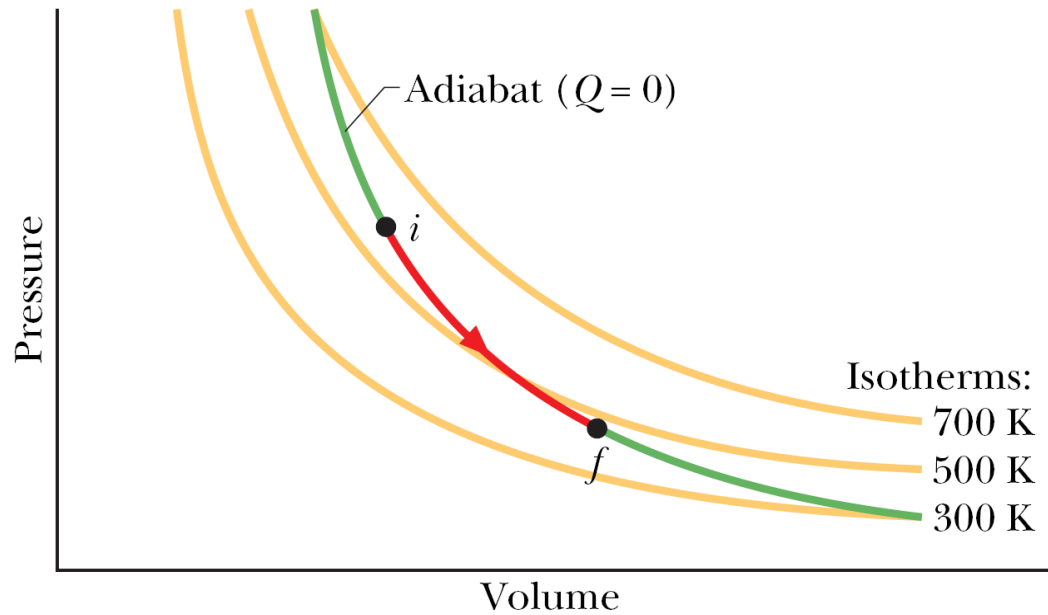
Proof

- the 1st law of thermodynamics:

$$n\,C_V\,\mathrm{d}\,T \Leftarrow \mathrm{d}\,E_{\mathrm{int}} = Q - p\,\mathrm{d}\,V = -p\,\mathrm{d}\,V \quad \Rightarrow \quad n\,\mathrm{d}\,T = -\frac{p}{C_V}\,\mathrm{d}\,V \quad \text{(a)}$$

- Now from the ideal gas law ($p\,V = n\,R\,T$) we have

$$n\,R\,\mathrm{d}\,T = p\,\mathrm{d}\,V + V\,\mathrm{d}\,p \quad \Rightarrow \quad n\,\mathrm{d}\,T = \frac{p\,\mathrm{d}\,V + V\,\mathrm{d}\,p}{C_p - C_V} \quad \text{(b)} \quad \Leftarrow \quad R = C_p - C_V$$

- Combining (a) & (b) gives

$$\frac{\mathrm{d}\,p}{p} + \frac{C_p}{C_V}\frac{\mathrm{d}\,V}{V} = 0 \quad \Rightarrow \quad \ln p + \gamma \ln V = \text{a const} \quad \Rightarrow \quad p\,V^{\gamma} = \text{a constant}$$

$$\left[-\frac{p}{C_V}\, \mathrm{d}\,V = \frac{p\,\mathrm{d}\,V + V\,\mathrm{d}\,p}{C_p - C_V} \right] \times (C_p - C_V)$$

$$\Rightarrow \quad -\frac{C_p - C_V}{C_V}\, p\,\mathrm{d}\,V = p\,\mathrm{d}\,V + V\,\mathrm{d}\,p$$

$$\Rightarrow \quad (1 - \gamma)\, p\,\mathrm{d}\,V = p\,\mathrm{d}\,V + V\,\mathrm{d}\,p$$

$$\Rightarrow \quad \gamma\, p\,\mathrm{d}\,V + V\,\mathrm{d}\,p = 0$$

$$\Rightarrow \quad \int_i^f \left[\frac{\mathrm{d}\,p}{p} + \gamma\, \frac{\mathrm{d}\,V}{V} = 0 \right]$$

$$\Rightarrow \quad \ln p + \gamma \ln V = \mathrm{const}$$

- Since the gas goes from an initial state i to a final state f,

$$p_i V_i^{\gamma} = p_f V_f^{\gamma} \quad \text{adiabatic process}$$

- use the ideal gas equation ($pV = nRT$) to eliminate p,

$$\frac{nRT}{V} V^{\gamma} = \text{a constant} \quad \Rightarrow \quad \boxed{T V^{\gamma-1} = \text{a constant}} \quad \text{adiabatic process}$$

- When the gas goes from an initial state i to a final state f, we can rewrite

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \quad \text{adiabatic process}$$

