

Chapter 17 Waves-II

Sound Waves

- 2 types of mechanical waves:

transverse waves: oscillations \perp the direction of wave travel

longitudinal waves: oscillations \parallel the direction of wave travel.

- **sound wave** is defined roughly as any longitudinal wave:

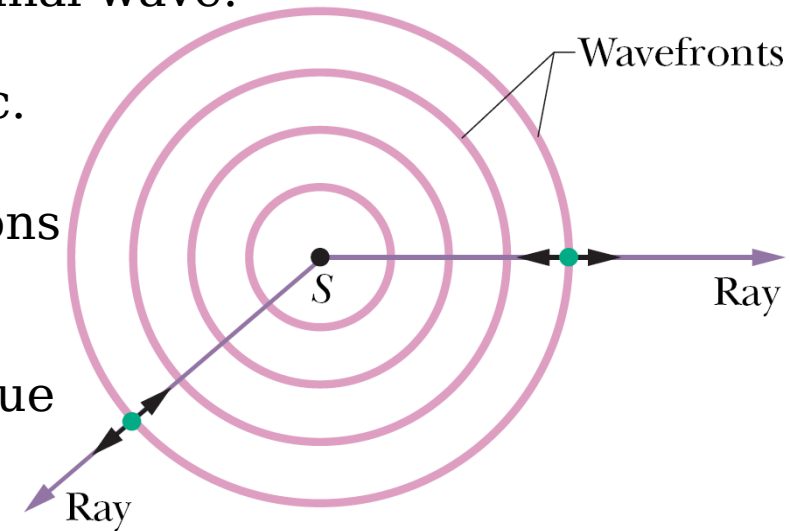
applications: seismic probing for oil; sonar;

ultrasound for medicine, music, etc.

- *Point source S*: emits sound waves in all directions

Wavefronts: surfaces of the oscillations due to the sound wave have the same value

Rays: directed lines \perp the wavefronts that indicate the direction of travel of the wavefronts.



- The wavefronts and rays indicate the direction of travel and the spread of the sound waves.

- Near a point source, the wavefronts are spherical and spread out in 3 dimensions, the waves are said to be *spherical*. Far from the source, the wavefronts look as planes, and the waves are said to be *planar*.

The Speed of Sound

- The speed of any mechanical wave depends on both an inertial property of the medium (to store kinetic energy) and an elastic property of the medium (to store potential energy)

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}} \quad \text{for transverse wave}$$

- For a longitudinal wave, the inertial property, corresponding to μ , is the volume density ρ of air; the elastic property, corresponding to τ , is the **bulk modulus** B ,

$$B = - \frac{\Delta p}{\Delta V/V} \quad \text{definition of bulk modulus}$$

because the potential energy is associated with periodic compressions and expansions of small volume elements of the air.

- The minus sign is to make sure B is always a positive quantity since the signs of Δp and ΔV are always opposite.

- Thus the speed of sound: $v = \sqrt{\frac{B}{\rho}}$ speed of sound

Formal Derivation

- Run along with the pulse at the sound speed (co-moving), then the pulse looks standing still, and air looks moving at speed v from left to right.

- As the fluid element enters the pulse, the leading face encounters a region of higher pressure, which slows the element to speed $v + \Delta v$, for $\Delta v < 0$.

- The time to complete the slowing when the rear face

reaches the pulse $\Delta t = \frac{\Delta x}{v}$

- the average net force on the element during Δt is

$$F = p A - (p + \Delta p) A = -\Delta p A$$

- the mass of the element

$$\Delta m = \rho \Delta V = \rho A \Delta x = \rho A v \Delta t$$

- The average acceleration of the element during Δt

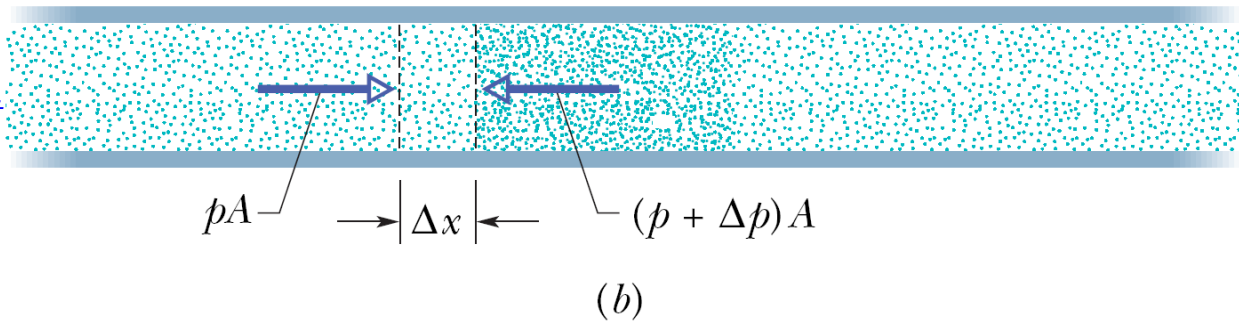
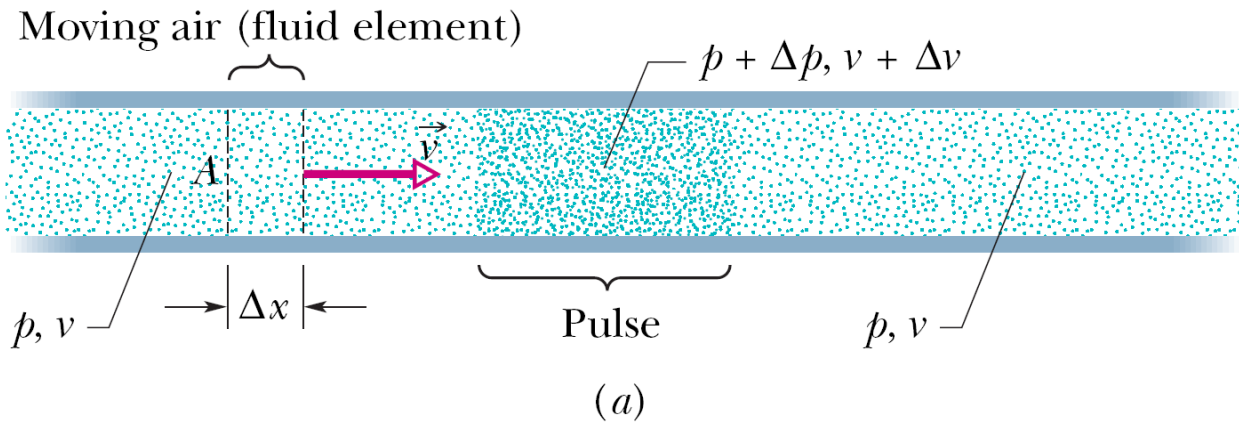
$$a = \frac{\Delta v}{\Delta t} \quad \text{acceleration}$$

