Chapter 19 The Kinetic Theory of Gases

- 3 variables associated with a gas:

(1) **volume**: a result of the freedom the atoms have to spread throughout the container
(2) **pressure**: a result of the collisions of the atoms with the container's walls
(3) **temperature**: related to the kinetic energy of the atoms

Avogadro's Number

One mole is the number of atoms in a 12*g* sample of carbon-12.

- **Avogadro's number** $N_A = 6.02 \times 10^{23}/\text{mole}$ Avogadro's number

- Avogadro suggested that all gases occupying the same volume under the same conditions of temperature and pressure contain the same number of atoms or molecules.

- The number of moles $n = \dfrac{N}{N_A}$

- The number of moles n in a sample from the mass M_{sam} of the sample and either the *molar mass* M (the mass of 1 mol) or the molecular mass m (the mass of one molecule):

$$n = \frac{M_{\text{sam}}}{M} = \frac{M_{\text{sam}}}{m\,N_A} \quad \Leftarrow \quad M = m\,N_A$$

Ideal Gases

- At low enough densities, all real gases tend to obey the relation

$$pV = nRT \quad \text{ideal gas law}$$

R is the **gas constant** $R = 8.31\ \mathrm{J/mol\cdot K}$.

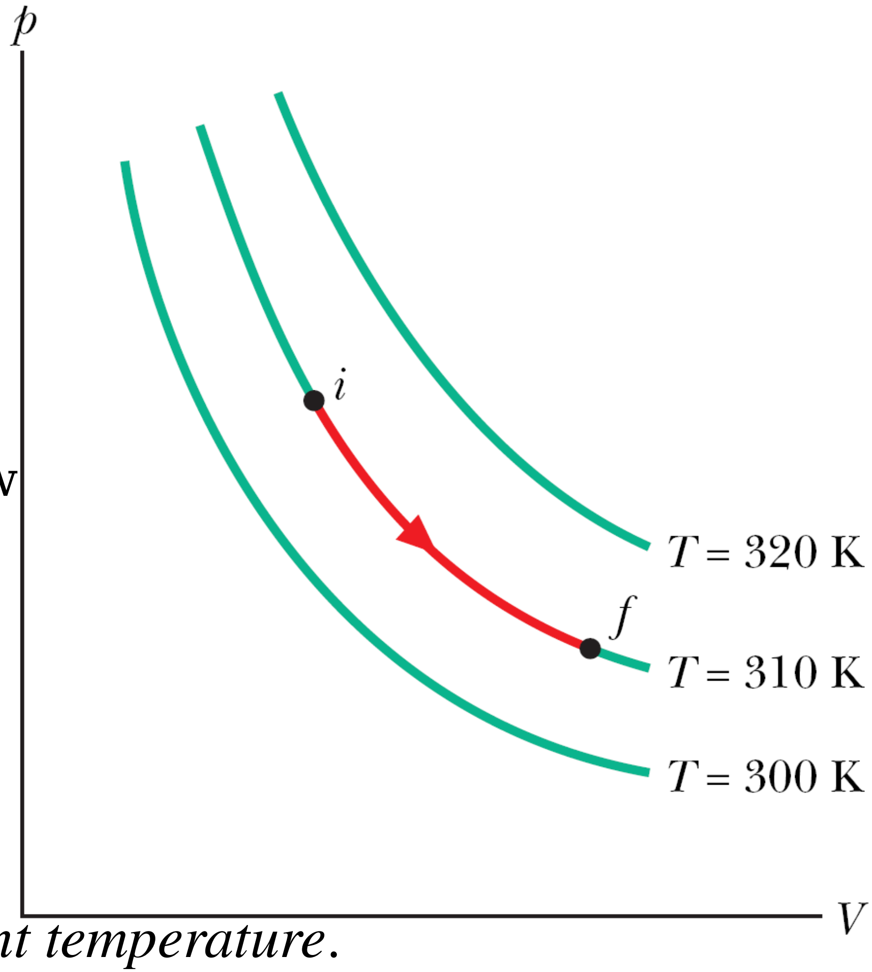
- We can rewrite the equation in terms of the **Boltzmann constant**,

$$k = \frac{R}{N_A} = \frac{8.31\ \mathrm{J/mol\cdot K}}{6.02\times10^{23}\ \mathrm{mol}^{-1}} = 1.38\times10^{-23}\ \mathrm{J/K}$$

$$\Rightarrow \quad nR = Nk \quad \Rightarrow \quad pV = NkT \quad \text{ideal gas law}$$

- An ideal gas is defined as a gas which behavior follows the above equation.

- *All real* gases approach the ideal state at low enough densities and at high enough temperature.



Work Done by an Ideal Gas at Constant Temperature

- **Isothermal expansion** (or **isothermal compression**): A process carried out from an initial volume V_i to a final volume V_f at *constant temperature*.

- On a *p*-*V* diagram, an *isotherm* is a curve that connects points that have the same temperature, ie, a graph of pressure versus volume for a gas whose temperature T is held constant: $p = \frac{nRT}{V} = \frac{\text{constant}}{V}$
- The isotherms with different temperatures doesn't cross each other.
- The work done by an ideal gas during an isothermal process

$$W = \int_{V_i}^{V_f} p\,\mathrm{d}V = \int_{V_i}^{V_f} \frac{nRT}{V}\,\mathrm{d}V \quad \Leftarrow \quad pV = nRT$$

- For an isothermal process: $T = \text{const} \quad \Rightarrow \quad W = nRT \int_{V_i}^{V_f} \frac{\mathrm{d}V}{V} = nRT \ln V \Big|_{V_i}^{V_f}$

$$\Rightarrow \quad W = nRT \ln \frac{V_f}{V_i} \quad \text{ideal gass, isothermal process}$$

- For $V_f > V_i$ (expansion), the work W done by an ideal gas is positive;
  For $V_f < V_i$ (compression), the work W done by an ideal gas is negative.

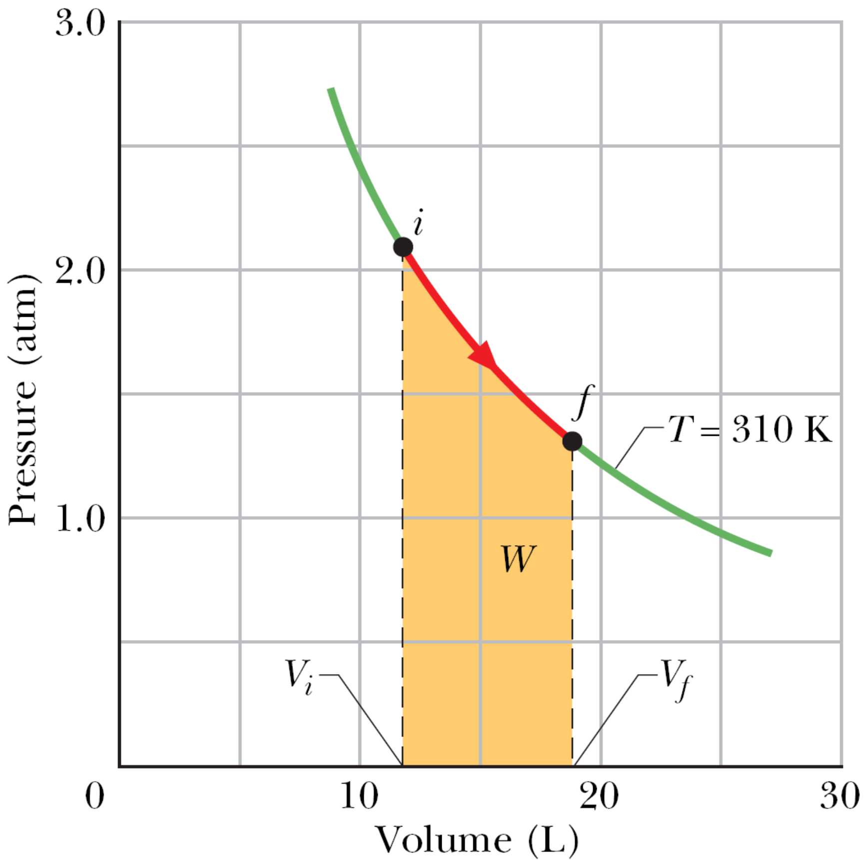
Work Done at Constant Volume and at Constant Pressure