- Understanding adiabatic processes allows you to understand why popping the cork on a cold bottle of champagne or the tab on a cold can of soda causes a slight fog to form at the opening of the container.
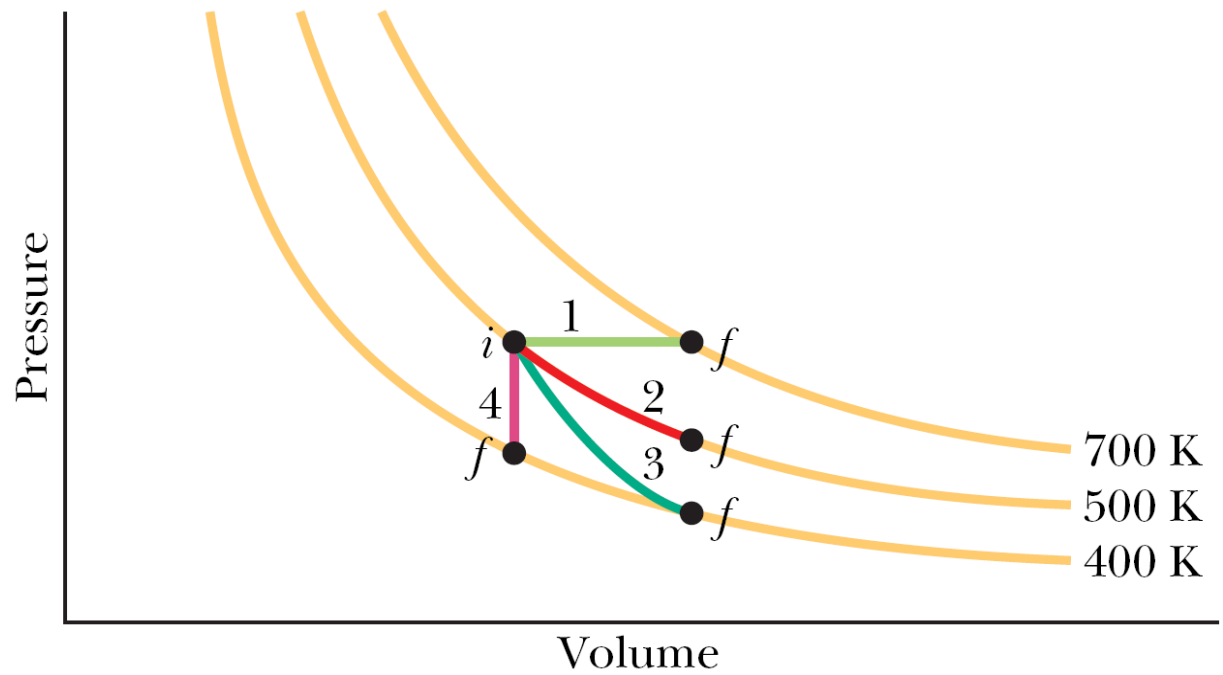
Free Expansions

- Since a free expansion is not in equilibrium, the equation of the adiabatic expansion can not apply to it.

- In a free expansion, because $\Delta E_{\text{int}} = 0$, the temperature of the final int state must be that of the initial state, ie, $T_i = T_f$, and $p_i V_i = p_f V_f$.

Problem 19-9, 19-10

Stopcock

Vacuum

Insulation
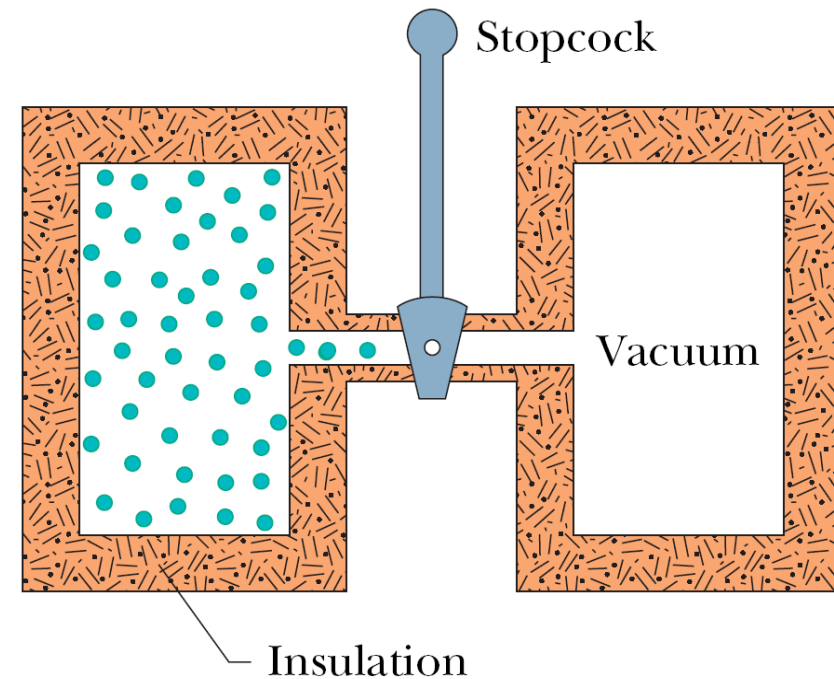
Four Special Processes

Some Special Results

Path	Constant Quantity	Process Type	$\Delta E_{int} = Q - W$ and $\Delta E_{int} = n C_V \Delta T$ for all paths
1	p	Isobaric	$Q = n C_p \Delta T ; \quad W = p \Delta V$
2	T	Isothermal	$Q = W = n R T \ln\left(V_f / V_i\right); \quad \Delta E_{int} = 0$
3	$p V^{\gamma}, T V^{\gamma-1}$	Adiabatic	$Q = 0 ; \quad W = -\Delta E_{int}$
4	V	Isochoric	$Q = \Delta E_{int} = n C_V \Delta T ; \quad W = 0$

Selected problems: 6, 14, 23, 41