- From Newton's 2nd law: $-\Delta p A = (\rho A v \Delta t) \frac{\Delta v}{\Delta t} \Rightarrow \rho v^2 = -\frac{\Delta p}{\Delta v/v}$

$$\left[\begin{array}{l} V = A v \Delta t \\ \Delta V = A \Delta v \Delta t \end{array} \Rightarrow \frac{\Delta V}{V} = \frac{A \Delta v \Delta t}{A v \Delta t} = \frac{\Delta v}{v} \right.$$

$$\Rightarrow \rho v^2 = -\frac{\Delta p}{\Delta v/v} = -\frac{\Delta p}{\Delta V/V} = B$$

$$\rho_{\text{water}} > \rho_{\text{air}} \quad \text{but} \quad v_{\text{water}} > v_{\text{air}} \Rightarrow B_{\text{water}} \gg B_{\text{air}}$$



Traveling Sound Waves

● We can produce a sinusoidal wave by sinusoidally moving a piston at the left end of the tube.

● The displacements are sinusoidal functions of x and t

$$s(x, t) = s_m \cos(kx - \omega t)$$

● λ is the distance in which the pattern of compression and expansion begins to repeat itself. Here we assume s_m is much less than λ .

● The pressure variation is

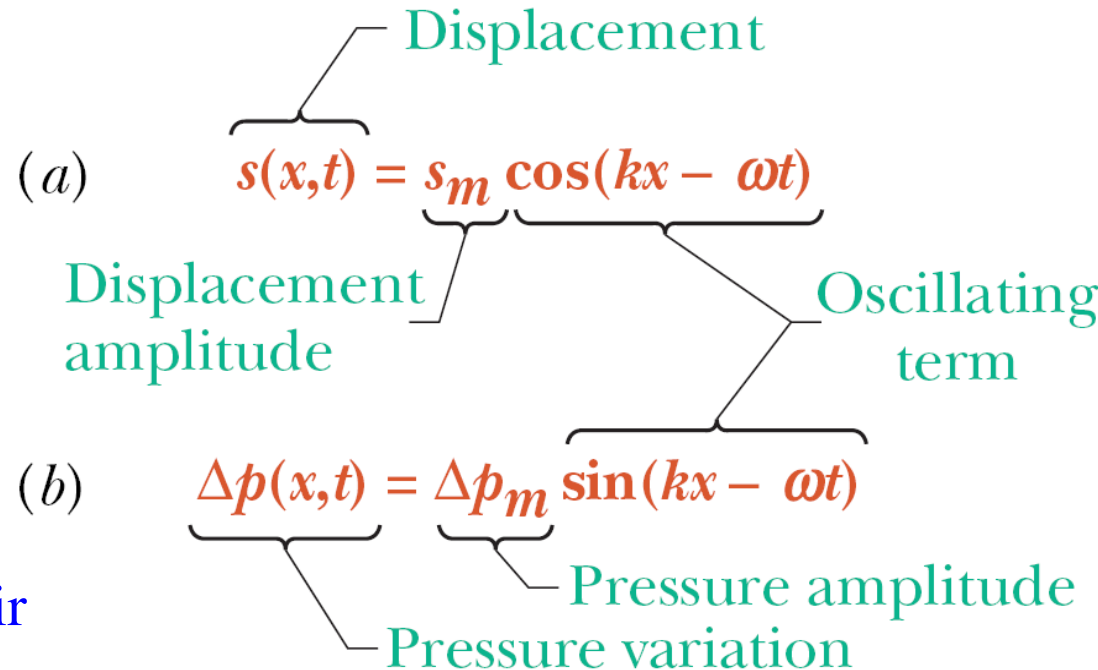
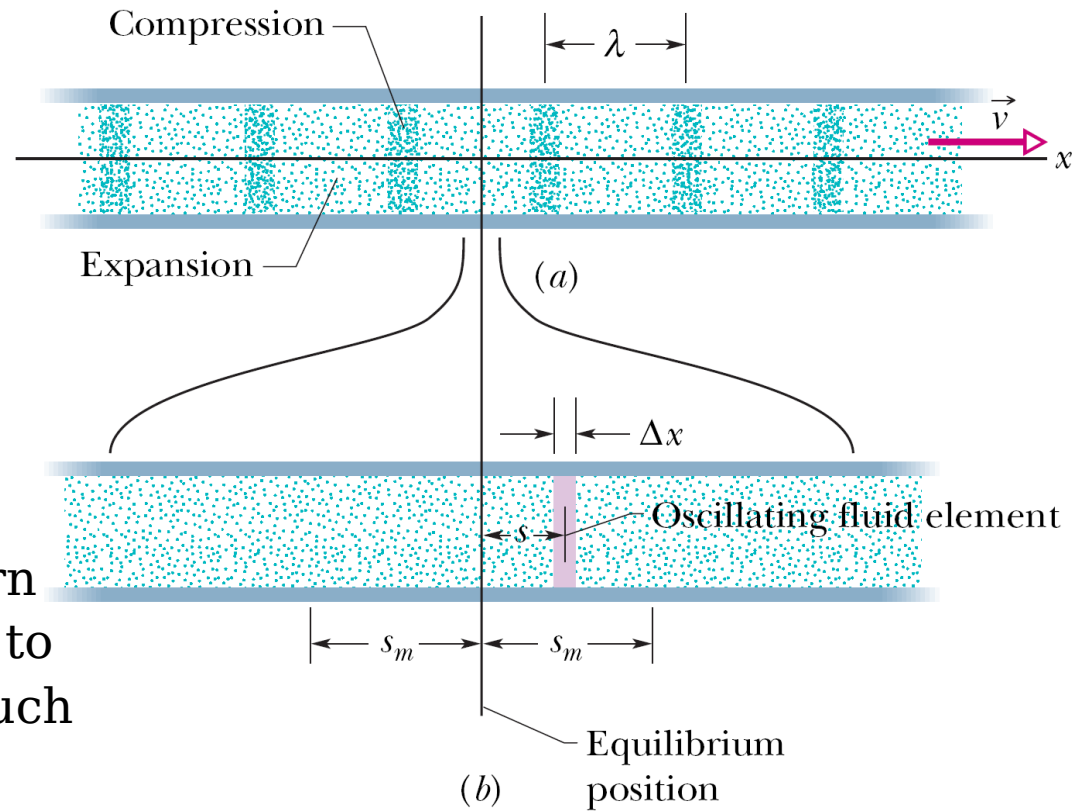
$$\Delta p(x, t) = \Delta p_m \sin(kx - \omega t)$$

where

$$\Delta p_m = (\nu \rho \omega) s_m \quad \text{pressure amplitude}$$

$$\Delta p_m \ll p$$

$\Rightarrow \Delta p < 0 \Rightarrow$ expansion of the air
 $\Delta p > 0 \Rightarrow$ compression of the air