- If the volume of the gas is constant, then $W = 0$ constant-volume process

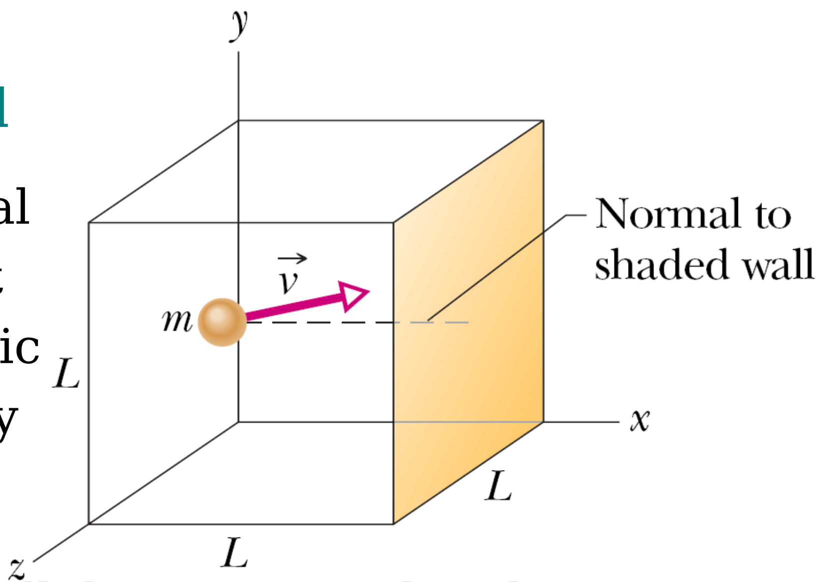
- If the volume changes while the pressure p of the gas is held constant, then

$$W = p\left(V_f - V_i\right) = p\,\Delta V \quad \text{constant-pressure process}$$

problem 19-1
problem 19-2



Pressure, Temperature, and RMS Speed



- n moles of an ideal gas are confined in a cubical box of volume V. The walls of the box are held at temperature T. We need to relate the macroscopic quantity - pressure p, to the microscopic quantity — the velocity of the molecule v.

- The molecules of gas in the box are moving in all directions and with various speeds, bouncing from the walls of the box.

- Ignore collisions of the molecules with one another and consider only elastic collisions with the walls.

- The change in the particle's momentum along the axis is

$$\Delta p_x = (-m v_x) - m v_x = -2 m v_x$$

- The momentum Δp_x delivered to the wall by the molecule is $+2mv_x$.

- The molecule of will hit the wall repeatedly. The time Δt between collisions is the time the molecule takes to travel to the opposite wall and back again (a distance $2L$) at speed v_x

$$\Delta t = \frac{2L}{v_x}$$

- The average rate at which momentum is delivered to the wall by this single molecule is

$$\frac{\Delta p_x}{\Delta t} = \frac{2\, m\, v_x}{2\, L / v_x} = \frac{m\, v_x^2}{L}$$

- From Newton's 2nd law, the rate at which momentum is delivered to the wall is the force acting on that wall.

- Dividing the magnitude of the total force F_x by the area of the wall ($= L^2$) gives the pressure p on that wall.

- We can write this pressure: $p = \frac{F_x}{L^2} = \frac{\frac{m\, v_{x\,1}^2}{L} + \cdots + \frac{m\, v_{x\,\mathrm{N}}^2}{L}}{L^2} = \frac{m}{L^3} \sum_{i=1}^{N} v_{x\,i}^2$

$$\left(v_x^2\right)_{\text{avg}} = \frac{1}{N} \sum_{i=1}^{N} v_{xi}^2\,, \quad N = n\, N_A \ \text{ and } \ M = N_A\, m \quad \text{the molar mass}$$

$$\Rightarrow \quad p = \frac{m}{L^3} \sum_{i=1}^{N} v_{x\,i}^2 = \frac{m}{V}\, N \left(v_x^2\right)_{\text{avg}} = \frac{n\, m\, N_A}{V} \left(v_x^2\right)_{\text{avg}} = \frac{n\, M \left(v_x^2\right)_{\text{avg}}}{V}$$

- For any molecule, $v^2 = v_x^2 + v_y^2 + v_z^2$. Because there are many molecules and because they are all moving in random directions, the average values of the squares of their velocity components are equal, so $v_x^2 = \frac{v^2}{3} \quad \Rightarrow \quad p = \frac{n\, M \left(v^2\right)_{\text{avg}}}{3\, V}$

- **Root-mean-square speed** $v_{\text{rms}} \equiv \sqrt{\left(v^2\right)_{\text{avg}}} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} v_i^2} \quad \Rightarrow \quad p = \frac{n\, M\, v_{\text{rms}}^2}{3\, V}$

- The equation tells how the pressure of the gas (a purely macroscopic quantity) depends on the speed of the molecules (a purely microscopic quantity).
- Using the ideal gas law ($p\,V = n\,R\,T$) gives $v_{\text{rms}} = \sqrt{\frac{3\,R\,T}{M}}$
- The rms speed is only a kind of average speed; many molecules move much faster than this, and some much slower.
- The speed of sound in a gas is closely related to the rms speed of the molecules of that gas.
- If molecules move so fast, why does it take as long as a minute or so before you can smell perfume when someone opens a bottle across a room?

problem 19-3

Translational Kinetic Energy

- Consider a single molecule of an ideal gas, Its average translational kinetic energy over the time is

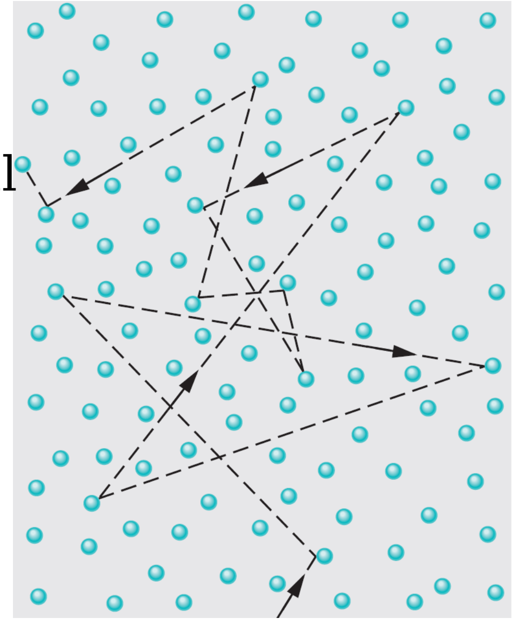
$$K_{\text{avg}} = \left(\frac{1}{2} m v^2\right)_{\text{avg}} = \frac{1}{2} m \left(v^2\right)_{\text{avg}} = \frac{1}{2} m v_{\text{rms}}^2 = \frac{1}{2} m \frac{3RT}{M} = \frac{3RT}{2N_A}$$

$$k = \frac{R}{N_A} \quad \Rightarrow \quad \boxed{K_{\text{avg}} = \frac{3}{2} kT}$$

At a given temperature T, all ideal gas molecules — no matter what their mass — have the same average translational kinetic energy — namely, $\frac{3}{2} kT$. When we measure the temperature of a gas, we are also measuring the average translational kinetic energy of its molecules.

Mean Free Path

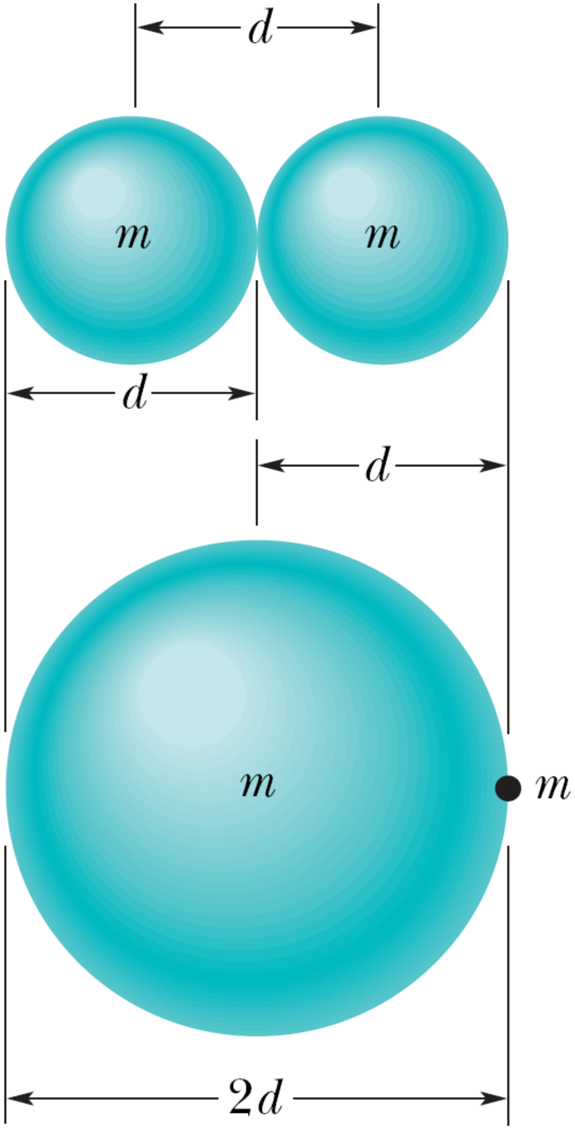
- The **mean free path** λ: is one useful parameter to describe this random motion of the molecules; it is the average distance traversed by a molecule between collisions.



