Chapter 16 Waves I

Types of Waves

- Waves are of 3 main types:

1. **Mechanical waves**: water waves, sound waves, seismic waves, etc.
   All these waves have 2 central features:
   (a) governed by Newton's laws;
   (b) only within a material medium, like water, air, rock.

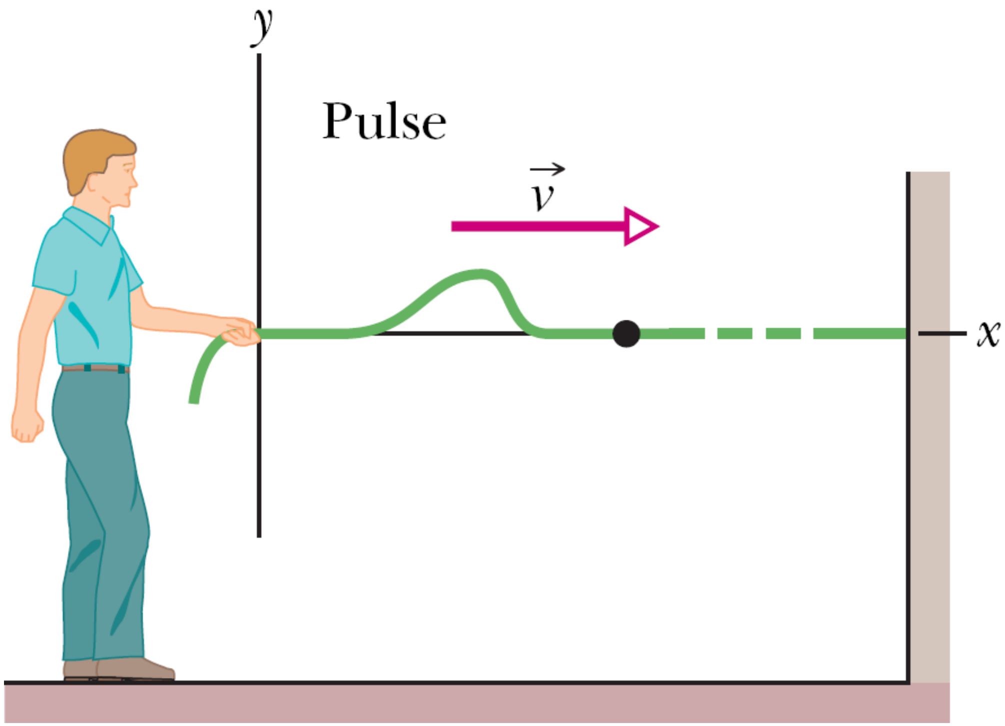
2. **Electromagnetic waves**: light, radio, microwaves, x rays, and radar waves.
   Require no material medium to exist. All EM waves travel through a vacuum at the same speed c.

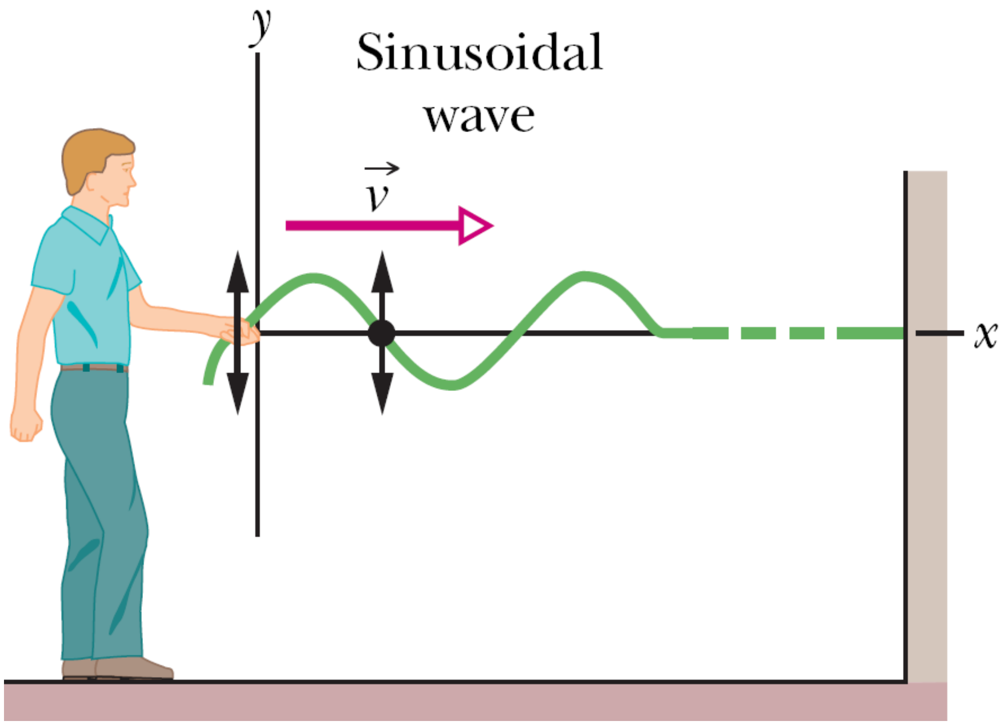
3. **Matter waves**: associated with electrons, protons, and other fundamental particles, and even atoms and molecules.

Transverse and Longitudinal Waves

- The motion is said to be **transverse**, and the wave is said to be a **transverse wave** if the displacement of every element is perpendicular to the direction of travel of the wave.

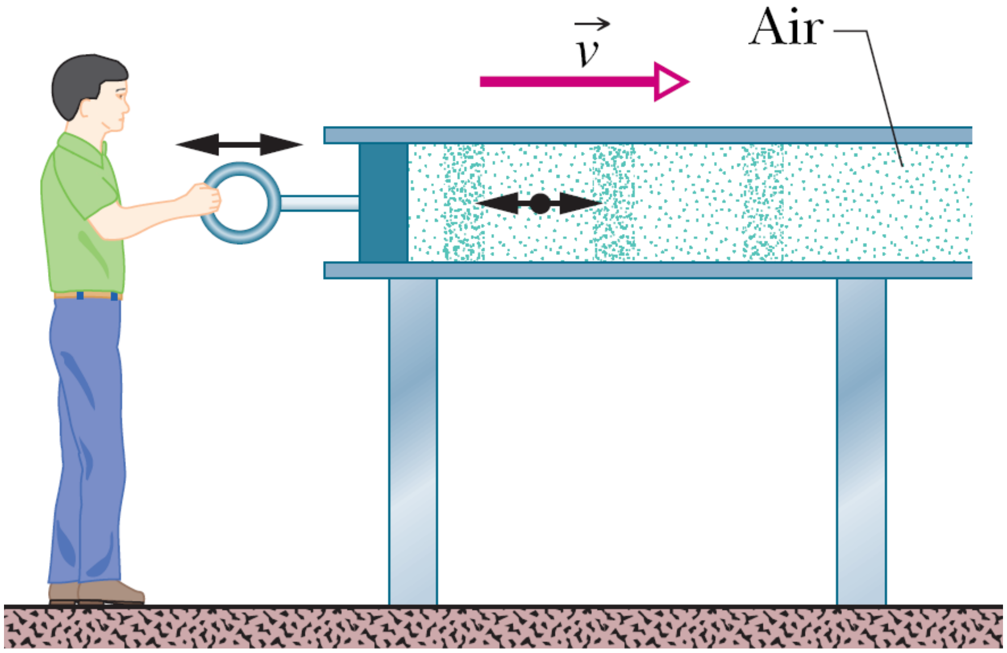
- The motion is said to be **longitudinal**, and the wave is said to be a **longitudinal wave** if the motion of every element is parallel to the direction of the wave's travel.





- Both a transverse wave and a longitudinal wave are said to be **traveling waves** because they both travel from one point to another.

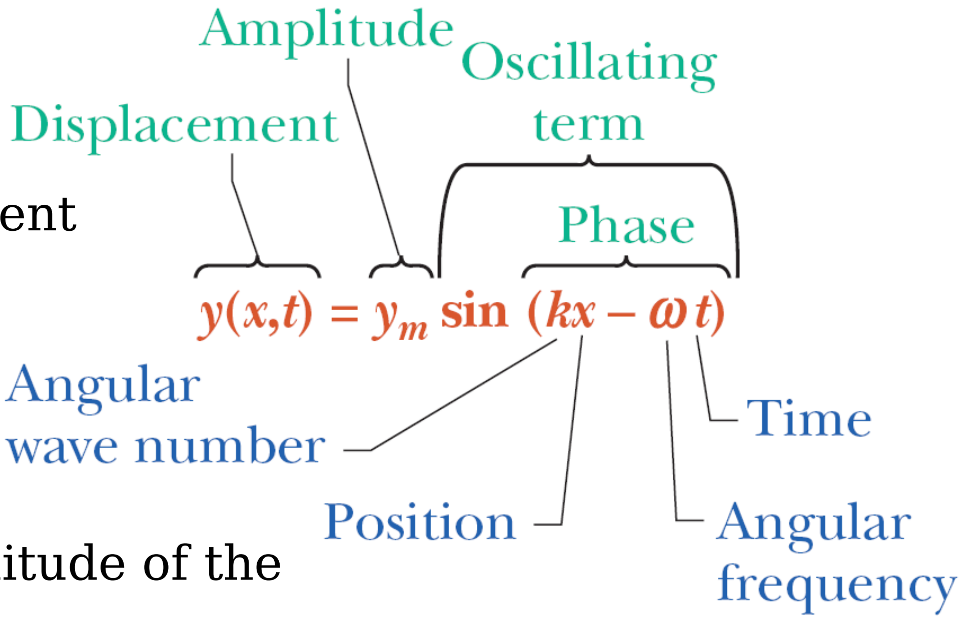
- Note that it is the wave that moves from end to end, not the material (string or air) through which the wave moves.