Derivation of the pressure variation

- The pressure variation $\Delta p = -B \frac{\Delta V}{V}$

$$\begin{cases} V = A \Delta x \\ \Delta V = A \Delta s \end{cases} \Rightarrow \Delta p = -B \frac{\Delta s}{\Delta x} = -B \frac{\partial s}{\partial x}$$

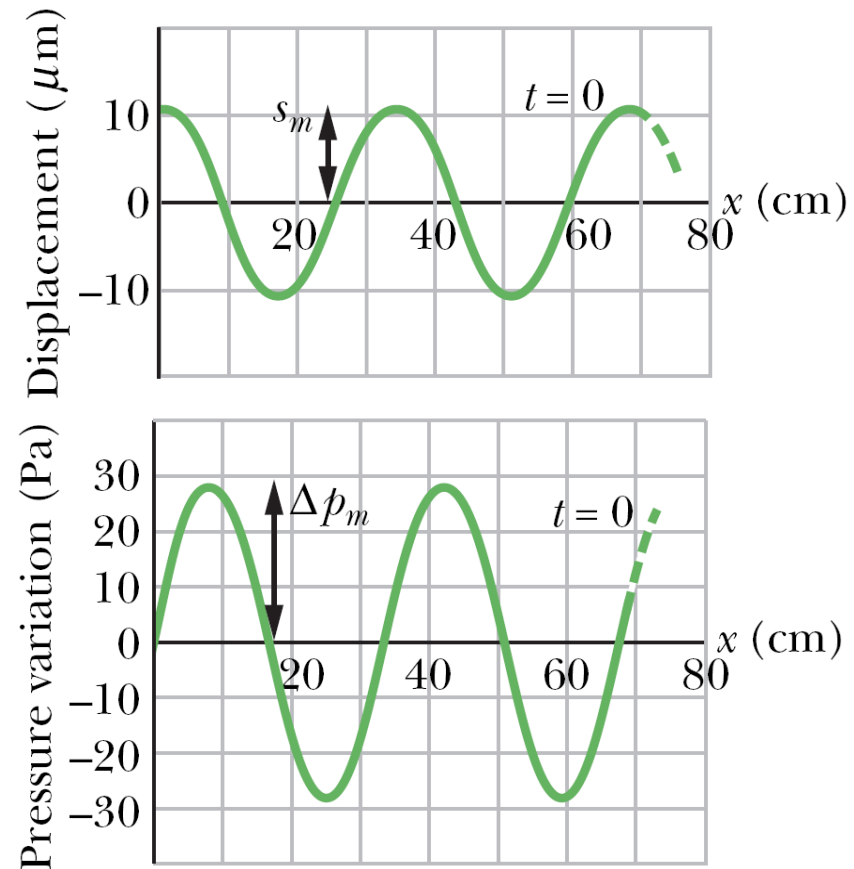
$$\frac{\partial s}{\partial x} = \frac{\partial}{\partial x} s_m \cos(kx - \omega t)$$

$$= -k s_m \sin(kx - \omega t)$$

$$\Rightarrow \Delta p = B k s_m \sin(kx - \omega t)$$

$$\Rightarrow \Delta p_m = B k s_m = v^2 \rho k s_m = v \rho \omega s_m$$

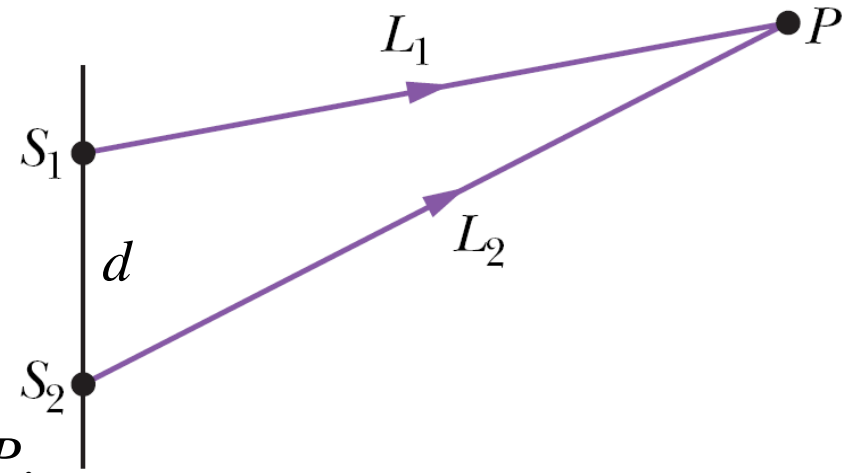
problem 17-1



Interference

- Consider the interference between 2 identical sound waves in phase traveling in the same direction.

- Assume $L_1 \gg d$, $L_2 \gg d$, we can approximate the waves as traveling in the same direction at P .



- The phase difference ϕ at P depends on their **path length difference**

$$\Delta L = |L_2 - L_1| \Rightarrow \frac{\phi}{2\pi} = \frac{\Delta L}{\lambda} \Rightarrow \phi = 2\pi \frac{\Delta L}{\lambda}$$

- Fully constructive interference occurs when ϕ is any integer multiple of 2π

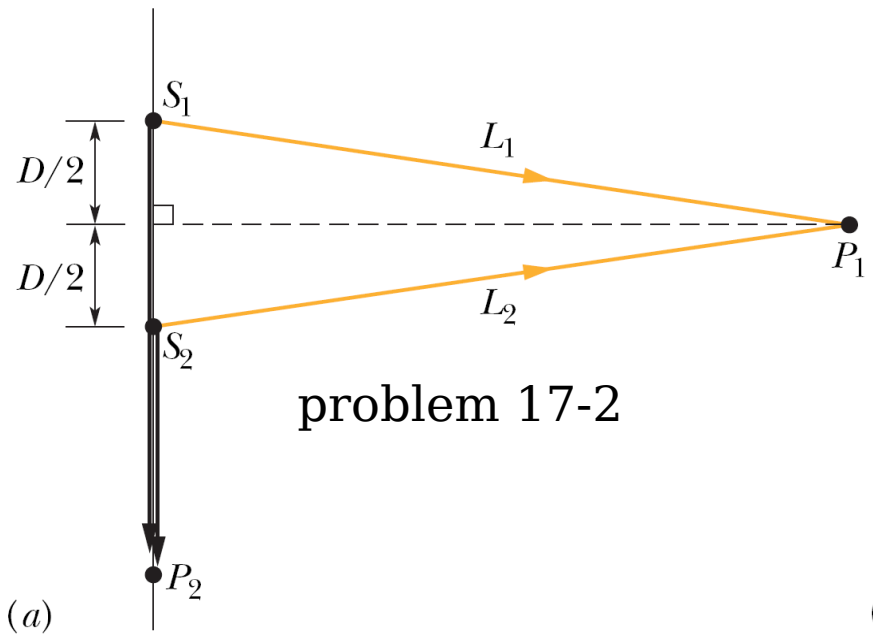
$$\phi = m(2\pi) \text{ for } m = 0, 1, 2, \dots \Rightarrow \frac{\Delta L}{\lambda} = 0, 1, 2, \dots \text{ fully constructive interference}$$

- For the fully constructive interference, the 2 waves is *exactly in phase* at P .