- We expect (1) $\lambda \propto \frac{V}{N}$ (2) $\lambda \propto \frac{1}{d^2}$,

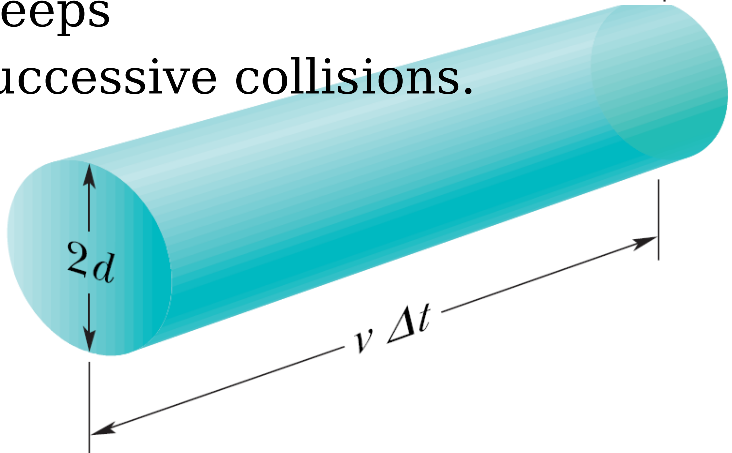
the cross section of a molecule determines its effective target area, thus

$$\boxed{\lambda = \frac{V}{\sqrt{2}\,\pi\, N\, d^2} \quad \text{mean free path}}$$



justification

- A collision will take place if the centers of 2 molecules come within a distance d of each other.
- As a single molecule zigzags through the gas, it sweeps out a cylinder of cross-sectional area πd^2 between successive collisions.



- For a time interval Δt, it moves a distance $v_{\text{avg}} \Delta t$.
- The volume of the cylinder is $(\pi d^2)(v_{\text{avg}} \Delta t)$.

- The number of collisions that occur in time Δt is equal to the number of molecules that lie within this cylinder,

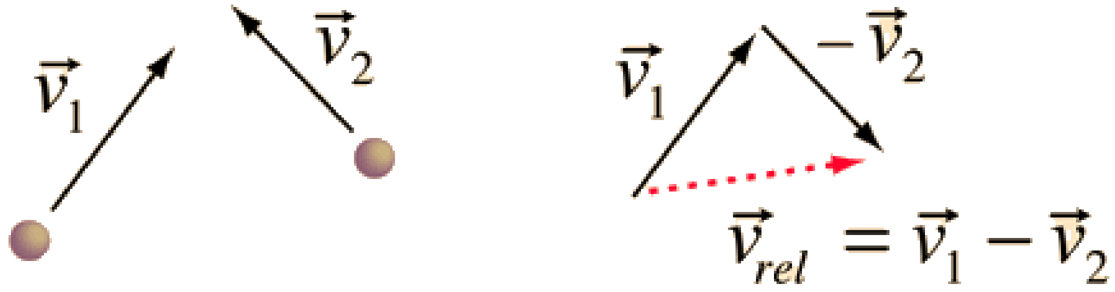
$$\lambda = \frac{\text{length of path during } \Delta t}{\text{number of collisions in } \Delta t} = \frac{v_{\text{avg}} \Delta t}{\pi d^2 v_{\text{rel}} \Delta t \, N / V} = \frac{v_{\text{avg}}}{\pi d^2 v_{\text{rel}} N / V}$$

v_{avg}: the mean speed of the molecules *relative to the container*;
v_{rel} : the mean speed of a single molecule *relative to the other molecules*.

- A detailed calculation, taking into account the actual speed distribution of the molecules, gives $v_{\text{rel}} = \sqrt{2}\, v_{\text{avg}} \quad \Rightarrow \quad \lambda = \dfrac{V}{\sqrt{2}\,\pi N d^2}$

- The mean free path of air molecules: 0.1μm for H=0; 16cm for H=100km; 20km for H=300km.

Random molecular velocities



$$v_{\text{rel}}^2 = \vec{v}_{\text{rel}}^{\,2} = (\vec{v}_1 - \vec{v}_2)\cdot(\vec{v}_1 - \vec{v}_2) = \vec{v}_1\cdot\vec{v}_1 - 2\,\vec{v}_1\cdot\vec{v}_2 + \vec{v}_2\cdot\vec{v}_2$$

$$\Rightarrow \quad \overline{v}_{\text{rel}} = \sqrt{\overline{\vec{v}_1\cdot\vec{v}_1 - 2\,\vec{v}_1\cdot\vec{v}_2 + \vec{v}_2\cdot\vec{v}_2}}$$

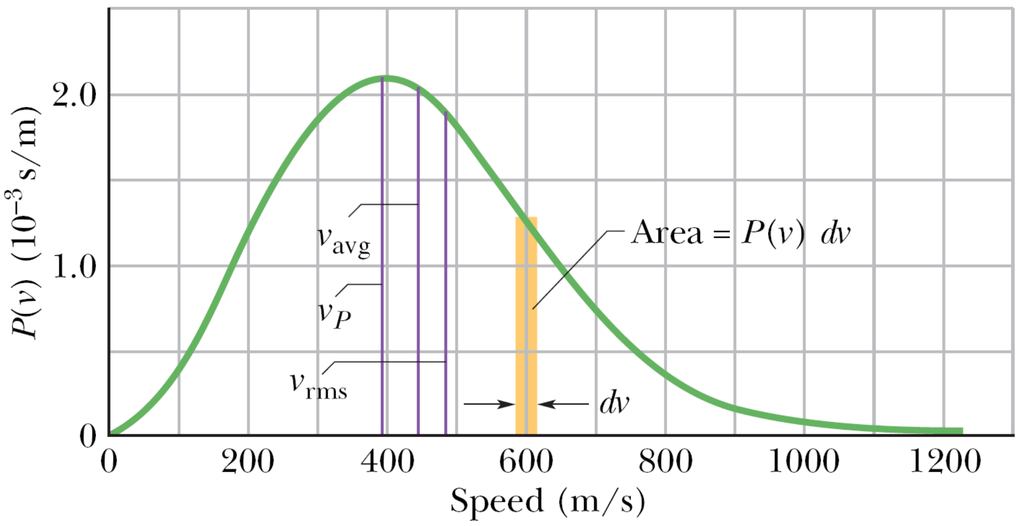
$$= \sqrt{\overline{\vec{v}_1\cdot\vec{v}_1} - \cancel{2\,\overline{\vec{v}_1\cdot\vec{v}_2}} + \overline{\vec{v}_2\cdot\vec{v}_2}}$$

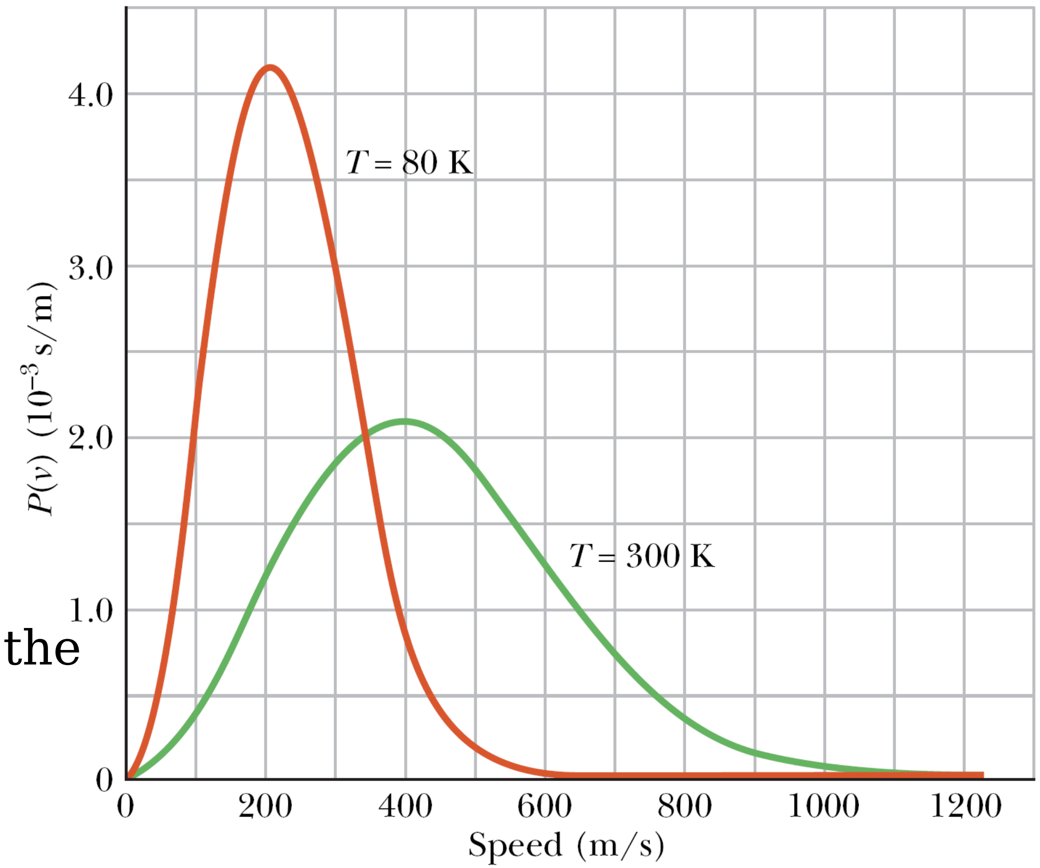
$$= \sqrt{\overline{v}_1^2 + \overline{v}_2^2}$$