Wavelength and Frequency

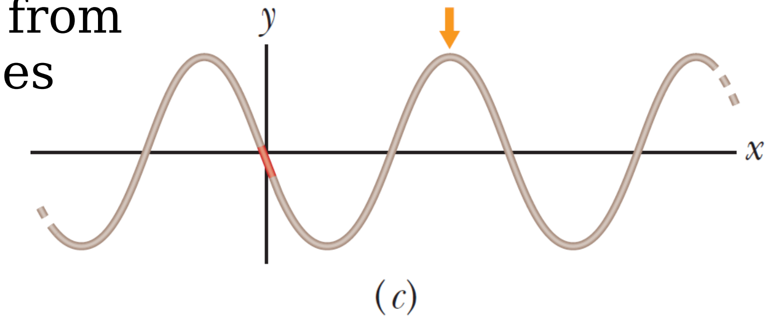
- At time t, the displacement y of the element located at position x is given by

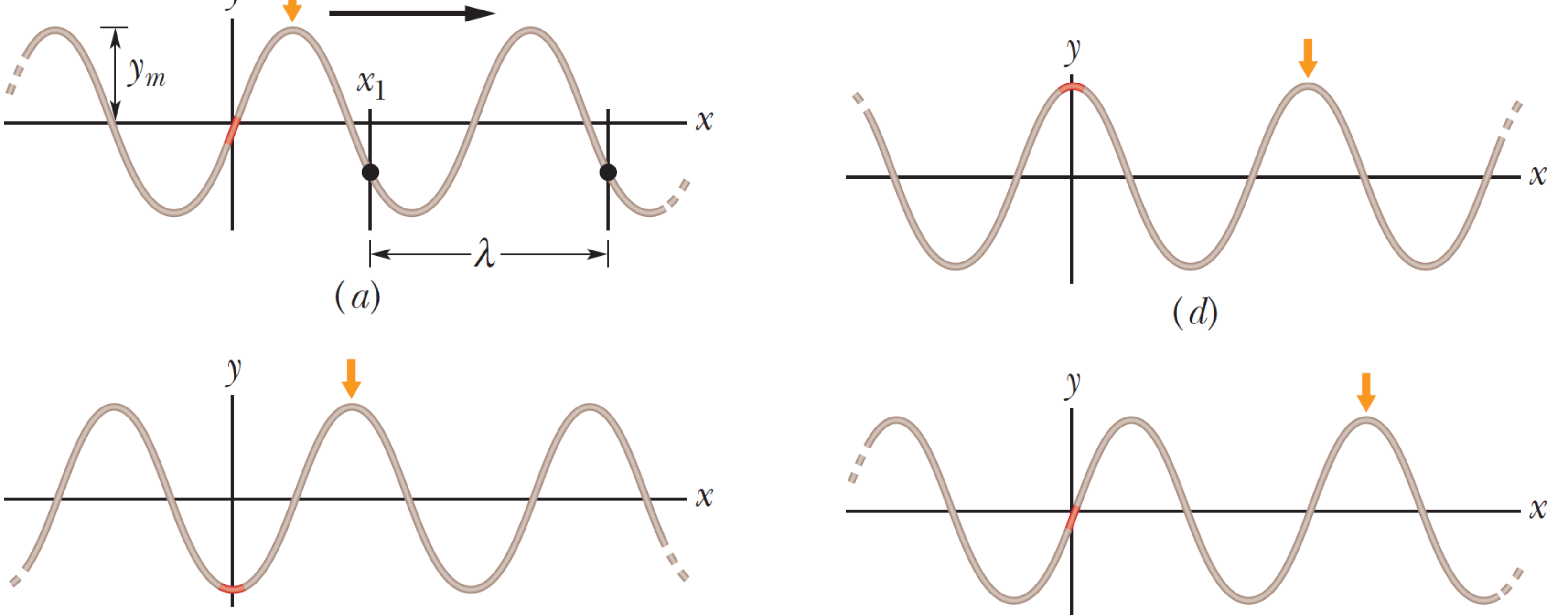
$$y(x,t) = y_m \sin(kx - \omega t)$$



Amplitude and Phase

- The **amplitude** y_m of a wave is the magnitude of the maximum displacement of the elements from their equilibrium positions as wave passes through them.




Watch this spot in this series of snapshots.

- Because y_m is a magnitude, it is always positive.

- The **phase** of the wave is the argument $kx - \omega t$ of the sine.

- The sine's extreme positive value (+1) corresponds to a peak of the wave moving through the element; at that instant the value of y at position x is y_m. Its extreme negative value (−1) corresponds to a valley of the wave moving through the element; at that instant the value of y at position x is $-y_m$.

- The sine function and the time-dependent phase of a wave correspond to the oscillation of a string element, and the amplitude of the wave determines the extremes of the element's displacement.

Wavelength and Angular Wave Number

- The **wavelength** λ of a wave is the distance between repetitions of the shape of the wave (or *wave shape*).

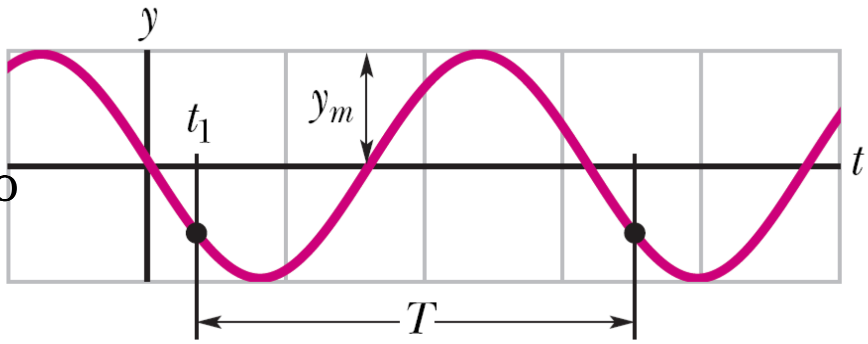
- By definition, the displacement y is the same at both ends of this wavelength that is, at $x = x_1$ and $x = x_1 + \lambda$

$$y_m \sin k x_1 = y_m \sin k (x_1 + \lambda) = y_m \sin (k x_1 + k\lambda)$$

$$\Rightarrow \quad k\lambda = 2\pi \quad \Rightarrow \quad \boxed{k = \frac{2\pi}{\lambda}} \quad \text{angular wave number}$$

- k is called the **angular wave number** of the wave; its SI unit is the radian per meter, or the inverse meter.

Period, Angular Frequency, and Frequency



- Define the **period** of oscillation T of a wave to be the time any string element takes to move through one full oscillation,

$$-y_m \sin \omega t_1 = -y_m \sin \omega (t_1 + T) = -y_m \sin (\omega t_1 + \omega T)$$

$$\Rightarrow \quad \omega T = 2\pi \quad \Rightarrow \quad \omega = \frac{2\pi}{T} \quad \text{angular frequency}$$

- ω is called the **angular frequency** of the wave; its SI unit is the radian per second.

- The **frequency** f of a wave is defined as $1/T$ and is related to the angular frequency ω by

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \quad \text{frequency}$$

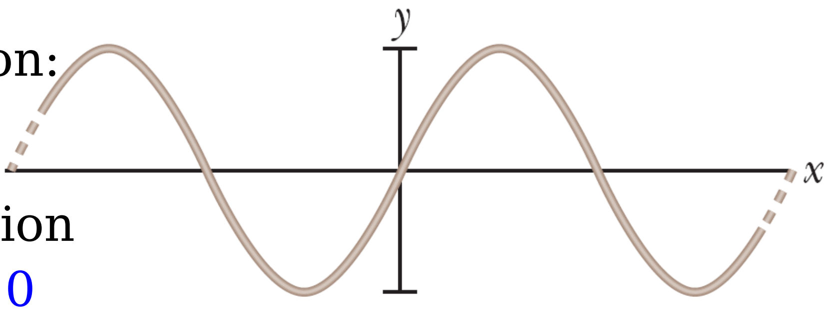
- The frequency f is a number of oscillations per unit time, it is usually measured in hertz or its multiples, such as kilohertz.

Phase Constant

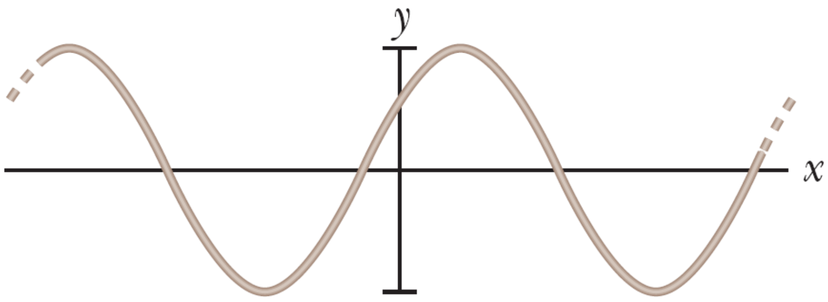
- Insert a **phase constant** ϕ in the wave function:

$$y = y_m \sin(k\,x - \omega\,t + \phi)$$

- The value of ϕ can be chosen so that the function gives some other displacement and slope at $x = 0$ when $t = 0$.