- Fully destructive interference occurs when ϕ is an odd integer multiple of π

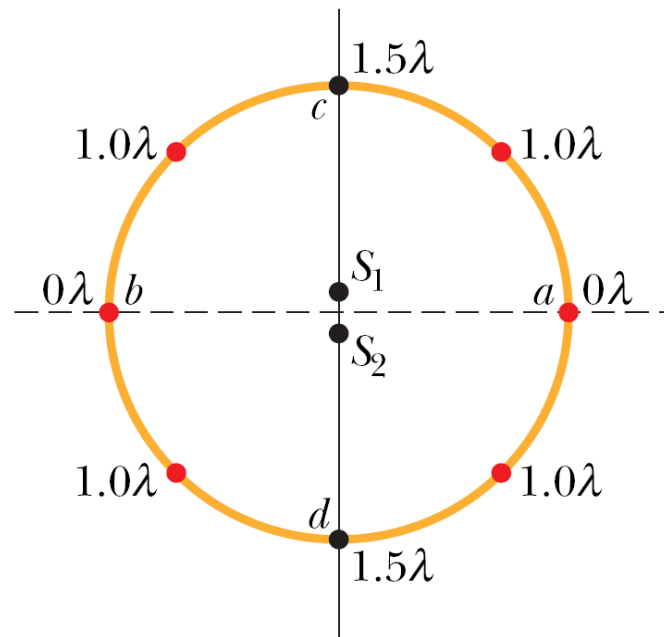
$$\phi = (2m + 1)\pi \text{ for } m = 0, 1, \dots \Rightarrow \frac{\Delta L}{\lambda} = 0.5, 1.5, \dots \text{ fully destructive interference}$$

- For the fully destructive interference, the 2 waves is *exactly out phase* at P .



problem 17-2

(a)



(b)

Intensity and Sound Level

- The **intensity** I of a sound wave at a surface is the average rate per unit area at which energy is transferred by the wave through or onto the surface

$$I = \frac{P}{A} \quad \leftarrow \quad \begin{array}{l} P : \text{power of the sound wave} \\ A : \text{area of the surface} \end{array}$$

- The kinetic energy dK of a slice of air $\frac{1}{2} dm v_s^2$, v_s is

not the speed of the wave but the speed of the oscillating element of air

$$v_s \equiv \frac{\partial s}{\partial t} = -\omega s_m \sin(kx - \omega t) \Rightarrow dK = \frac{1}{2} (\rho A dx) (-\omega s_m)^2 \sin^2(kx - \omega t)$$

$$\Rightarrow \frac{dK}{dt} = \frac{1}{2} \rho A v \omega^2 s_m^2 \sin^2(kx - \omega t)$$

$$\Rightarrow \left(\frac{dK}{dt} \right)_{\text{avg}} = \frac{1}{4} \rho A v \omega^2 s_m^2 \Rightarrow \left(\frac{dU}{dt} \right)_{\text{avg}} = \frac{1}{4} \rho A v \omega^2 s_m^2 \quad \leftarrow \text{similar argument}$$

$$I \equiv \frac{P}{A} = \frac{1}{A} \left(\frac{dK}{dt} + \frac{dU}{dt} \right)_{\text{avg}} \Rightarrow I = \frac{1}{2} \rho v \omega^2 s_m^2 \quad \leftarrow s_m \propto \frac{1}{r} \text{ if spherical}$$



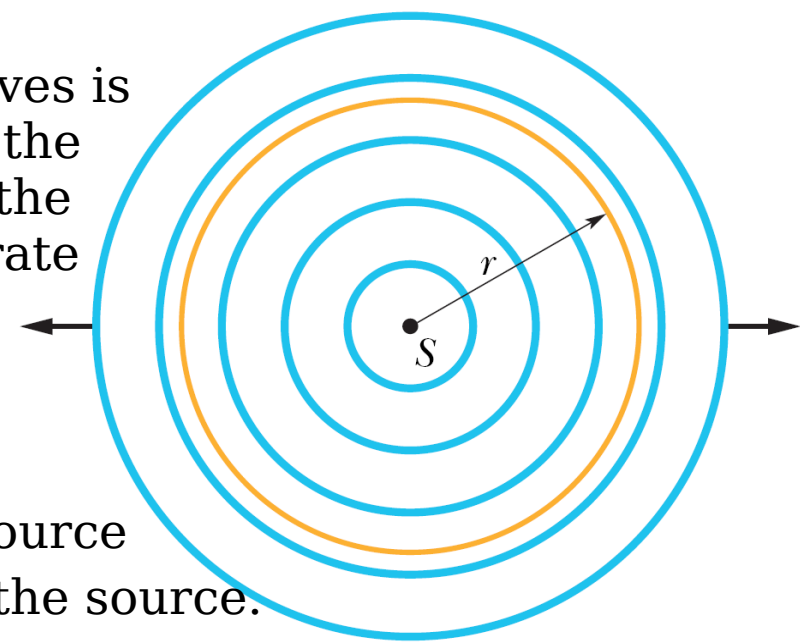
Variation of Intensity with Distance

- Assume the sound source is a point source that emits the sound with equal intensity in all directions (*isotropically*).

- Assume the mechanical energy of the sound waves is conserved as they spread from this source. Thus, the time rate at which energy is transferred through the surface by the sound waves must equal the time rate at which energy is emitted by the source. Thus, the intensity at the sphere must be

$$I = \frac{P_s}{4\pi r^2}$$

- The intensity of sound from an isotropic point source decreases with the square of the distance r from the source.