$$= \sqrt{\overline{v}^2 + \overline{v}^2}$$

$$= \sqrt{2}\,\overline{v}$$

The Distribution of Molecular Speeds





- **Maxwell's speed distribution law**, the speed distribution of gas molecules, is

$$P(v) = \sqrt{\frac{2M^3}{\pi R^3 T^3}}\, v^2 e^{-\frac{M v^2}{2RT}}$$

- The quantity $P(v)$ is a probability distribution function: For any speed v, the product $P(v)\mathrm{d}v$ (a dimensionless quantity) is the fraction of molecules with speeds in the interval $\mathrm{d}v$ centered on speed v,

$$\int_0^{\infty} P(v)\,\mathrm{d}\,v = 1$$

- The fraction (frac) of molecules with speeds in an interval of v_1 to v_2 is

$$\text{frac} = \int_{v_1}^{v_2} P(v)\,\mathrm{d}\,v$$

Average, RMS, and Most Probable Speeds

- find the **average speed** v_{avg} of the molecules in a gas with the distribution law

$$v_{\text{avg}} = \int_0^\infty v\,P(v)\,\mathrm{d}v$$

- Substituting for $P(v)$ from the Maxwell's speed distribution law and find

$$\boxed{v_{\text{avg}} = \sqrt{\frac{8RT}{\pi M}}} \quad \text{average speed}$$

- Similarly, $(v^2)_{\text{avg}} = \int_0^\infty v^2 P(v)\,\mathrm{d}v = \frac{3RT}{M} \Rightarrow \boxed{v_{\text{rms}} = \sqrt{(v^2)_{\text{avg}}} = \sqrt{\frac{3RT}{M}}}$ rms speed

- The **most probable speed** v_P is the speed at which $P(v)$ is maximum, therefore

solve it to find $\frac{\mathrm{d}P}{\mathrm{d}v} = 0 \quad \Rightarrow \quad \boxed{v_P = \sqrt{\frac{2RT}{M}}}$ most probable speed

- Some molecules will have speeds that are many times of v_P. These molecules form the *high-speed tail* of a distribution curve, useful for rain & sunshine.

problem 19-5, 19-6

Gaussian Integral

$$\int_{-\infty}^{+\infty} e^{-x^2}\,\mathrm{d}x = 2\int_{0}^{\infty} e^{-x^2}\,\mathrm{d}x = \sqrt{\pi}$$

Proof:

$$\left(\int_{-\infty}^{+\infty} e^{-x^2}\,\mathrm{d}x\right)^2 = \int_{-\infty}^{+\infty} e^{-x^2}\,\mathrm{d}x \int_{-\infty}^{+\infty} e^{-y^2}\,\mathrm{d}y = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} e^{-(x^2+y^2)}\,\mathrm{d}x\,\mathrm{d}y$$

$$= \int_{0}^{\infty}\int_{0}^{2\pi} e^{-r^2}(r\,\mathrm{d}\theta)(\mathrm{d}r) \quad \Leftarrow \quad \begin{aligned} & r^2 = x^2 + y^2 \\ & \mathrm{d}x\,\mathrm{d}y = (r\,\mathrm{d}\theta)(\mathrm{d}r) = r\,\mathrm{d}r\,\mathrm{d}\theta \end{aligned}$$

$$= \int_{0}^{2\pi}\mathrm{d}\theta \int_{0}^{\infty} e^{-r^2} r\,\mathrm{d}r = 2\pi\cdot\frac{1}{2}\int_{0}^{\infty} e^{-q}\,\mathrm{d}q \quad \Leftarrow \quad q \equiv r^2$$

$$= -\pi e^{-q}\Big|_{0}^{\infty} = \pi$$

$$s \equiv \sqrt{\frac{2\,M^3}{\pi\,R^3\,T^3}}\,, \quad u^2 = \frac{2\,R\,T}{M} \quad \Rightarrow \quad P(v) = s\,v^2\,e^{-\frac{v^2}{u^2}}$$

$$\int_0^\infty P(v)\,\mathrm{d}\,v = s\int_0^\infty v^2\,e^{-\frac{v^2}{u^2}}\,\mathrm{d}\,v = \frac{s}{2}\int_0^\infty v\,e^{-\frac{v^2}{u^2}}\,\mathrm{d}\,v^2 = -\frac{s\,u^3}{2}\int_0^\infty \frac{v}{u}\,\mathrm{d}\,e^{-\frac{v^2}{u^2}}$$

$$= -\frac{s\,u^3}{2}\left(\cancel{\frac{v}{u}\,e^{-\frac{v^2}{u^2}}}\bigg|_0^\infty - \int_0^\infty e^{-\frac{v^2}{u^2}}\,\mathrm{d}\,\frac{v}{u}\right) = \frac{s\,u^3}{2}\int_0^\infty e^{-\frac{v^2}{u^2}}\,\mathrm{d}\,\frac{v}{u} = \frac{s\,u^3}{2}\,\frac{\sqrt{\pi}}{2} = 1$$

$$\bar{v} = \int_0^\infty v\,P(v)\,\mathrm{d}\,v = s\int_0^\infty v^3\,e^{-\frac{v^2}{u^2}}\,\mathrm{d}\,v = \frac{s\,u^4}{2}\int_0^\infty q\,e^{-q}\,\mathrm{d}\,q \quad \Leftarrow \quad q \equiv \frac{v^2}{u^2}$$

$$= -\frac{s\,u^4}{2}\left(\cancel{q\,e^{-q}}\bigg|_0^\infty - \int_0^\infty e^{-q}\,\mathrm{d}\,q\right) = -\frac{s\,u^4}{2}\,e^{-q}\bigg|_0^\infty = \sqrt{\frac{8\,R\,T}{\pi\,M}}$$

$$\overline{v^2} = \int_0^\infty v^2\,P(v)\,\mathrm{d}\,v = s\int_0^\infty v^4\,e^{-\frac{v^2}{u^2}}\,\mathrm{d}\,v = \frac{s\,u^5}{2}\int_0^\infty x^3\,e^{-x^2}\,\mathrm{d}\,x^2 \quad \Leftarrow \quad x \equiv \frac{v}{u}$$

$$= -\frac{s\,u^5}{2}\left(\cancel{x^3\,e^{-x^2}}\bigg|_0^\infty - \frac{3}{2}\int_0^\infty x\,e^{-x^2}\,\mathrm{d}\,x^2\right) = -\frac{3\,s\,u^5}{4}\int_0^\infty x\,\mathrm{d}\,e^{-x^2} = \frac{3\,s\,u^5}{4}\,\frac{2}{s\,u^3} = \frac{3\,R\,T}{M}$$

The Molar Specific Heats of an Ideal Gas

Internal Energy $\mathscr{E}_{\text{int}}$

- Assume that our ideal gas is a *monatomic gas* and the internal energy $\mathscr{E}_{\text{int}}$ of the ideal gas is simply the sum of the translational kinetic energies of its atoms.