(a)

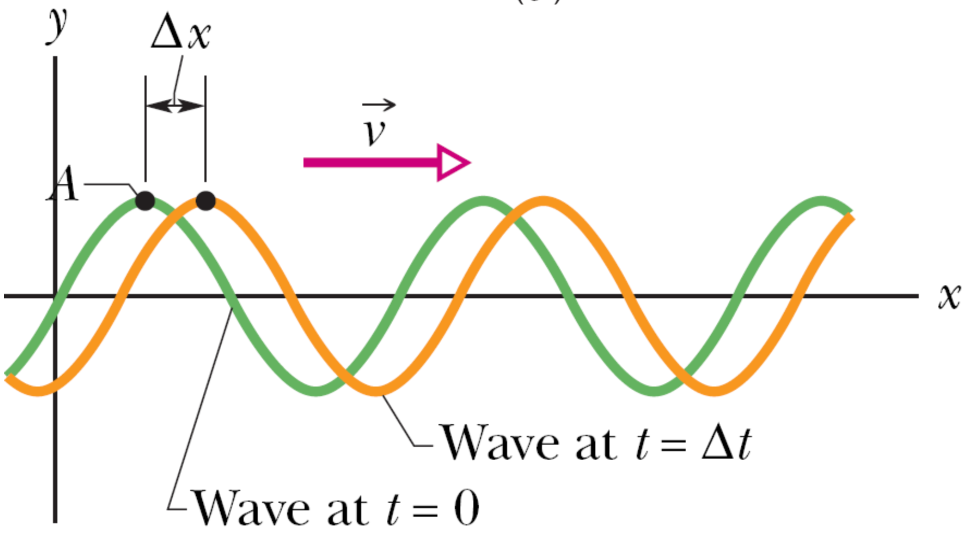


(b)

The Speed of a Traveling Wave

- The ratio $\Delta x/\Delta t$ (or, in the differential limit, $\mathrm{d}x/\mathrm{d}t$) is the **wave speed** v.

- If a point retains its displacement as it moves, the phase giving it that displacement must remain a constant: $k\,x - \omega\,t = \text{a constant}$



- Note although this argument is constant, both x and t are changing.

- Taking the derivative gives

$$k\frac{\mathrm{d}\,x}{\mathrm{d}\,t} - \omega = 0 \quad \text{or} \quad \frac{\mathrm{d}\,x}{\mathrm{d}\,t} = v = \frac{\omega}{k}$$

$$\Rightarrow \boxed{v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda\,f} \quad \text{wave speed} \quad \Leftarrow \quad k = \frac{2\pi}{\lambda}, \quad \omega = \frac{2\pi}{T}$$

- The wave speed is one wavelength per period; the wave moves a distance of one wavelength in one period of oscillation.

- We can find the equation of a wave traveling in the opposite direction by replacing t with $-t$. This corresponds to the condition $kx+\omega t = \text{a constant}$

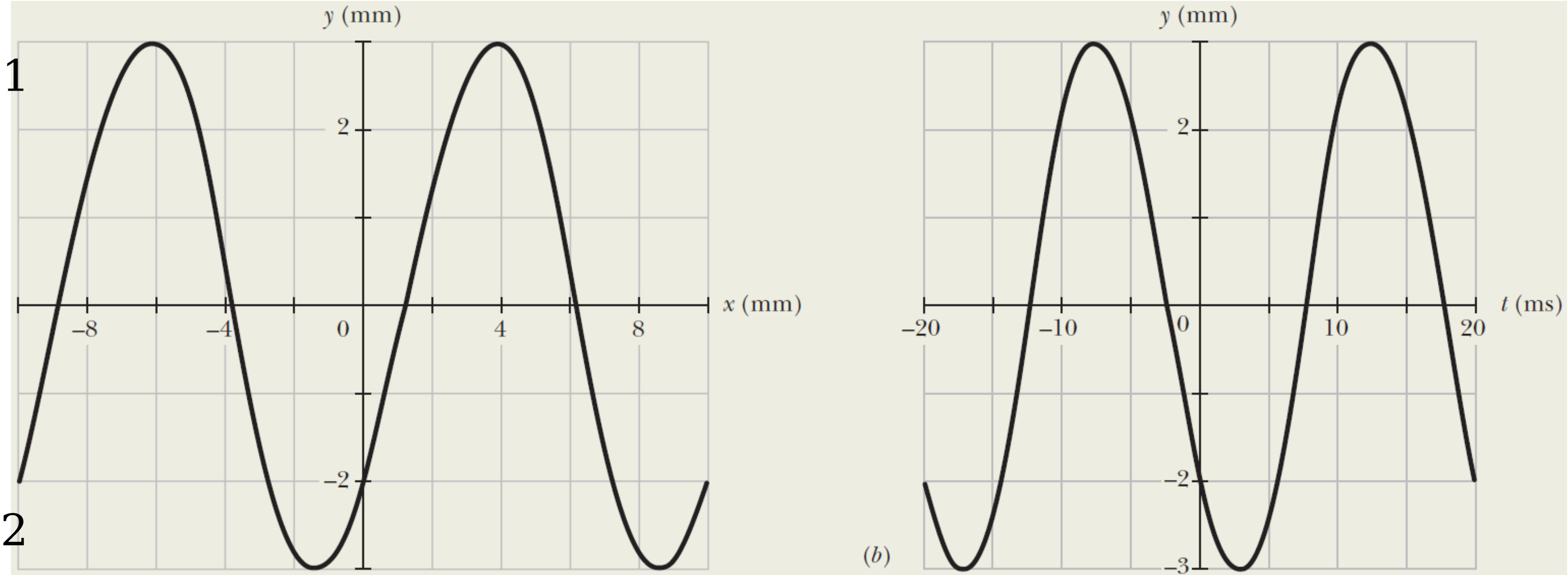
- A wave traveling in the negative direction of x is described by the equation

$$y(x,t) = y_m \sin(kx+\omega t)$$ and its velocity $$\frac{dx}{dt} = -\frac{\omega}{k}$$

- The minus sign verifies that the wave is indeed moving in the negative direction of x and justifies our switching the sign of the time variable.

- Consider a wave of arbitrary shape, given by $y(x,t) = h(kx \pm \omega t)$ where h represents any function, the sine function being one possibility.

Problem 16-1



problem 16-2

Wave Speed on a Stretched String

- The speed of a wave is related to the wave's wavelength and frequency, but *it is set by the properties of the medium*.

- For the medium to oscillate as a wave passes, the medium must possess both mass (for the kinetic energy) and elasticity (for the potential energy) ⇒ the medium's mass and elasticity properties determine the speed of a wave.

Dimensional Analysis

- In dimensional analysis we examine the dimensions of all the physical quantities involved to determine the quantities they produce.

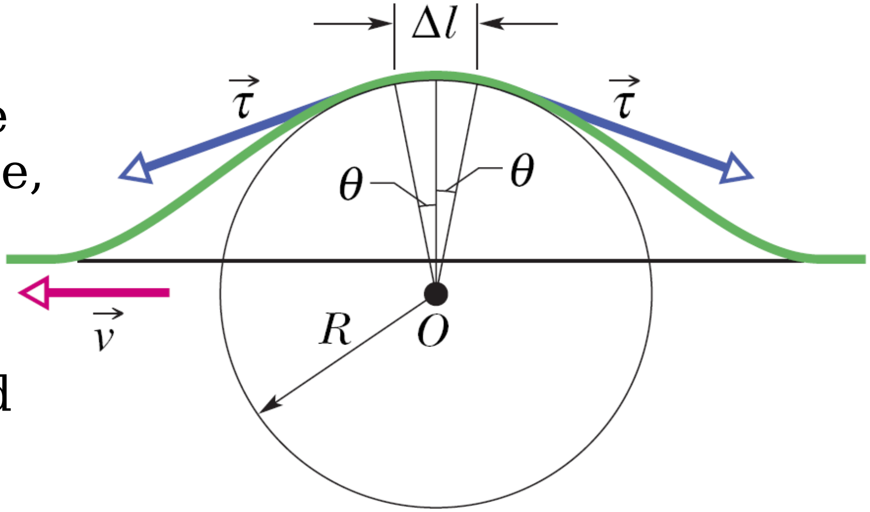
- We would like to examine mass m (M) and elasticity to find a speed v (LT^{-1}).

- For the mass m (M) of a string element ℓ (L), the *linear density* μ of the string is used to represent it by $\mu = m/\ell$ (ML^{-1}).

- Tension τ (MLT^{-2}) is needed to send a wave along a string. As a wave travels along the string, it displaces elements of the string by causing additional stretching, with adjacent sections of string pulling on each other because of the tension. Thus, tension is related with the elasticity of the string.

- Want to generate v by combining μ with τ $\Rightarrow$ $v = C\sqrt{\dfrac{\tau}{\mu}}$, in which C is a dimensionless constant that cannot be determined with dimensional analysis.

Derivation from Newton's 2nd Law



- Choose a reference frame in which the pulse remains stationary; ie, run along with the pulse, keeping it constantly in view.