The Decibel Scale

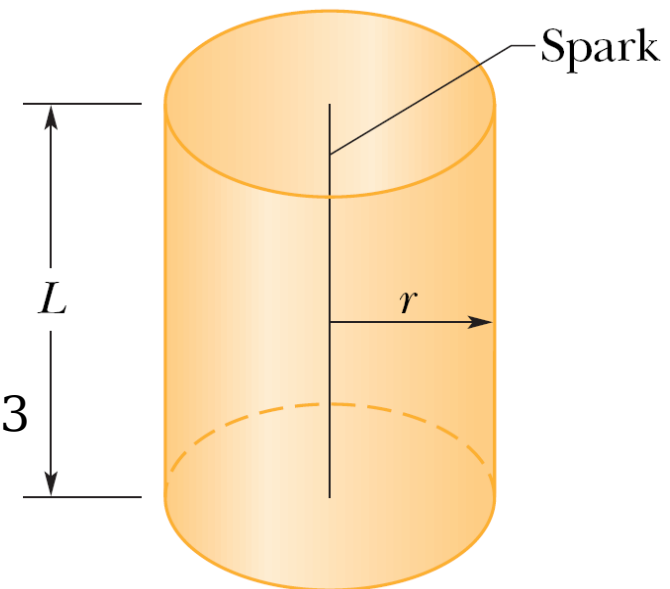
- The sound level of a sound wave is defined as $\beta = (10 \text{ dB}) \log \frac{I}{I_0}$

here **dB** is the abbreviation for **decibel**, the unit of sound level. I_0 is a standard reference intensity ($=10^{-12} \text{W/m}^2$), chosen because it is near the lower limit of the human range of hearing

$$\beta = \begin{cases} 0 \text{ dB} & \leftarrow I \leq I_0 \\ 40 \text{ dB} & \leftarrow I = 10^4 I_0 \\ 120 \text{ dB} & \leftarrow I = 10^{12} I_0 \end{cases}$$

problem 17-4

problem 17-3



Sources of Musical Sound

- The wavelength required of forming a standing waves is one that corresponds to a *resonant frequency*.

- The advantage of setting up standing waves is that the string oscillates with a large, sustained amplitude, pushing back and forth against the surrounding air and generating a noticeable sound wave with the same frequency.

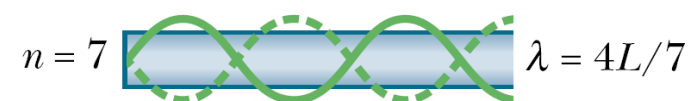
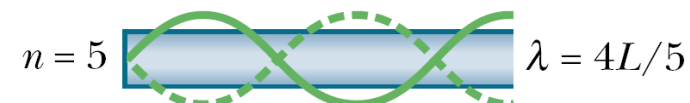
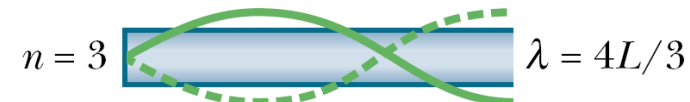
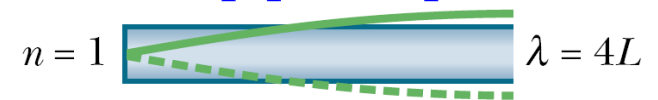
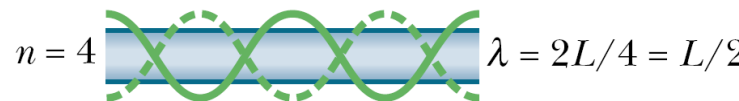
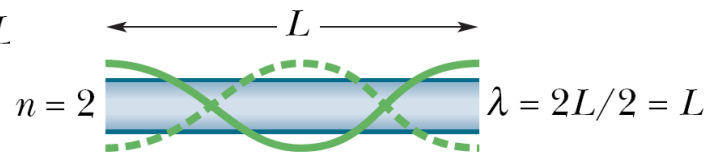
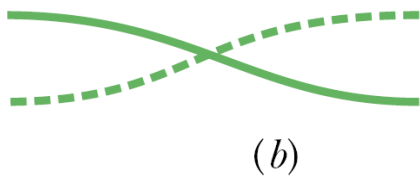
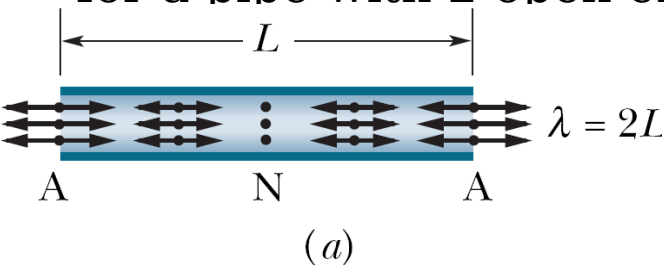
- The standing wave pattern for 2 open ends, $\lambda=2L$, is called the *fundamental mode* or *1st harmonic*.

- $\lambda=L$ for *2nd harmonic*. $\lambda=\frac{2L}{3}$ for *3rd harmonic*.

- For 2 open ends: $\lambda = \frac{2L}{n}$, for $n = 1, 2, 3, \dots$, n : the *harmonic number*.

- The resonant frequencies for a pipe with 2 open ends:

$$f = \frac{v}{\lambda} = \frac{nv}{2L} \text{ for } n = 1, 2, 3, \dots \text{ pipe, 2 open ends}$$



- The simplest wave pattern for 1 open end is $\lambda=4L$.

The next one is $\lambda = \frac{4L}{3}$.

- For 1 open end: $\lambda = \frac{4L}{n}$ for $n = 1, 3, 5, \dots$

Note: the harmonic number n must be an odd number.