- The average translational kinetic energy of a single atom depends only on the gas temperature, ie, $K_{\text{avg}} = \frac{3}{2} k T$. The internal energy of a sample of n moles of the gas is $\mathscr{E}_{\text{int}} = (n N_A) K_{\text{avg}} = (n N_A)\left(\frac{3}{2} k T\right) = (n N_A)\left(\frac{3R}{2N_A} T\right)$

$$\Rightarrow \quad \mathscr{E}_{\text{int}} = \frac{3}{2} n R T \quad \text{monatomic ideal gas}$$

The internal energy $\mathscr{E}_{\text{int}}$ of an ideal gas is a function of the gas temperature only; it does not depend on any other variable.

- 2 types of the molar specific heat of an ideal gas:
  C_V: molar specific heat at constant volume;
  C_p: molar specific heat at constant pressure.

Molar Specific Heat at Constant Volume

- The heat Q is related to the temperature change ΔT by

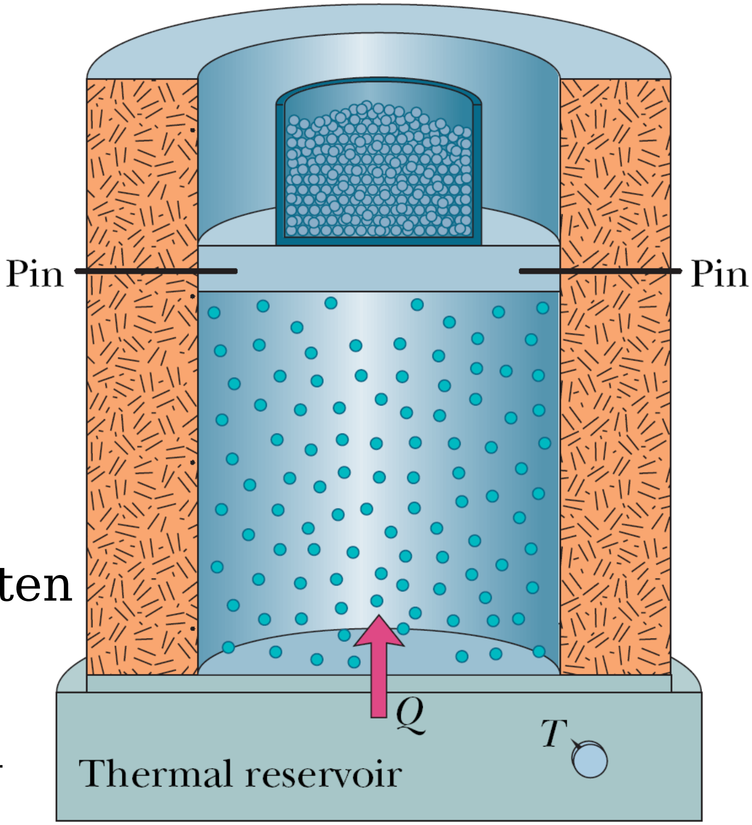
$$Q = n C_V \Delta T \quad \text{constant volume}$$

- The 1st law of thermodynamics gives

$$\Delta \mathscr{E}_{\text{int}} = Q - W = n\, C_V\, \Delta T - W = n\, C_V\, \Delta T$$

$$\Rightarrow \quad C_V = \frac{\Delta E_{\text{int}}}{n\, \Delta T} = \frac{3\, n\, R\, \Delta T/2}{n\, \Delta T}$$

$$\Rightarrow \quad C_V = \frac{3}{2} R = 12.5\ \text{J}/\text{mol}\cdot\text{K} \quad \text{monatomic gas}$$



- The internal energy of any ideal gas can be rewritten

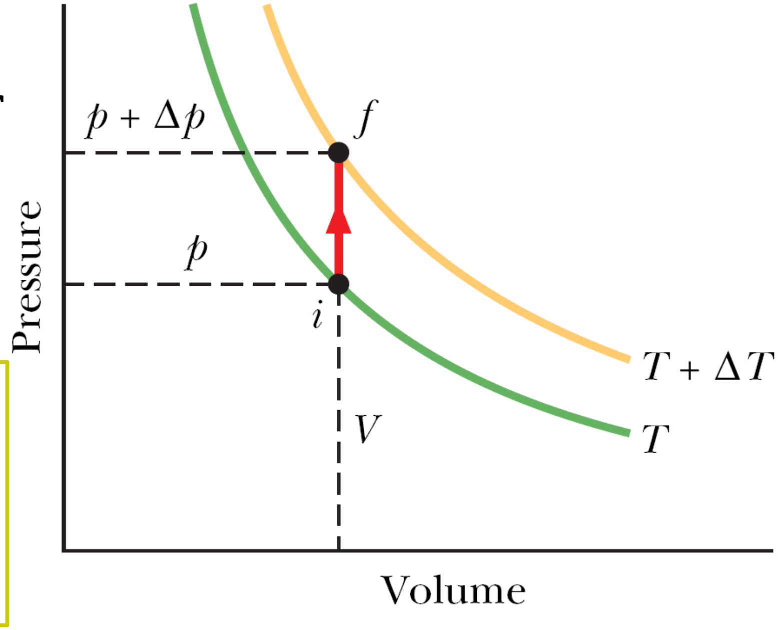
$$\mathscr{E}_{\text{int}} = n\, C_V\, T \quad \text{any idea gas}$$

- This equation applies to monatomic, diatomic, and polyatomic ideal gases, provided the appropriate value of C_V is used.

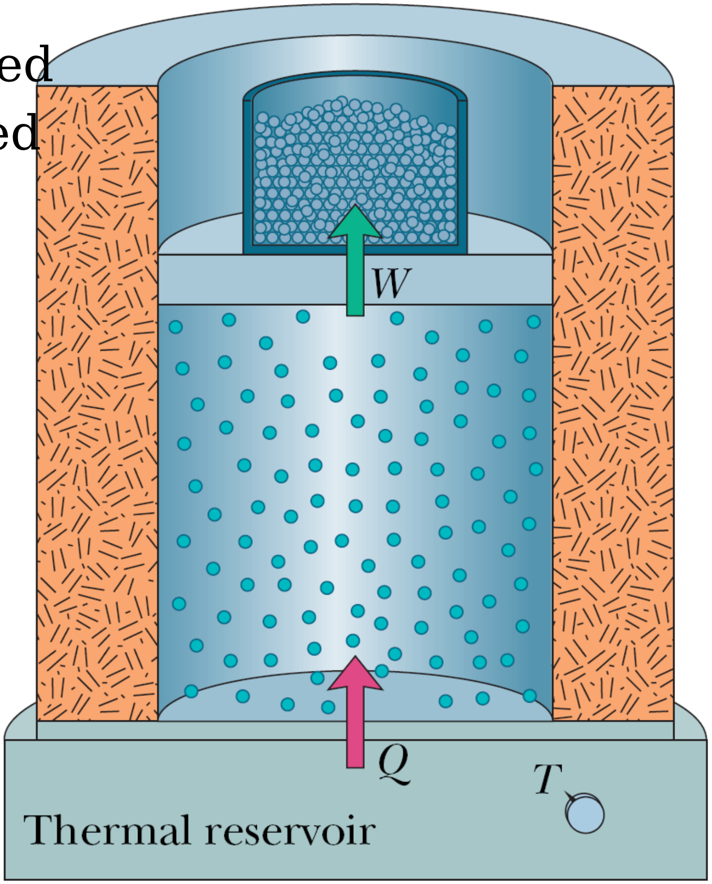
- When an ideal gas that is confined to a container undergoes a temperature change ΔT, then the change in its internal energy as is

$$\Delta \mathscr{E}_{\text{int}} = n\, C_V\, \Delta T \quad \text{ideal gas, any process}$$

A change in the internal energy $\mathscr{E}_{\text{int}}$ of a confined ideal gas depends on the change in the gas temperature only; it does not depend on what type of process produces the change in the temperature.



- Although the values of heat Q and work W associated with these 3 paths differ, the values of $\Delta\mathcal{E}_{\text{int}}$ associated with them are identical because they all involve the same temperature change ΔT.