- The horizontal components of these tension forces cancel, but the vertical components add to form a radial restoring force

$$F = 2(\tau \sin\theta) \approx \tau(2\theta) = \tau \frac{\Delta \ell}{R} \quad \text{force}$$

- The mass of the element $\Delta m = \mu \Delta \ell \quad \text{mass}$

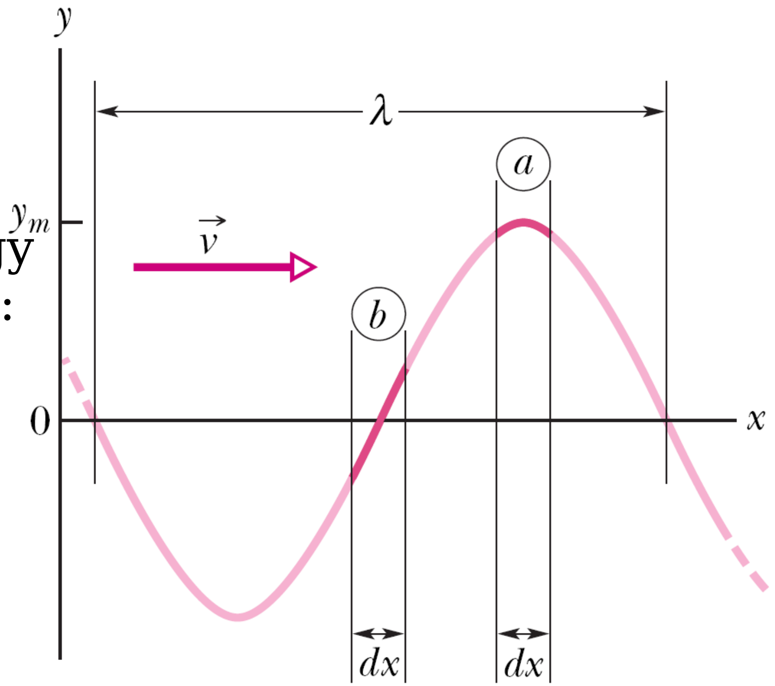
- The string element $\Delta\ell$ has a centripetal acceleration to the center of the circle

$$\text{acceleration} \ \ a = \frac{v^2}{R} \quad \Rightarrow \quad \text{force} = \text{mass} \times \text{acceleration}$$

$$\Rightarrow \quad \tau \frac{\Delta \ell}{R} = \mu \Delta \ell \frac{v^2}{R} \quad \Rightarrow \quad \boxed{v = \sqrt{\frac{\tau}{\mu}}} \ \ \text{speed} \quad \Rightarrow \quad C = 1$$

The speed of a wave along a stretched ideal string depends only on the tension and linear density of the string and not on the frequency of the wave.

Energy and Power of a Wave Traveling Along a String



- As the wave moves away, it transports that energy as both kinetic energy and elastic potential energy:

Kinetic Energy

- A string element of mass dm, oscillating transversely in simple harmonic motion as the wave passes through it, has kinetic energy associated with its transverse velocity $\vec{u}$.

$$\left[\begin{array}{ll} \text{the tranverse velocity \& the kinetic energy reach their maxima at} & y=0 \\ \text{the tranverse velocity \& the kinetic energy vanish} \quad \text{at} & y=y_m \end{array}\right.$$

Elastic Potential Energy

- A string element of length dx, oscillating transversely, must increase and decrease its length in a periodic way to fit the sinusoidal wave form. Elastic potential energy is associated with these length changes, just as for a spring.

$$\left[\begin{array}{ll} \text{the length change \& the potential energy reach their maxima at} & y=0 \\ \text{the length change \& the potential energy vanish} \quad \text{at} & y=y_m \end{array}\right.$$

Energy Transport

- The oscillating string element has both its maximum kinetic energy and its maximum elastic potential energy at $y = 0$.

- The regions of the string at maximum displacement have no energy, the regions at zero displacement have maximum energy.

- As the wave travels along the string, forces due to the tension in the string continuously do work to transfer energy from regions with energy to regions with no energy. Thus the wave *transports* the energy along the string.

The Rate of Energy Transmission

- The kinetic energy associated with a string element of mass: $\mathrm{d}K = \frac{1}{2}\,\mathrm{d}m\,u^2$
- The transverse speed $u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$

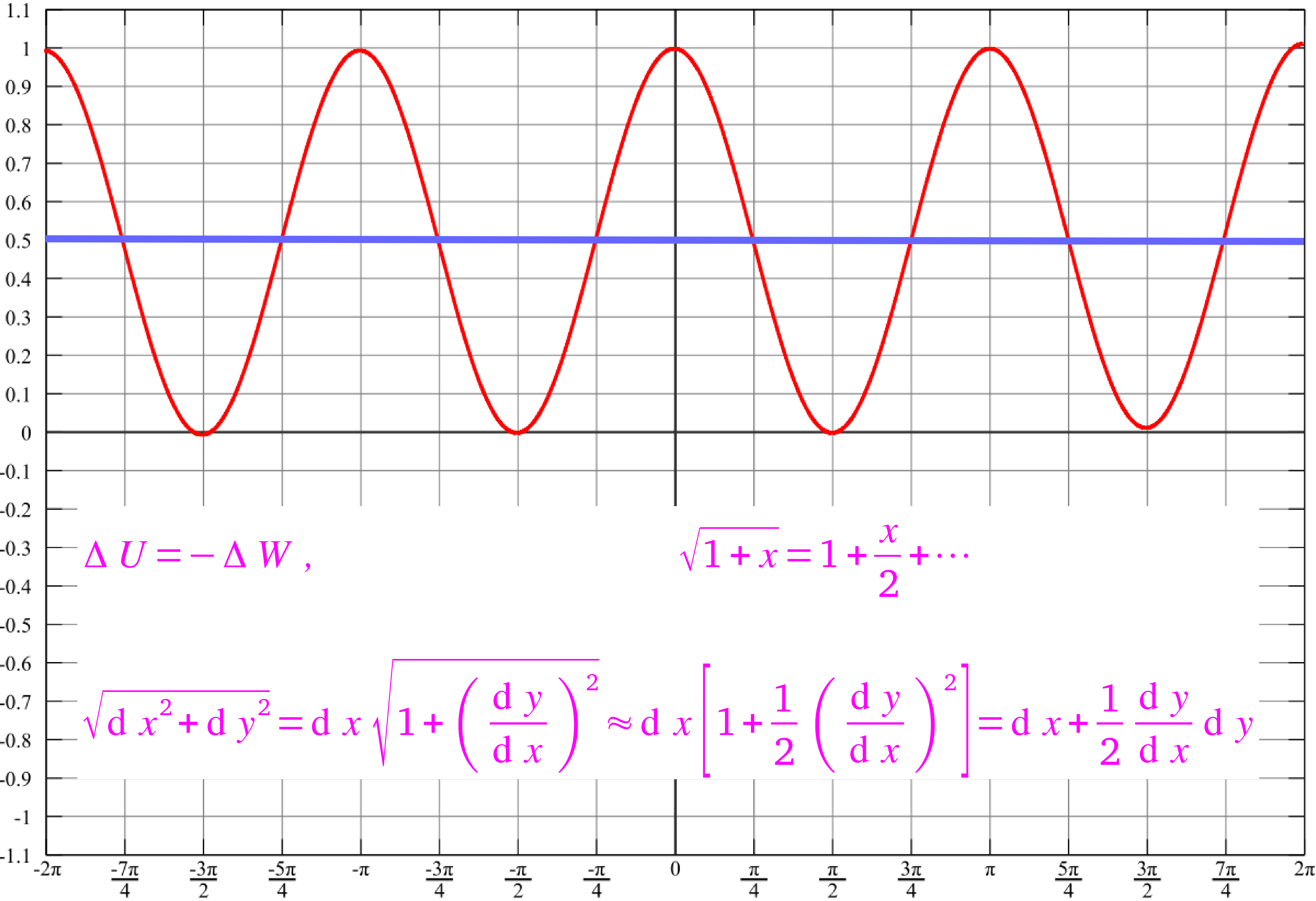
$$\Rightarrow \quad \mathrm{d}K = \frac{1}{2}(\mu\,\mathrm{d}x)(-\omega y_m)^2 \cos^2(kx - \omega t) \quad \Leftarrow \quad \mathrm{d}m = \mu\,\mathrm{d}x$$

$$\Rightarrow \quad \frac{\mathrm{d}K}{\mathrm{d}t} = \frac{1}{2}\mu v \omega^2 y_m^2 \cos^2(kx - \omega t) \qquad \Leftarrow \quad v = \frac{\mathrm{d}x}{\mathrm{d}t}$$

- The *average* rate at which kinetic energy is transported is

$$\left.\frac{\mathrm{d}K}{\mathrm{d}t}\right|_{\mathrm{avg}} = \frac{\mu v}{2}\omega^2 y_m^2 \left[\cos^2(kx - \omega t)\right]_{\mathrm{avg}} = \frac{\mu v}{4}\omega^2 y_m^2$$

$$\text{where} \quad \left[\cos^2\theta\right]_{\mathrm{avg}} = \frac{1}{2\pi}\int_0^{2\pi} \cos^2\theta\,\mathrm{d}\theta = \frac{1}{2}$$



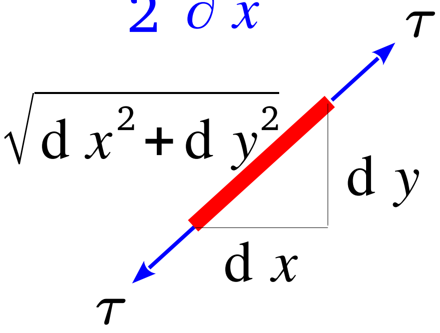
$$\Delta U = -\Delta W ,$$

$$\sqrt{1+x} = 1 + \frac{x}{2} + \cdots$$

$$\sqrt{d x^2 + d y^2} = d x \sqrt{1 + \left(\frac{d y}{d x}\right)^2} \approx d x \left[1 + \frac{1}{2}\left(\frac{d y}{d x}\right)^2\right] = d x + \frac{1}{2}\frac{d y}{d x} d y$$

- For the elastic potential energy

$$dU = -\tau\left(\sqrt{dx^2+dy^2}-dx\right) \approx -\tau\, dx\left[1+\frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2\right]+\tau\, dx = -\frac{\tau}{2}\frac{\partial y}{\partial x}dy$$



$$\Rightarrow \quad \frac{dU}{dt} = -\frac{\tau}{2}\frac{\partial y}{\partial x}\frac{\partial y}{\partial t} = \frac{1}{2}\tau k \omega y_m^2 \cos^2(kx-\omega t)$$

$$= \frac{1}{2}\mu v \omega^2 y_m^2 \cos^2(kx-\omega t) \quad \Leftarrow \quad \sqrt{\frac{\tau}{\mu}} = v = \frac{\omega}{k}$$

$$\Rightarrow \quad \left.\frac{dU}{dt}\right|_{\text{avg}} = \frac{1}{2}\mu v \omega^2 y_m^2 \left[\cos^2(kx-\omega t)\right]_{\text{avg}} = \frac{1}{4}\mu v \omega^2 y_m^2$$

- The average kinetic energy and the average potential energy are equal.