- The resonant frequencies for a pipe with 1 open end

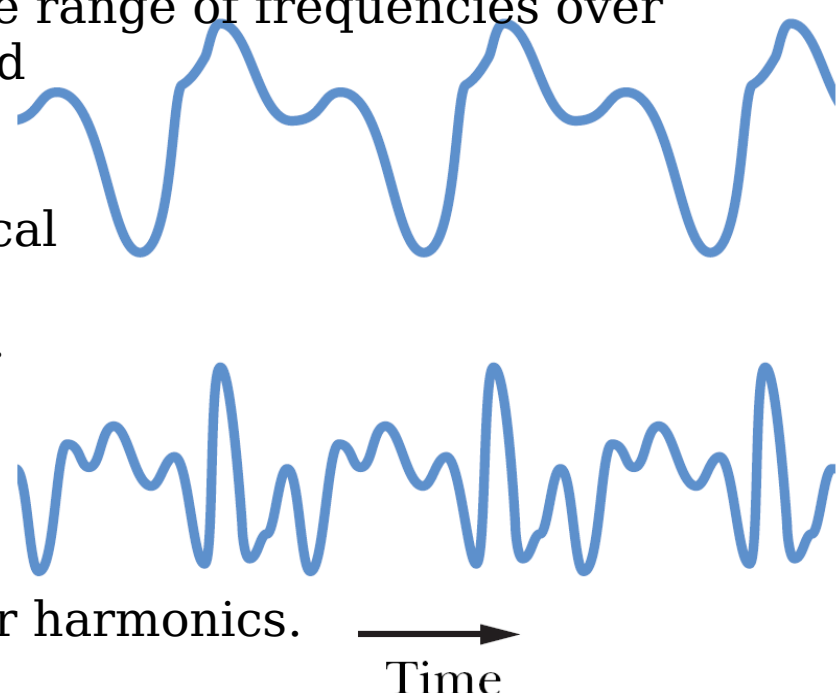
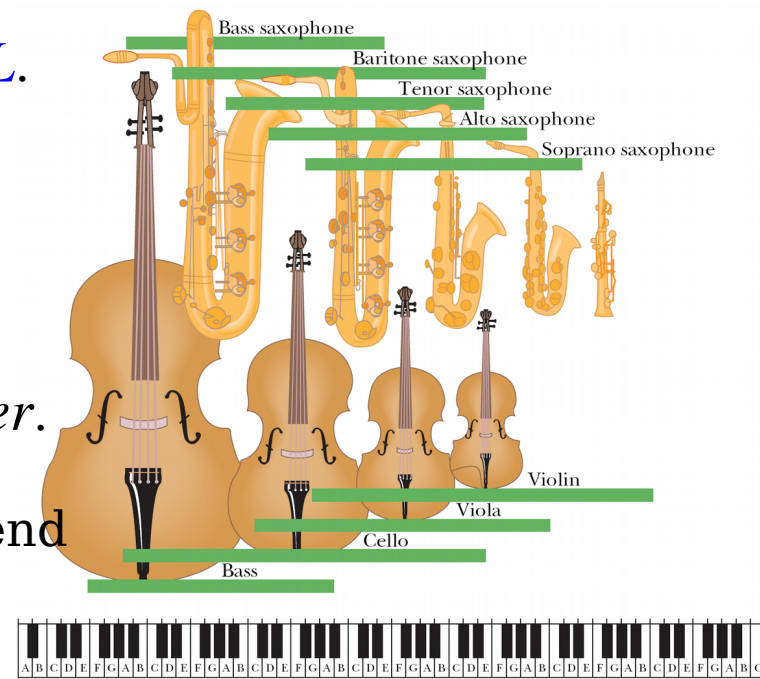
$$f = \frac{v}{\lambda} = \frac{nv}{4L} \text{ for } n = 1, 3, 5, \dots \text{ pipe, 1 open end}$$

- Only odd harmonics can exist in a pipe with 1 open end.

- The length of a musical instrument reflects the range of frequencies over which the instrument is designed to function, and smaller length implies higher frequencies.

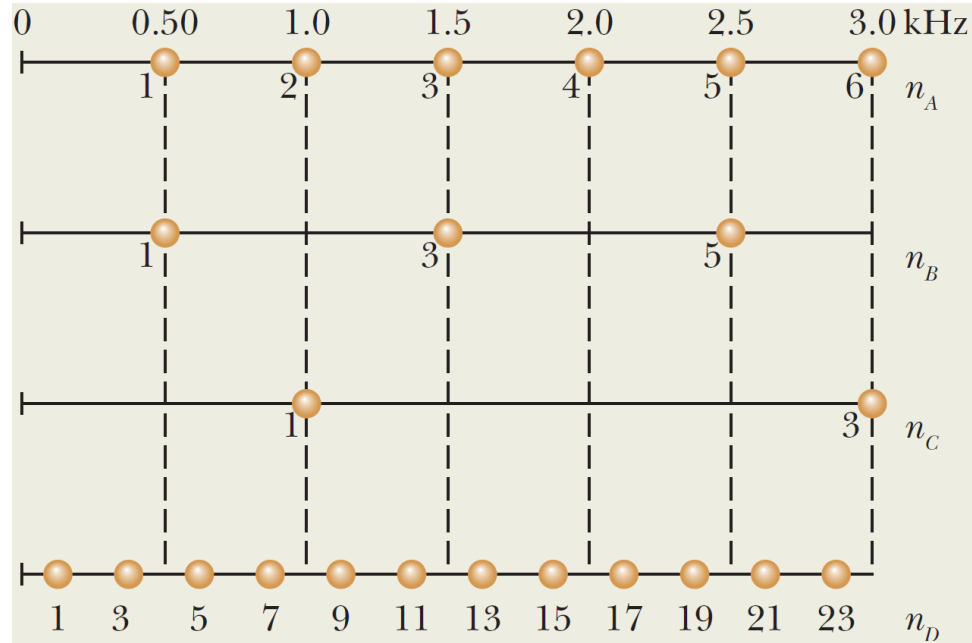
- In any oscillating system giving rise to a musical sound, the fundamental and one or more of the higher harmonics are generated simultaneously. Thus, you hear them together as a net wave.

- When different instruments are played at the same note, they produce the same fundamental frequency but different intensities for the higher harmonics.



- Thus, because different instruments produce different net waves, they sound different even when they are played at the same note.

Problem 17-5



Beats

- **beats**: the difference between the 2 combining frequencies.

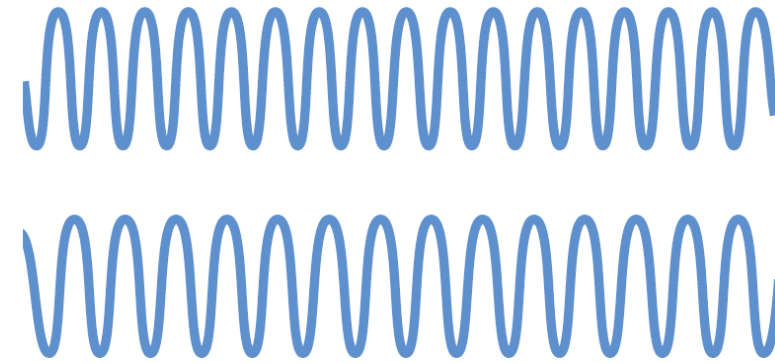
- 2 sound waves with different frequencies

$$s_1 = s_m \cos \omega_1 t, \quad s_2 = s_m \cos \omega_2 t \quad \Leftarrow \quad \omega_1 > \omega_2$$

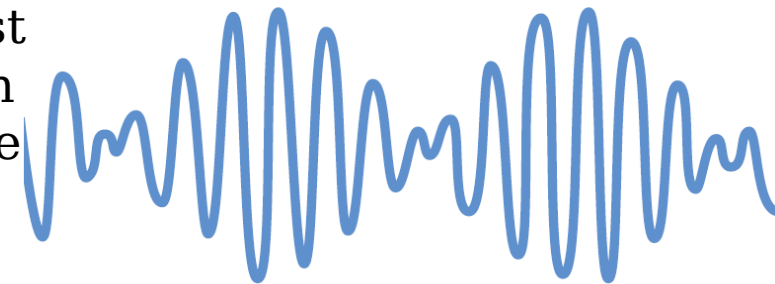
$$\Rightarrow s = s_1 + s_2 = s_m (\cos \omega_1 t + \cos \omega_2 t)$$

$$= 2 s_m \cos \frac{\omega_1 - \omega_2}{2} t \cos \frac{\omega_1 + \omega_2}{2} t$$

$$\Rightarrow s = [2 s_m \cos \omega' t] \cos \omega t \quad \Leftarrow \quad \omega' = \frac{\omega_1 - \omega_2}{2}, \quad \omega = \frac{\omega_1 + \omega_2}{2} \quad \xrightarrow{\text{Time}}$$



- Assume that the 2 angular frequencies are almost equal, ie, $\omega \gg \omega'$. It is regarded as a cosine function with an angular frequency ω and with an amplitude (varying with angular frequency ω') is $|2s_m \cos \omega' t|$.