Molar Specific Heat at Constant Pressure

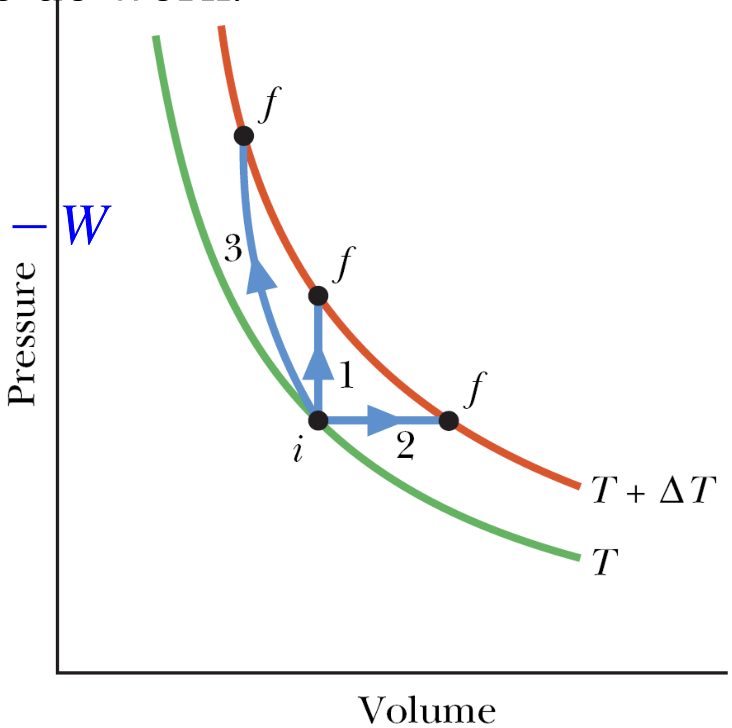
- Q is related to the temperature change ΔT by

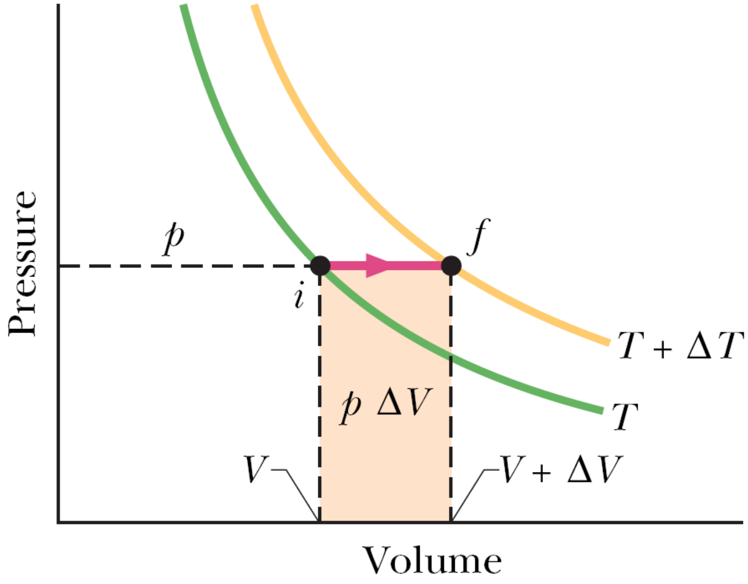
$$Q = n\,C_p\,\Delta T \quad \text{constant pressure}$$

- This C_p is greater than the molar specific heat at constant volume C_V, because energy must now be supplied not only to raise the temperature of the gas but also for the gas to do work.

- From the 1st law of Thermodynamics

$$n\,C_V\,\Delta T \Leftarrow \Delta E_{\text{int}} = Q - W$$
$$= n\,C_p\,\Delta T - p\,\Delta V$$
$$= n\,C_p\,\Delta T - n\,R\,\Delta T$$
$$\Rightarrow \quad C_V = C_p - R$$
$$\Rightarrow \quad C_p = C_V + R$$





Problem 19-7

Degrees of Freedom and Molar Specific Heats

- To explain the discrepancy by considering the possibility that molecules with more than one atom can store internal energy in forms other than translational kinetic energy.

- Assume that all 3 types(monatomic, diatomic, polyatomic) of molecules can have translational motions, and rotational motions.

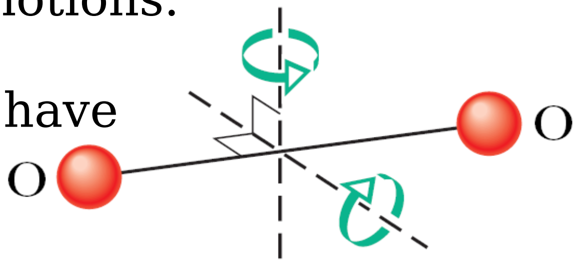
- Assume that the diatomic and polyatomic molecules can have oscillatory motions.

- The theorem of the **equipartition of energy**:

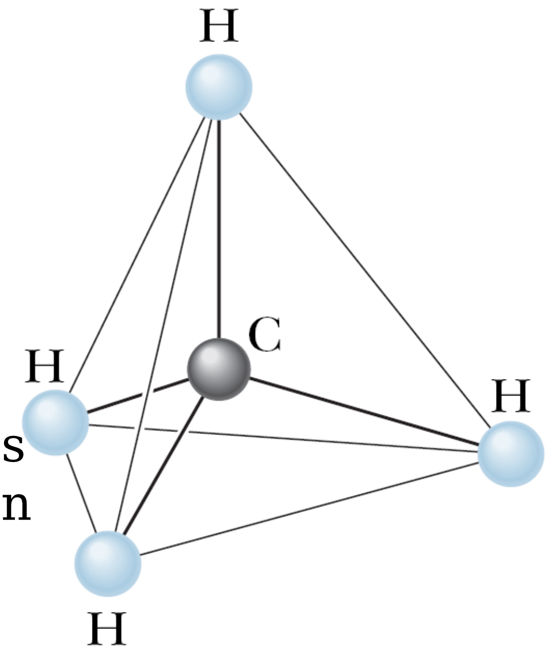
Every kind of molecule has a certain number f, *degrees of freedom*, which are independent ways in which the molecule can store energy. Each such degree of freedom has associated with it - on average - an energy of $\frac{kT}{2}$ per molecule (or $\frac{RT}{2}$ per mole).

- For the translational motion, the molecules have velocity components along xyz axes. Thus, gas molecules of all types have 3 degrees of translational freedom and, on average, an associated energy of $3\left(\frac{kT}{2}\right)$ per molecule.

(a) **He**



(b) $\mathbf{O_2}$



(c) $\mathbf{CH_4}$

- A monatomic gas molecule does not rotate and so has no rotational energy.

- A diatomic molecule can rotate only about axes perpendicular to the line connecting the atoms and have only 2 degrees of rotational freedom and a rotational energy of only $2\left(\frac{kT}{2}\right)$ per molecule.

- Only the polyatomic molecules are able to rotate with an angular velocity component along each of the 3 axes, and have 3 degrees of rotational freedom, and an energy of $3\left(\frac{kT}{2}\right)$ per molecule.

- Now the internal energy: $E_{\text{int}} = \frac{f}{2} nRT \quad \Rightarrow \quad C_V = \frac{f}{2} R = 4.16\, f \text{ J/mol}\cdot\text{K}$

where f is the number of degrees of freedom.

- $f = 3$ for monatomic gases, $f = 5$ for diatomic gases, $f = 6$ for polyatomic gases.

Degrees of Freedom for Various Molecules

		Degree of Freedom			specific heat	
Molecule	Example	Translational	Rotational	Total (f)	C_V	C_p
monatomic	He	3	0	3	$\frac{3}{2}R$	$\frac{5}{2}R$
diatomic	O_2	3	2	5	$\frac{5}{2}R$	$\frac{7}{2}R$
polyatomic	CH_4	3	3	6	$3R$	$4R$