- The **average power**, the average rate of the total energy transmitted by the wave,

$$P_{\text{avg}} = \left[\frac{dK}{dt}+\frac{dU}{dt}\right]_{\text{avg}} = \left.\frac{dK}{dt}\right|_{\text{avg}} + \left.\frac{dU}{dt}\right|_{\text{avg}} = 2\left.\frac{dK}{dt}\right|_{\text{avg}}$$

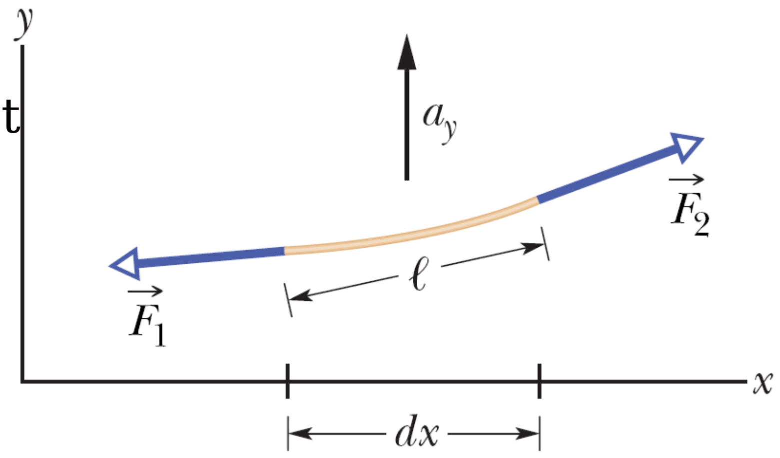
$$\Rightarrow \quad P_{\text{avg}} = \frac{1}{2}\mu v \omega^2 y_m^2 \quad \text{average power}$$

- The dependence of the average power of a wave on the square of its amplitude and also on the square of its angular frequency is a general result, true for waves of all types.

problem 16-3

The Wave Equation

- Assume that the wave amplitude is small so that the element can be tilted only slightly from the x axis as the wave passes.



(a)

- Because of the slight curvature of the element, the 2 forces produce a net force that causes the element to have an upward acceleration a_y

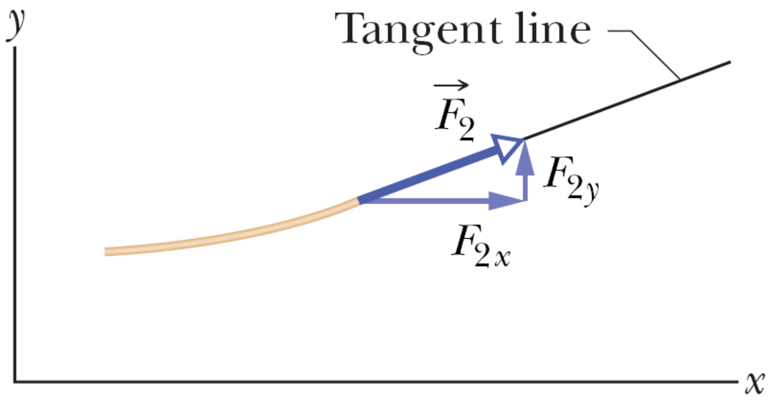
$$F_{\text{net},y} = F_{2y} - F_{1y} = \mathrm{d}m\, a_y = (\mu\, \mathrm{d}x)\frac{\mathrm{d}^2 y}{\mathrm{d}t^2}$$

- For the forces, the string slopes are defined as

$$S_1 \equiv \frac{F_{1y}}{F_{1x}}, \quad S_2 \equiv \frac{F_{2y}}{F_{2x}}$$

and the magnitudes

$$\sqrt{F_{1x}^2 + F_{1y}^2} \Leftarrow F_1 = \tau = F_2 \Rightarrow \sqrt{F_{2x}^2 + F_{2y}^2}$$



(b)

- Since the element is only slightly tilted $\left[\begin{array}{lll} F_{1y} \| F_{1x} & \Rightarrow & F_{1x} \approx \tau, \quad F_{1y} \approx S_1 \tau \\ F_{2y} \| F_{2x} & \Rightarrow & F_{2x} \approx \tau, \quad F_{2y} \approx S_2 \tau \end{array}\right.$

Thus $S_2 \tau - S_1 \tau = (\mu\, \mathrm{d}x)\dfrac{\mathrm{d}^2 y}{\mathrm{d}t^2} \quad \Rightarrow \quad \dfrac{\mu}{\tau}\dfrac{\mathrm{d}^2 y}{\mathrm{d}t^2} = \dfrac{S_2 - S_1}{\mathrm{d}x} = \dfrac{\mathrm{d}S}{\mathrm{d}x} \quad \Leftarrow \quad \mathrm{d}S \equiv S_2 - S_1$

- Since S is the slope at any point, $S=\frac{\mathrm{d}\,y}{\mathrm{d}\,x} \quad \Rightarrow \quad \frac{\mathrm{d}\,S}{\mathrm{d}\,x}=\frac{\mathrm{d}\left(\frac{\mathrm{d}\,y}{\mathrm{d}\,x}\right)}{\mathrm{d}\,x} \Rightarrow \frac{\partial^2 y}{\partial x^2}=\frac{\mu}{\tau}\frac{\partial^2 y}{\partial t^2}$

$$\Rightarrow \quad \frac{\partial^2 y}{\partial x^2}=\frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} \quad \text{wave equation} \quad \Leftarrow \quad v=\sqrt{\frac{\tau}{\mu}}$$

Note: we switched to the notation of partial derivatives because we differentiate only with respect to x or to t.

The Principle of Superposition for Waves

- If 2 waves travel simultaneously along the same stretched string, the displacement of the string when the waves overlap is then the algebraic sum

$$y'(x,t)=y_1(x,t)+y_2(x,t)$$

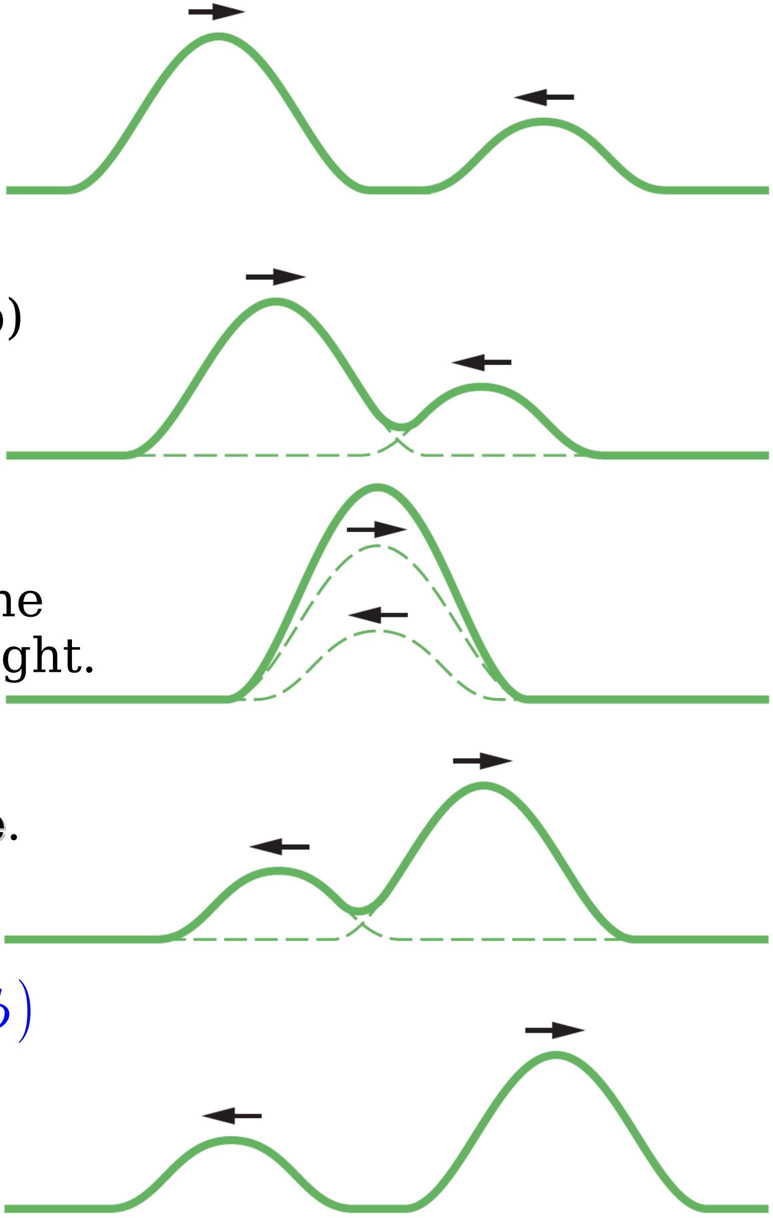
- This summation of displacements along the string means that

Overlapping waves algebraically add to produce a **resultant wave** (or **net wave**).

- **principle of superposition**: when several effects occur simultaneously, their net effect is the sum of the individual effects.