- A maximum amplitude will occur whenever $\cos \omega' t$ has the value +1 or -1, which happens twice in each repetition of the cosine function.

- Thus the angular frequency ω_{beat} at which beats occur is $\omega_{\text{beat}} = 2\omega'$

$$\omega_{\text{beat}} = 2 \omega' = \omega_1 - \omega_2 \quad \Rightarrow \quad f_{\text{beat}} = f_1 - f_2 \quad \text{beat frequency} \quad \Leftarrow \quad \omega = 2 \pi f$$

- Musicians use the beat phenomenon in tuning instruments.

The Doppler Effect

- The motion-related frequency changes are related the **Doppler effect**.
- The Doppler effect holds not only for sound waves but also for electromagnetic waves, with different principles.
- we shall measure the speeds of a source S of sound waves and a detector D of those waves *relative to that body of air*. Assume S and D move directly toward or directly away from each other, at speeds less than the speed of sound.

- The emitted frequency f and the detected frequency f' are related by

$$f' = f \frac{v \pm v_D}{v \pm v_S} \quad \text{general Doppler effect}$$

- The choice of + or – signs is set by this rule:

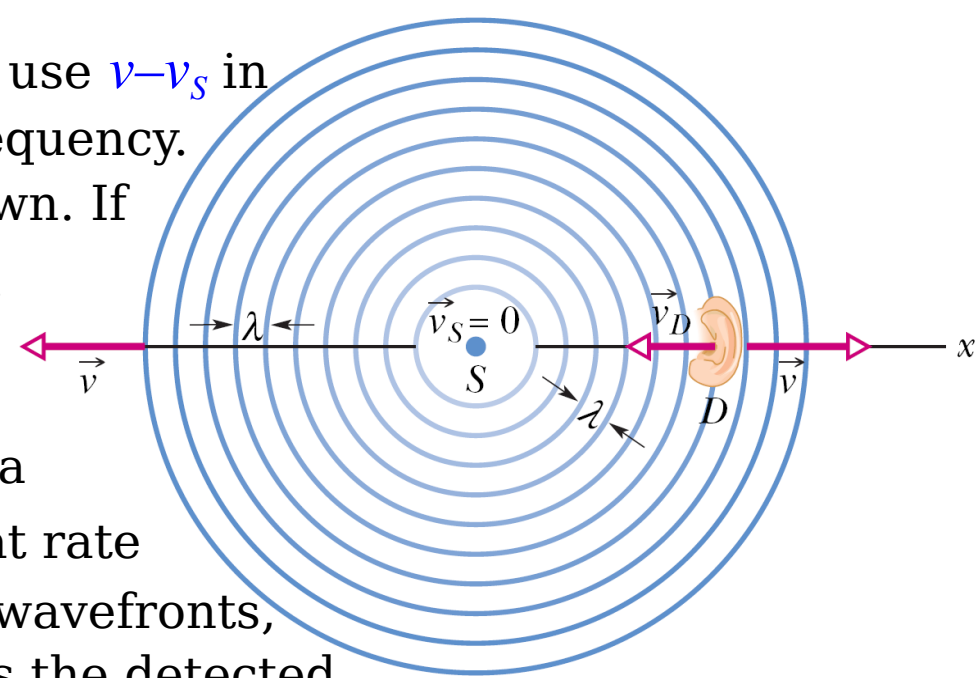
When the motion of detector or source is toward the other, the sign on its speed must give an upward shift in frequency. When the motion of detector or source is away from the other, the sign on its speed must give a downward shift in frequency.

- If the detector moves toward the source, use $v+v_D$ in the numerator to get a shift up in the frequency. If it moves away, use $v-v_D$ to get a shift down. If it is stationary, substitute 0 for v_D .

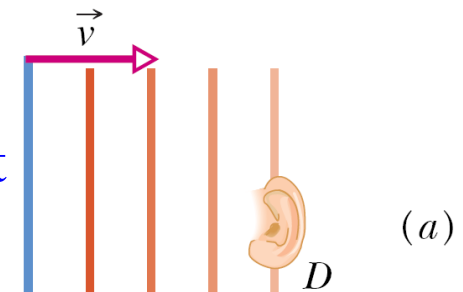
- If the source moves toward the detector, use $v-v_s$ in the denominator to get a shift up in the frequency. If it moves away, use $v+v_s$ to get a shift down. If the source is stationary, substitute 0 for v_s .

Detector Moving, Source Stationary

- A detector is moving at speed v_D toward a stationary source. If D were stationary, that rate would be f , but since D is moving into the wavefronts, the rate of interception is greater, and thus the detected frequency f' is greater than f .

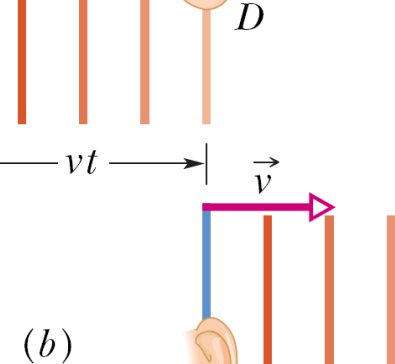


- Consider the situation in which D is stationary $f = \frac{vt/\lambda}{t} = \frac{v}{\lambda}$ no Doppler effect



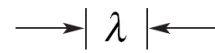
- Consider D moves in the direction opposite the wavefront velocity

$$f' = \frac{(vt + v_D t)/\lambda}{t} = \frac{v + v_D}{\lambda} = \frac{v + v_D}{v/f} = f \frac{v + v_D}{v} \leftarrow \lambda = \frac{v}{f}$$