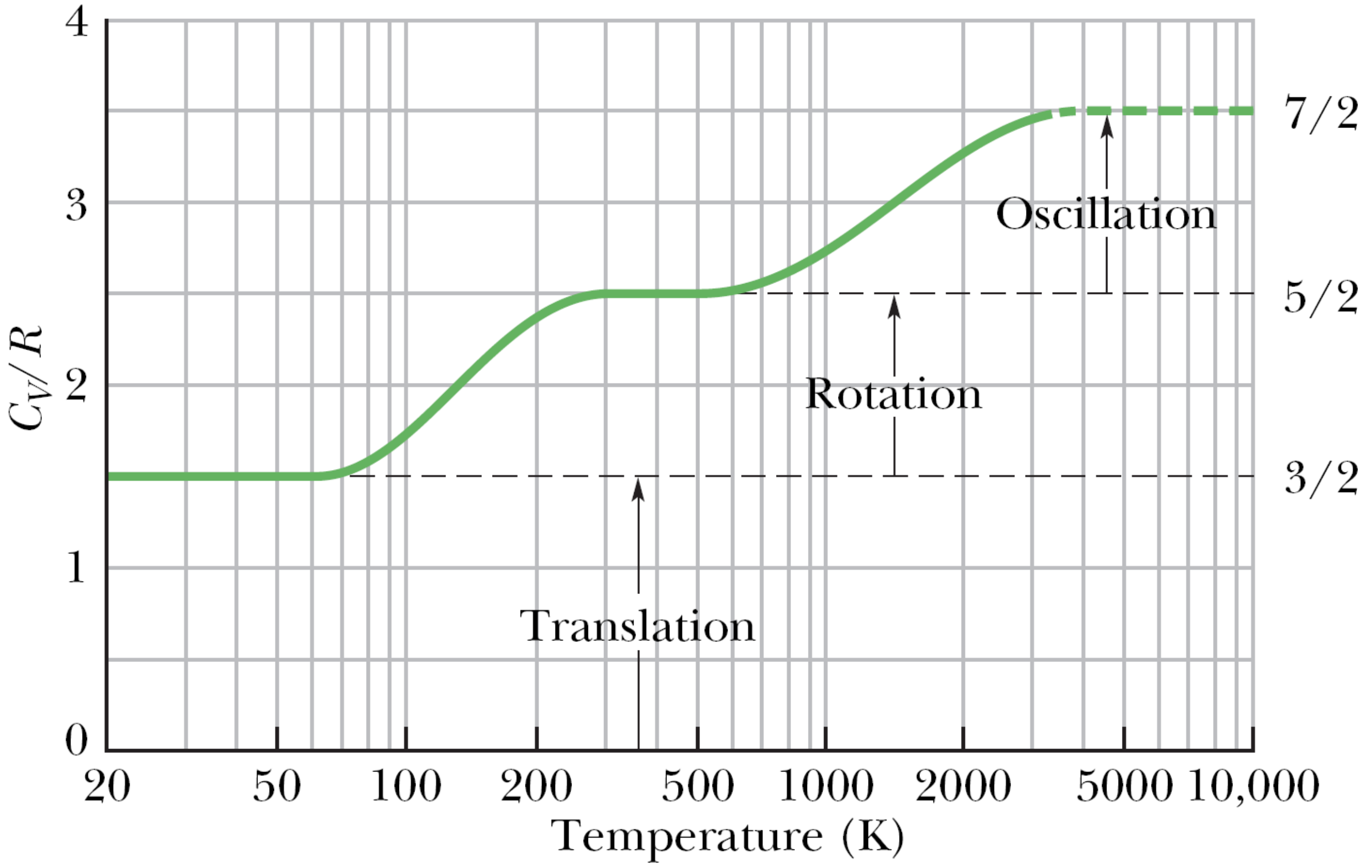
problem 19-8

A Hint of Quantum Theory

- Improve the agreement of kinetic theory by including the oscillations of the atoms in a gas of diatomic or polyatomic molecules.

- Experiment shows that such oscillations are "turned on" only when the gas molecules have relatively large energies.

- Quantum theory shows that rotational motion of the molecules requires a certain minimum amount of energy; oscillatory motion of the molecules requires a certain (higher) minimum amount of energy.



The Adiabatic Expansion of an Ideal Gas

- Ensure that $Q = 0$ either by carrying out the process very quickly (as in sound waves) or by doing it (at any rate) in a well-insulated container.

- The relation between the pressure and the volume during an adiabatic process

$$\boxed{p V^{\gamma} = \text{a constant}} \quad \text{adiabatic process}$$

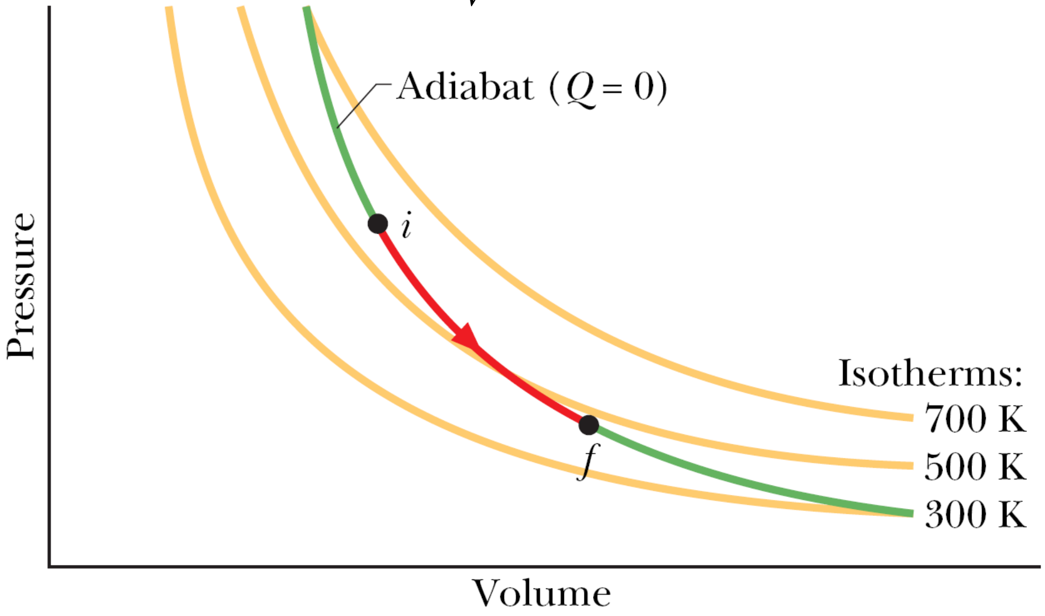
in which $\gamma = \dfrac{C_p}{C_V}$, the ratio of the molar specific heats for the gas.

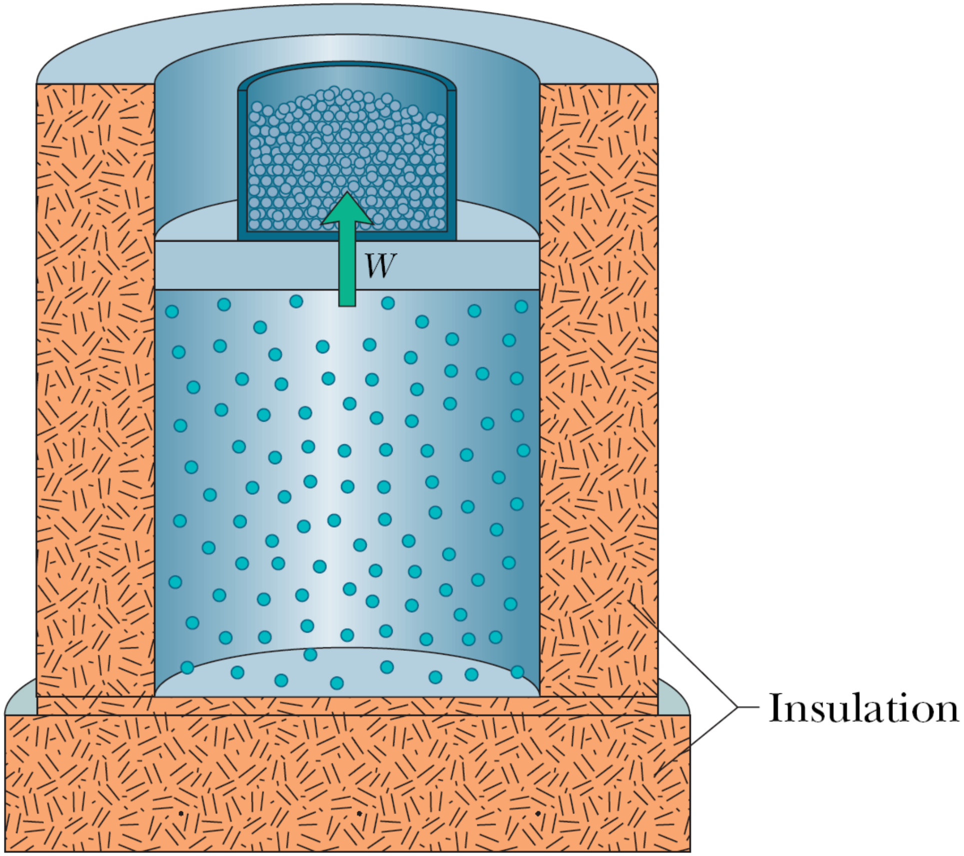
Proof

- The 1st law of thermodynamics:

$$n C_V \,\mathrm{d}T \Leftarrow \mathrm{d}E_{\text{int}} = Q - p\,\mathrm{d}V = -p\,\mathrm{d}V$$

$$\Rightarrow \quad n\,\mathrm{d}T = -\frac{p}{C_V}\,\mathrm{d}V \qquad \text{(a)}$$





- Now from the ideal gas law ($pV = nRT$) we have

$$nR\,\mathrm{d}T = p\,\mathrm{d}V + V\,\mathrm{d}p \quad \Rightarrow \quad n\,\mathrm{d}T = \frac{p\,\mathrm{d}V + V\,\mathrm{d}p}{C_p - C_V} \quad \text{(b)} \quad \Leftarrow \quad R = C_p - C_V$$