Overlapping waves do not in any way alter the travel of each other.

Interference of Waves



- For 2 sinusoidal waves of the same wavelength and amplitude in the same direction along a stretched string, the resultant wave depends on the extent to which the waves are *in phase* (in step) with respect to each other.

- If the waves are exactly in phase, they combine to double the displacement of either wave acting alone. If they are exactly out of phase, they combine to cancel everywhere, and the string remains straight.

- We call this phenomenon of combining waves **interference**, and the waves are said to **interfere**.

- For these 2 waves $y_1(x,t)=y_m\sin(kx-\omega t)$
  $y_2(x,t)=y_m\sin(kx-\omega t+\phi)$

- These 2 waves are said to be *out of phase* by ϕ or to have a *phase difference* of ϕ, or one wave is said to be *phase-shifted* from the other by ϕ.

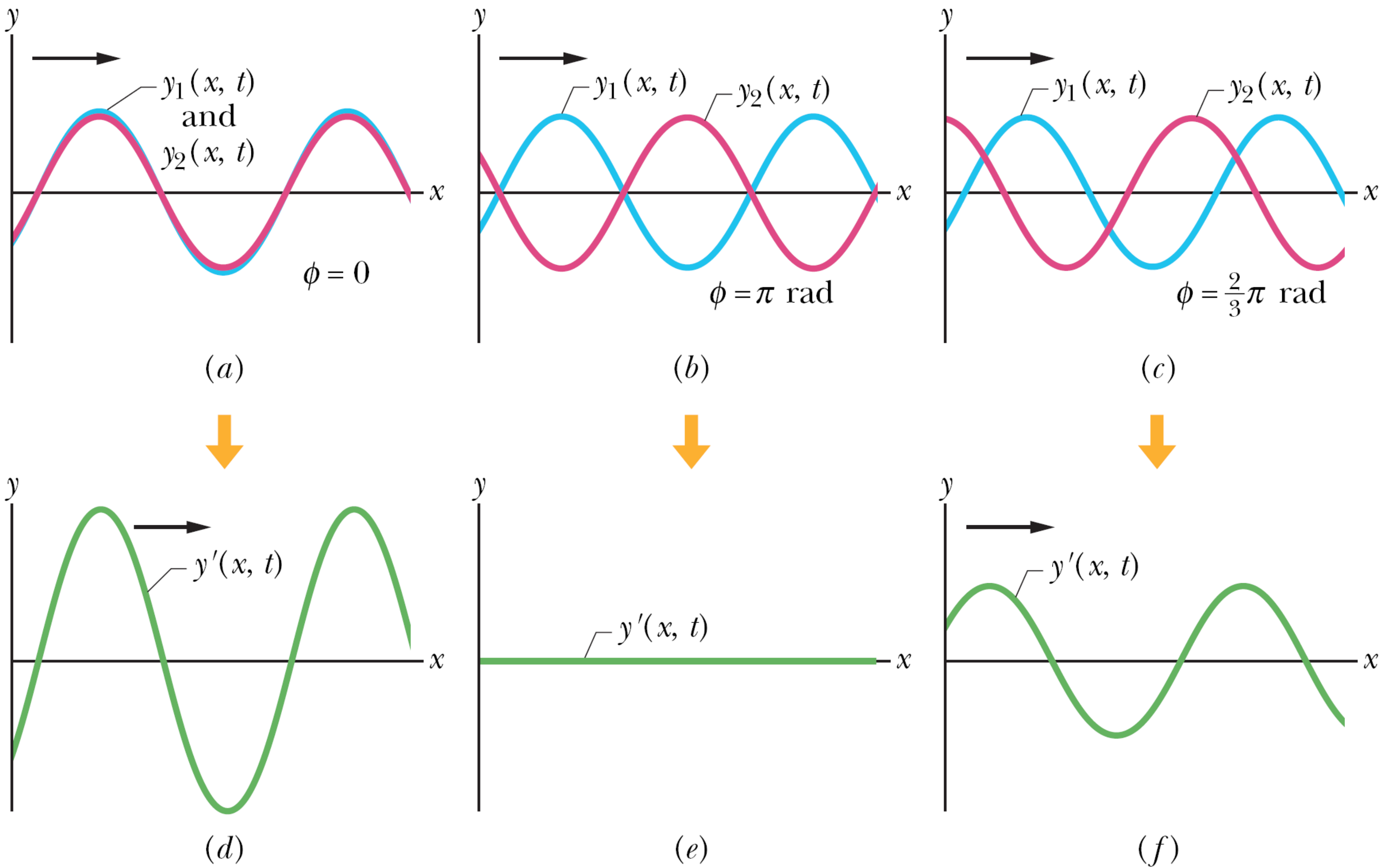
- From the principle of superposition, the resultant wave

$$y'(x,t)=y_1(x,t)+y_2(x,t)=y_m\sin(kx-\omega t)+y_m\sin(kx-\omega t+\phi)$$

$$\sin\alpha+\sin\beta=2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\Rightarrow\quad \boxed{y'(x,t)=\left(2y_m\cos\frac{\phi}{2}\right)\sin\left(kx-\omega t+\frac{\phi}{2}\right)}$$

$$\overbrace{y'(x,t)}^{\text{Displacement}}=\underbrace{[2y_m\cos\tfrac{1}{2}\phi]}_{\substack{\text{Magnitude}\\\text{gives}\\\text{amplitude}}}\;\underbrace{\sin(kx-\omega t+\tfrac{1}{2}\phi)}_{\substack{\text{Oscillating}\\\text{term}}}$$



(a) (b) (c)

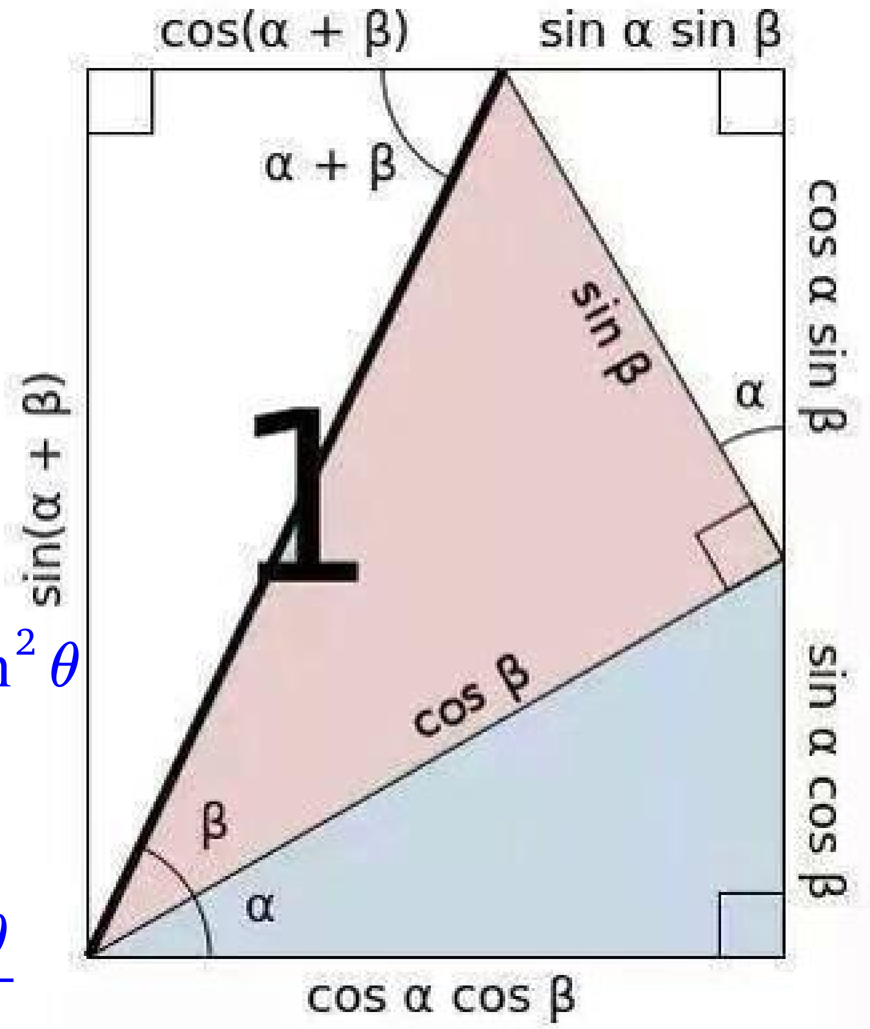
(d) (e) (f)

Trignometric Identities

$$\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$$

$$\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$$

$$\text{If } \theta = \phi \quad \Rightarrow \begin{cases} \sin 2\theta = 2\sin\theta\cos\theta \\ 0 = 0 \\ \cos 2\theta = \cos^2\theta - \sin^2\theta \\ \qquad\quad = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta \\ 1 = \cos^2\theta + \sin^2\theta \end{cases}$$



$$\Rightarrow \quad \cos^2\theta = \frac{1 + \cos 2\theta}{2}, \quad \sin^2\theta = \frac{1 - \cos 2\theta}{2}$$

$$\text{Let} \quad \begin{matrix} \theta = \dfrac{A+B}{2} \\ \phi = \dfrac{A-B}{2} \end{matrix} \quad \Rightarrow \quad \begin{matrix} \sin A \pm \sin B = +2\sin\dfrac{A \pm B}{2}\cos\dfrac{A \mp B}{2} \\ \cos A + \cos B = +2\cos\dfrac{A+B}{2}\cos\dfrac{A-B}{2} \\ \cos A - \cos B = -2\sin\dfrac{A+B}{2}\sin\dfrac{A-B}{2} \end{matrix}$$

If 2 sinusoidal waves of the same amplitude and wavelength travel in the same direction along a stretched string, they interfere to produce a resultant sinusoidal wave traveling in that direction.