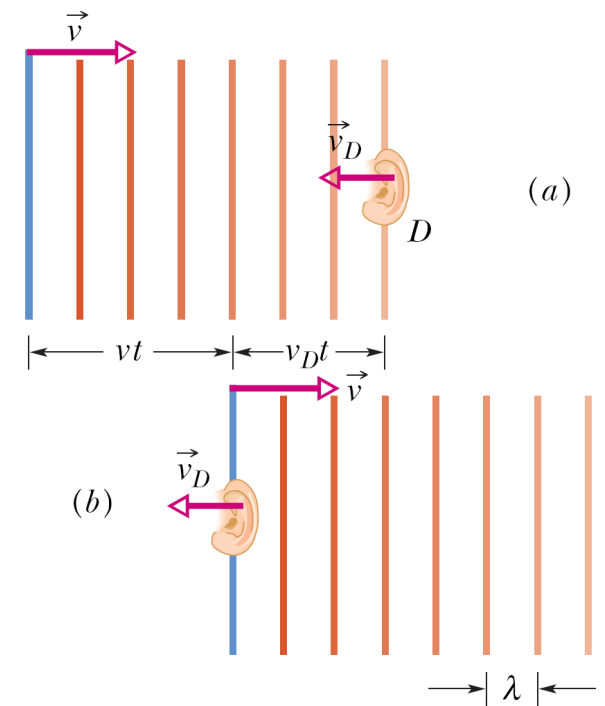
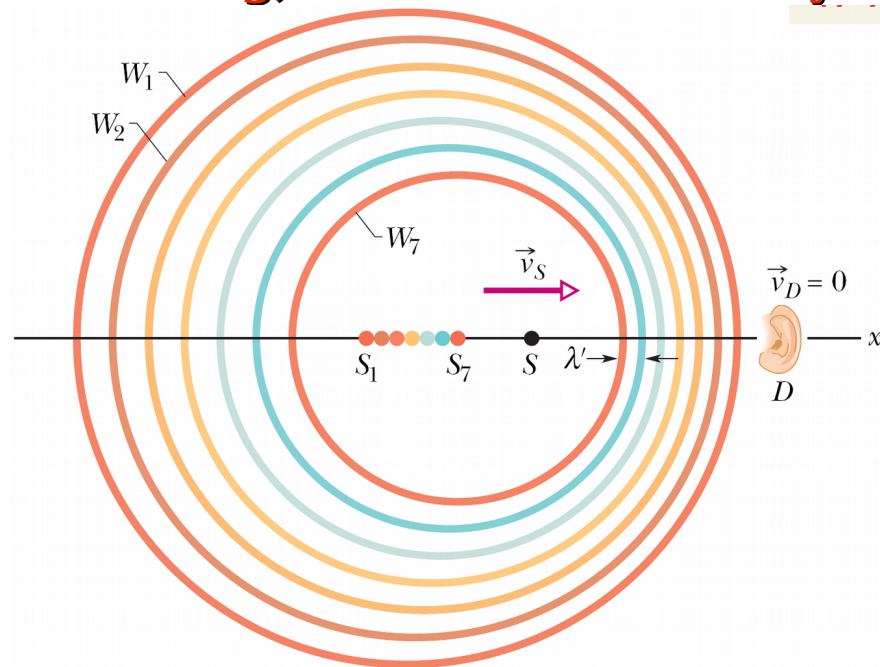


- If D moves away from the source $f' = f \frac{v - v_D}{v} \leq f$

- Thus $f' = f \frac{v \pm v_D}{v}$ detector moving, source stationary



Source Moving, Detector Stationary



- Let source S move toward D at speed v_s

$$\lambda' = vT - v_s T \Rightarrow f' = \frac{v}{\lambda'} = \frac{v}{vT - v_s T} = \frac{v}{v/f - v_s/f} = f \frac{v}{v - v_s} \geq f$$

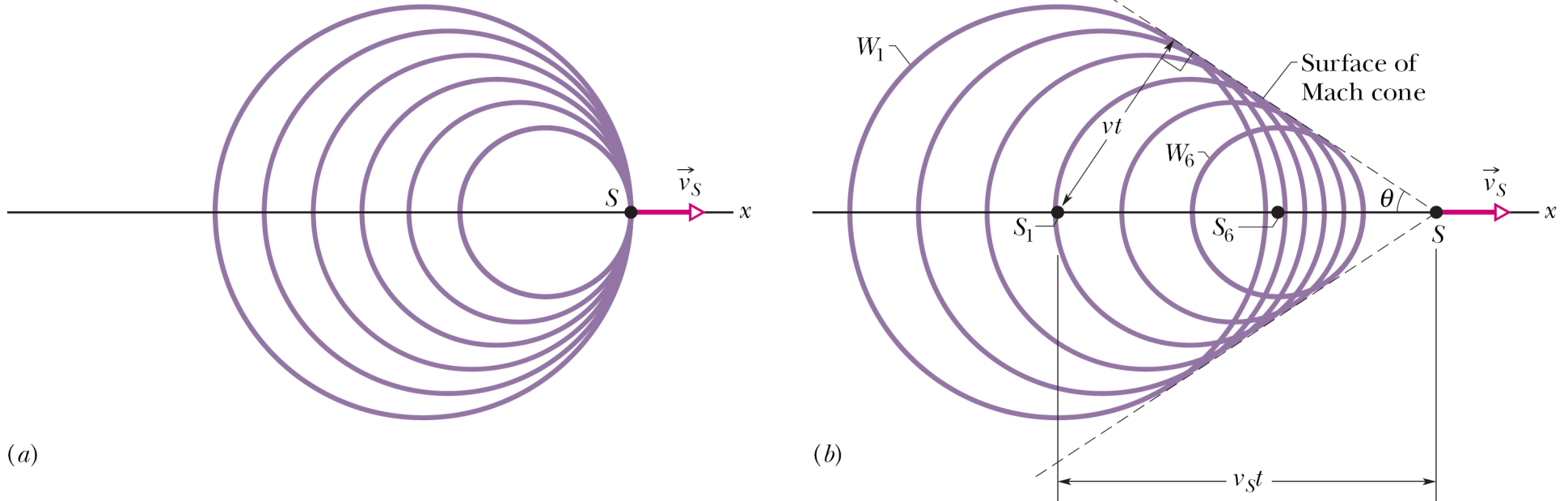
- If S move away from D : $\lambda' = vT + v_s T \Rightarrow f' = f \frac{v}{v + v_s} \leq f$

Thus $f' = f \frac{v}{v \pm v_s}$ source moving, detector stationary

General Doppler Effect Equation

- We can now derive the general Doppler effect equation by combing the 2 special cases together.

Supersonic Speeds, Shock Waves



- If a source is moving toward a stationary detector at a speed equal to the speed of sound, i.e., $v_s = v$, the Doppler equation predicts that the detected frequency f' will be infinitely great.
- For such *supersonic* speeds, the Doppler equation no longer applies.
- the bunching of the wavefront actually forms a cone called the *Mach cone*.
- A *shock wave* is said to exist along the surface of this cone, because the bunching of wavefronts causes an abrupt rise and fall of air pressure as the surface passes through any point.

- the *Mach cone angle*: the half-angle θ of

the cone $\sin \theta = \frac{v t}{v_s t} = \frac{v}{v_s}$ Mach cone angle

- The ratio $\frac{v_s}{v}$ is called the *Mach number*.

- The shock wave generated by a supersonic aircraft or projectile produces a burst of sound, called a *sonic boom*, in which the air pressure first suddenly increases and then suddenly decreases below normal before returning to normal.

The chosen problems: 18, 31, 55, 57