- Combining (a) & (b) gives

$$\left(-\frac{p}{C_V}\mathrm{d}V = \frac{p\,\mathrm{d}V + V\,\mathrm{d}p}{C_p - C_V}\right) \times \left(C_p - C_V\right) \quad \Rightarrow \quad -\frac{C_p - C_V}{C_V}\,p\,\mathrm{d}V = p\,\mathrm{d}V + V\,\mathrm{d}p$$

$$\Rightarrow \quad (1-\gamma)\,p\,\mathrm{d}V = p\,\mathrm{d}V + V\,\mathrm{d}p \quad \Rightarrow \quad \gamma\,p\,\mathrm{d}V + V\,\mathrm{d}p = 0$$

$$\Rightarrow \quad \int_i^f \left(\frac{\mathrm{d}p}{p} + \gamma\frac{\mathrm{d}V}{V} = 0\right) \quad \Rightarrow \quad \ln p + \gamma \ln V = \text{const} \quad \Rightarrow \quad pV^\gamma = \text{a constant}$$

- Since the gas goes from an initial state i to a final state f,

$$p_i V_i^\gamma = p_f V_f^\gamma \quad \text{adiabatic process}$$

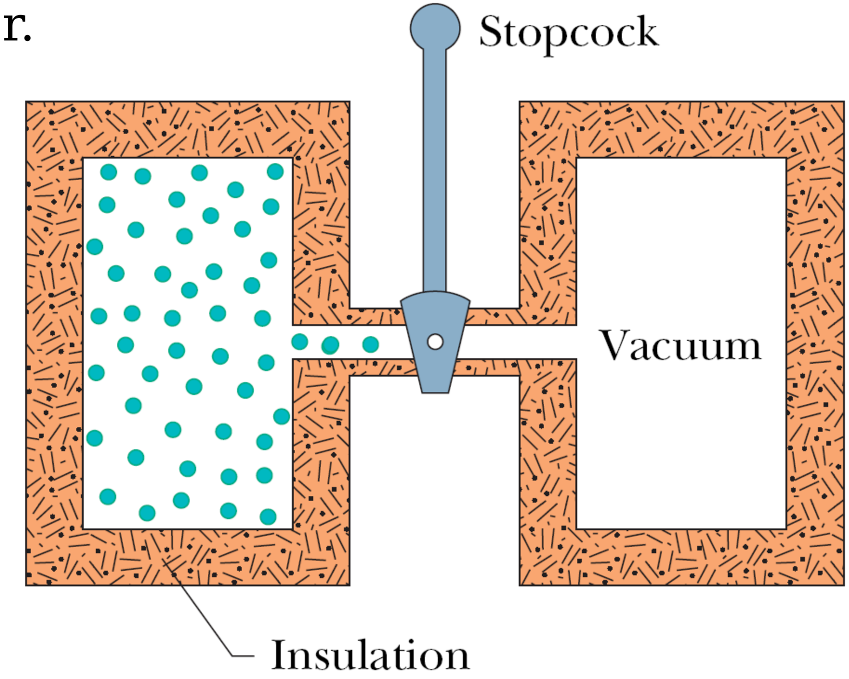
- Use the ideal gas equation ($pV = nRT$) to eliminate p,

$$\frac{nRT}{V}V^\gamma = \text{a constant} \quad \Rightarrow \quad TV^{\gamma-1} = \text{a constant} \quad \text{adiabatic process}$$

- When the gas goes from an initial state i to a final state f, we can rewrite

$$T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1} \quad \text{adiabatic process}$$

- Understanding adiabatic processes allows you to understand why popping the cork on a cold bottle of champagne or the tab on a cold can of soda causes a slight fog to form at the opening of the container.



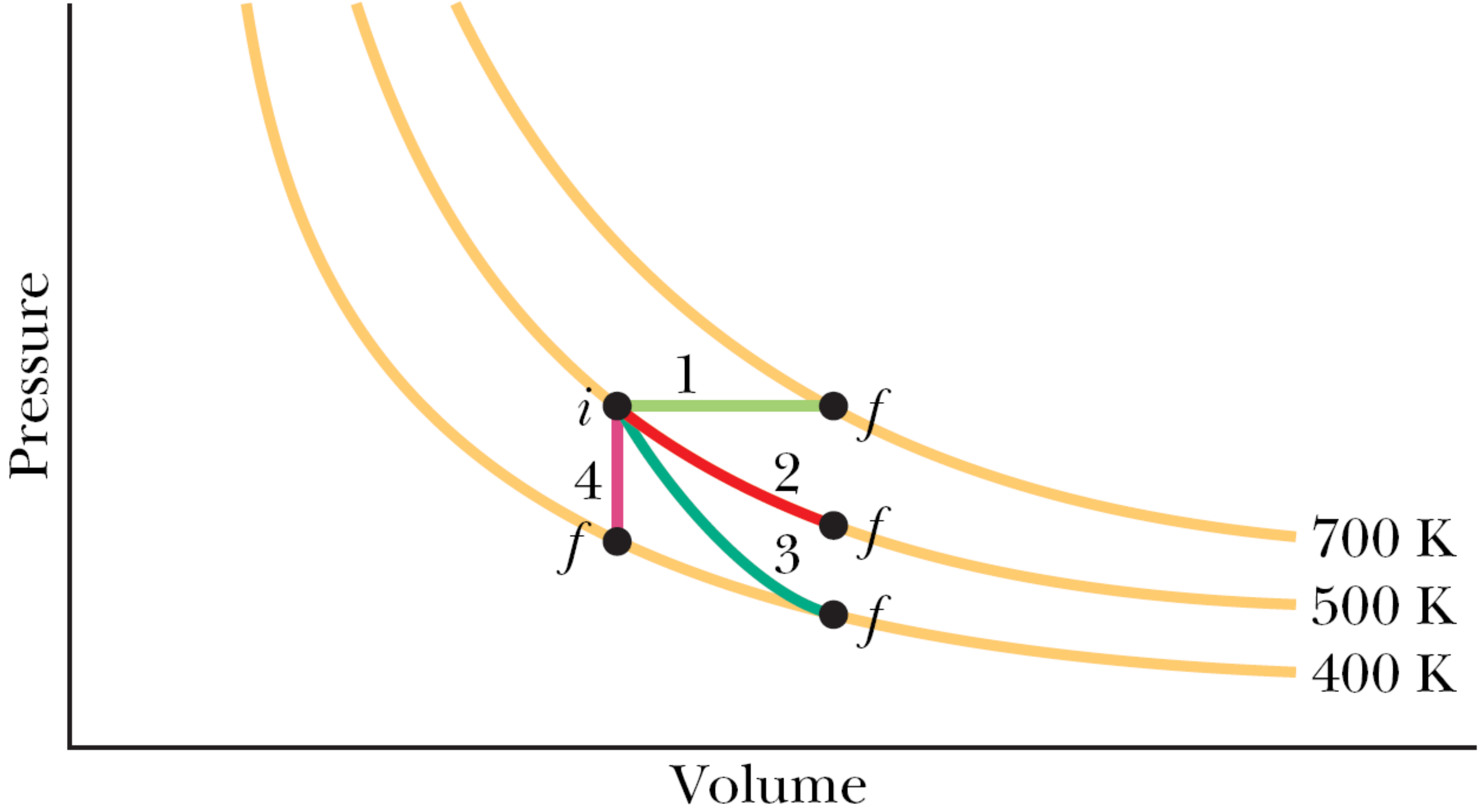
Free Expansions

- Since a free expansion is not in equilibrium, the equation of the adiabatic expansion can not apply to it.

- In a free expansion, because $\Delta\mathcal{E}_{\text{int}}=0$, the temperature of the final int state must be that of the initial state, ie, $T_i=T_f$, and $p_iV_i=p_fV_f$.

Four Special Processes

			Some Special Results
Path	Constant Quantity	Process Type	$\Delta\mathcal{E}_{\text{int}}=Q-W$ and $\Delta\mathcal{E}_{\text{int}}=nC_V\Delta T$ for all paths
1	p	Isobaric	$Q=nC_p\Delta T;\quad W=p\Delta V$
2	T	Isothermal	$Q=W=nRT\ln\left(V_f/V_i\right);\quad \Delta\mathcal{E}_{\text{int}}=0$
3	$pV^{\gamma},\ TV^{\gamma-1}$	Adiabatic	$Q=0;\quad W=-\Delta\mathcal{E}_{\text{int}}$
4	V	Isochoric	$Q=\Delta\mathcal{E}_{\text{int}}=nC_V\Delta T;\quad W=0$



Problem 19-9, 19-10

Selected problems: 6, 14, 23, 41