- The resultant wave differs from the interfering waves in 2 respects:

(1) its phase constant: $\frac{\phi}{2}$; (2) its amplitude: $y'_m = \left| 2\, y_m \cos\frac{\phi}{2} \right|$ amplitude

- If $\phi = 0$, the interfering waves are exactly in phase, $y'(x,t) = 2\, y_m \sin(kx - \omega t)$ the amplitude of the resultant wave is the greatest amplitude, twice the amplitude of either interfering wave. Interference that produces the greatest possible amplitude is called *fully constructive interference*.

- If $\phi = \pi$, the interfering waves are exactly out of phase, $y'(x,t) = 0$ the amplitude of the resultant wave is the minimal one, 0. This type of interference is called *fully destructive interference*.

- Phase difference can be described in terms of wavelengths as well as angles

Because $\theta = \frac{2\pi L}{\lambda}$.

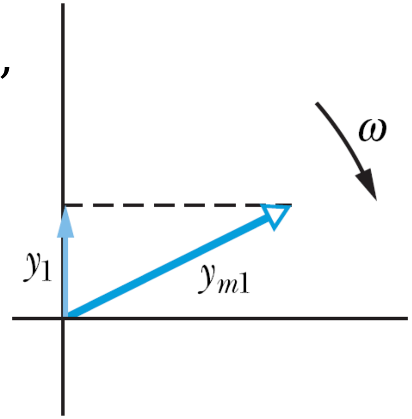
- When interference is neither fully constructive nor fully destructive, it is called *intermediate interference*. The amplitude of the resultant wave is intermediate between 0 and $2y_m$.

Problem 16-4

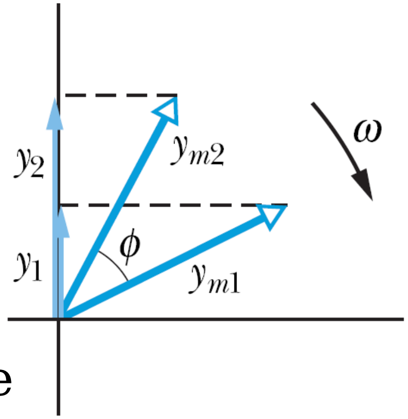
Phasors

- A **phasor** is a vector that has a magnitude equal to the amplitude of the wave and that rotates around an origin; the angular speed of the phasor is equal to the angular frequency of the wave. Thus, we can represent a wave vectorially with a phasor.

- When 2 waves travel along the same string in the same direction, we can represent them and their resultant wave in a *phasor diagram*.



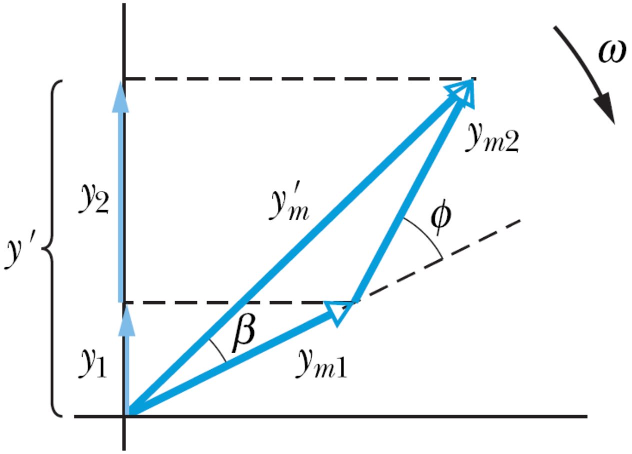
- If ϕ is a *positive* quantity, then the phasor for wave 2 *lags* the phasor for wave 1 as they rotate. If ϕ is a *negative* quantity, then the phasor for wave 2 *leads* the phasor for wave 1.



- To obtain the resultant wave, we vectorially add the 2 phasors at any instant during their rotation. The magnitude of the vector sum equals the resultant amplitude, The angle between the vector sum and wave 1 equals the resultant phase constant: $y'(x,t) = y'_m \sin(kx - \omega t + \beta)$

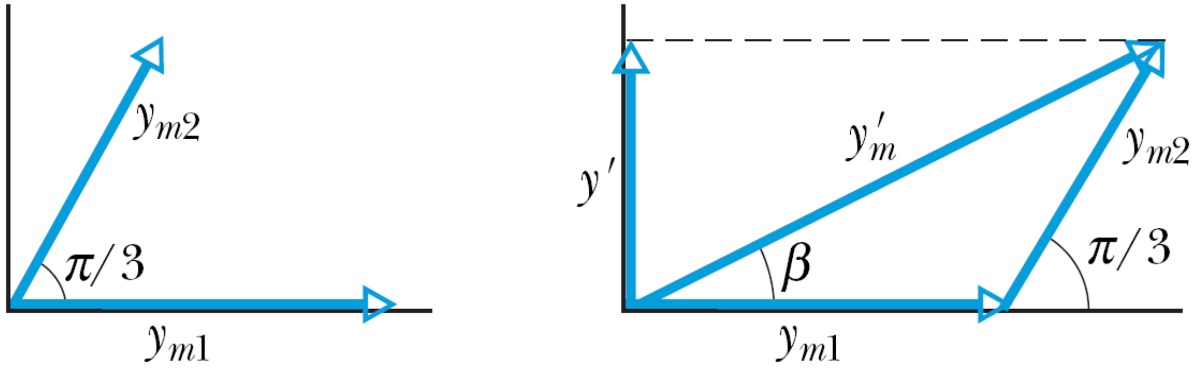
- In contrast to the method of the previous section:

We can use phasors to combine waves even if their amplitudes are different.



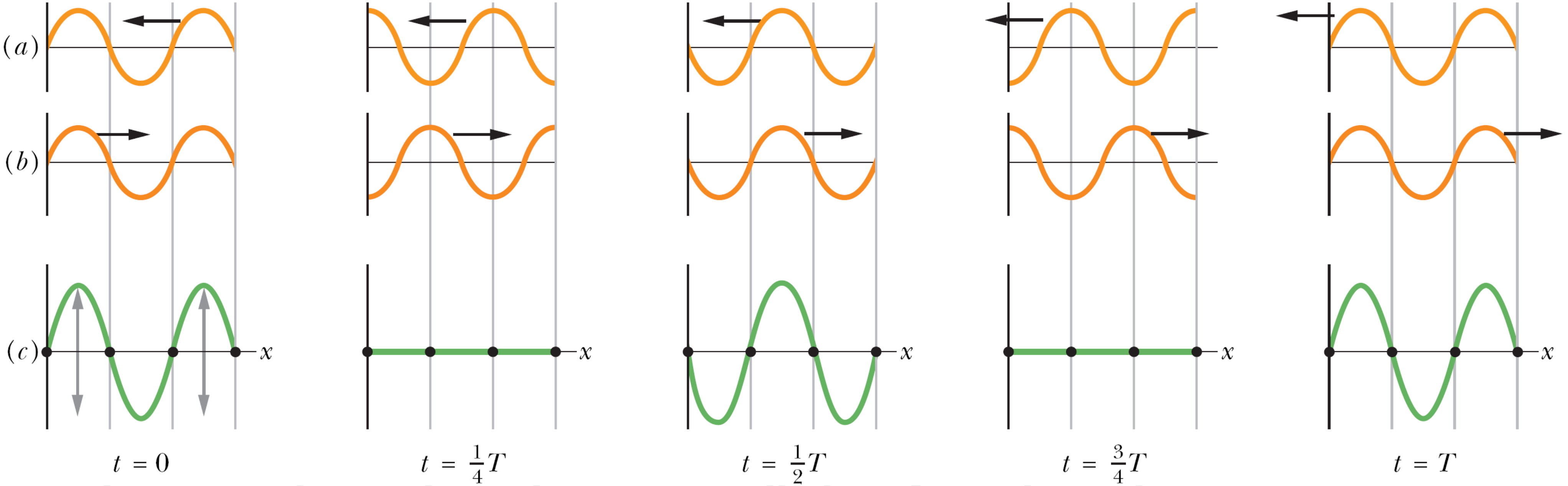
Standing Waves

- For interference, 2 sinusoidal waves of the same wavelength and amplitude traveling *in the same direction*.



problem 16-5

- If in opposite directions:



- There are places along the string, called **nodes**, where the string never moves.

- Halfway between adjacent nodes are **antinodes**, where the amplitude of the resultant wave is a maximum.

- **Standing waves:** the wave patterns do not move left or right; the locations of the maxima and minima do not change.

If 2 sinusoidal waves of the same amplitude and wavelength travel in opposite directions along a stretched string, their interference with each other produces a standing wave.

- To analyze a standing wave,

$$\begin{cases} y_1(x,t) = y_m \sin(kx - \omega t) \\ y_2(x,t) = y_m \sin(kx + \omega t) \end{cases}$$

$$\Rightarrow \quad y'(x,t) = y_1(x,t) + y_2(x,t)$$

$$= y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t)$$

$$\Rightarrow \quad y'(x,t) = [2 y_m \sin kx] \cos \omega t$$

$$\overbrace{y'(x,t)}^{\text{Displacement}} = \underbrace{[2y_m \sin kx]}_{\substack{\text{Magnitude} \\ \text{gives} \\ \text{amplitude} \\ \text{at position } x}} \underbrace{\cos \omega t}_{\substack{\text{Oscillating} \\ \text{term}}}$$

- The quantity $2 y_m \sin kx$ can be viewed as the amplitude of oscillation of the string element that is located at position x. Since an amplitude is always positive and $\sin kx$ can be negative, we take $|2 y_m \sin kx|$ to be the amplitude at x.

- In the standing wave, the amplitude is 0 (nodes) for

$$\sin kx = 0 \quad \Rightarrow \quad kx = n\pi \quad \Rightarrow \quad x = \frac{n\lambda}{2} \quad \text{for } n = 0,\ 1,\ 2, \cdots \quad \Leftarrow \quad k = \frac{2\pi}{\lambda}$$

- The amplitude of the standing wave has a maximum value of $2y_m$ (antinodes) for

$$|\sin kx| = 1 \quad \Rightarrow \quad kx = \left(n + \frac{1}{2}\right)\pi \quad \Rightarrow \quad x = \frac{2n+1}{4}\lambda \quad \text{for } n = 0,\ 1,\ 2, \cdots$$

- The antinodes are separated by $\dfrac{\lambda}{2}$, located halfway between pairs of nodes.

Reflections at a Boundary

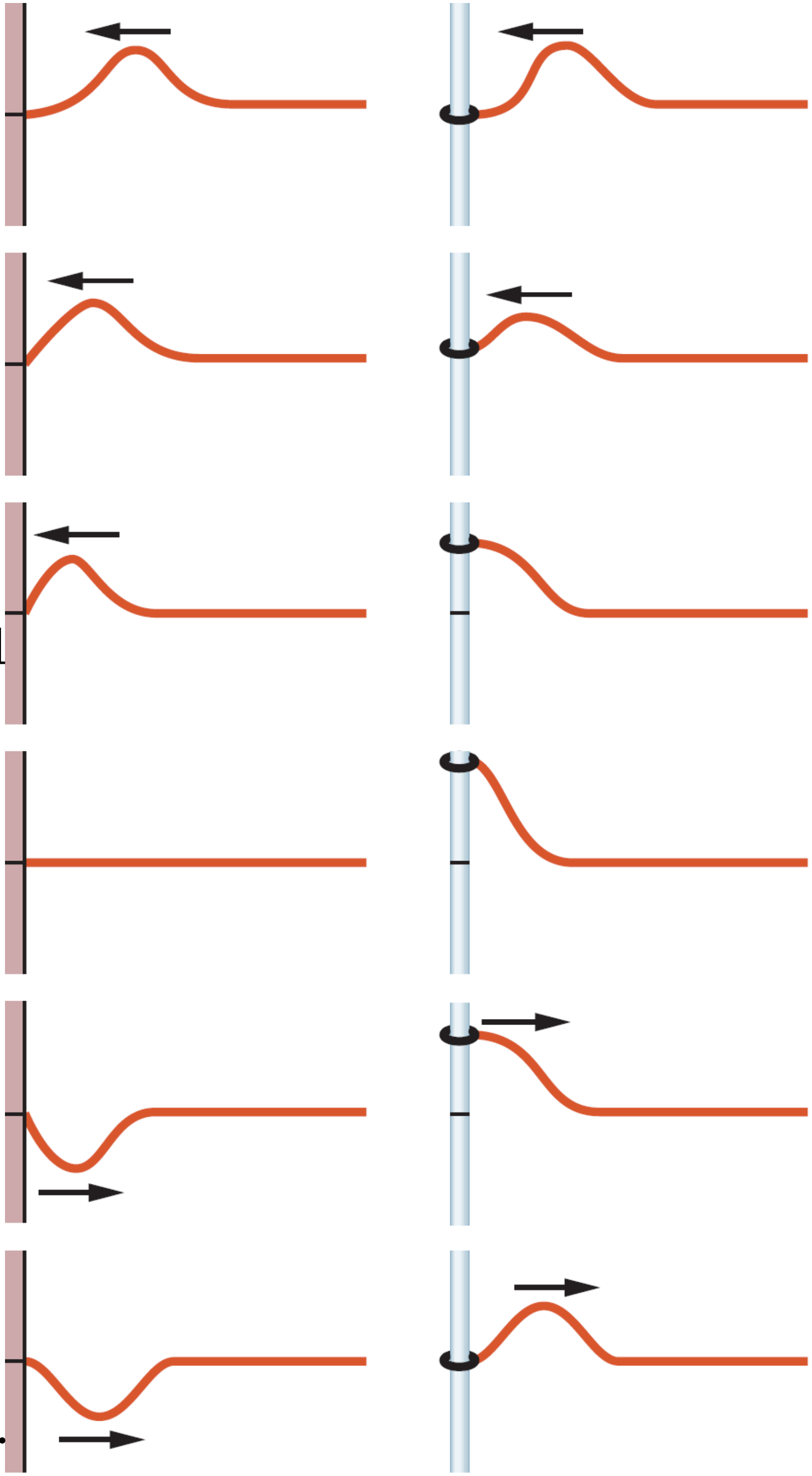
- The incident wave and the reflected wave can be described by

$$y_1(x,t)=y_m\sin(kx-\omega t)$$

$$y_2(x,t)=y_m\sin(kx+\omega t)$$

- In a “hard” reflection, there is a node at the support because the string is fixed there. The reflected and incident pulses must have opposite signs, so as to cancel each other at that point, due to Newton's 3rd law.

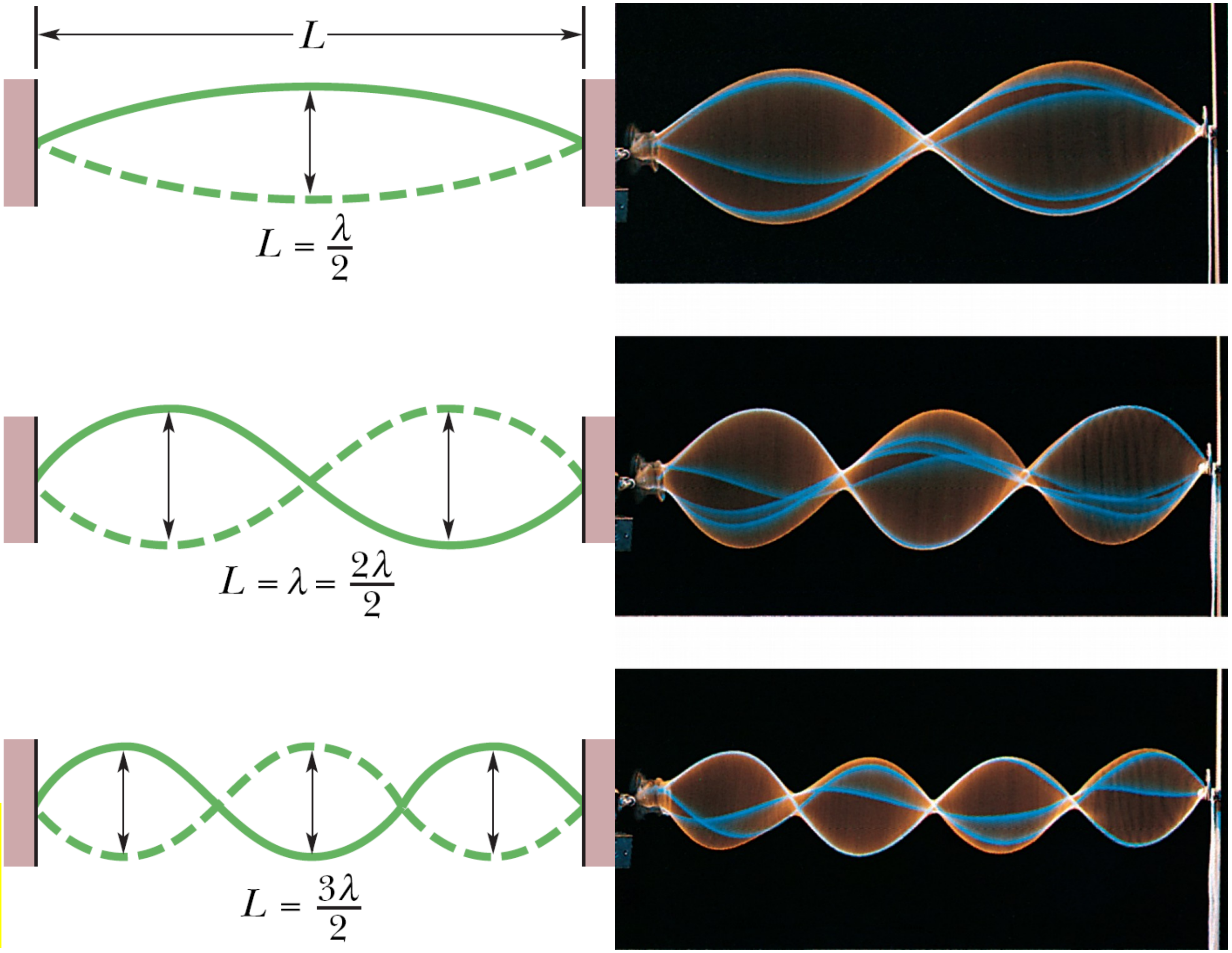
- In a “soft” reflection, the incident and reflected pulses reinforce each other, creating an antinode at the end of the string; the maximum displacement of the ring is twice the amplitude of either of these pulses, due to the tension becoming half at the end point.



Standing Waves and Resonance

- For certain frequencies, the interference produces a standing wave pattern (or **oscillation mode**) with nodes and large antinodes.

- Such a standing wave is said to be produced at **resonance**, and the string is said to resonate at these frequencies, called **resonant frequencies**.



- If the string is oscillated at other than a resonant frequency, a standing wave is not set up. Then the results in only small oscillations.

- A standing wave can be set up on a string of length L by a wave with a wavelength

$$\lambda = \frac{2L}{n} \quad \text{for } n = 1,\ 2,\ 3,\cdots$$

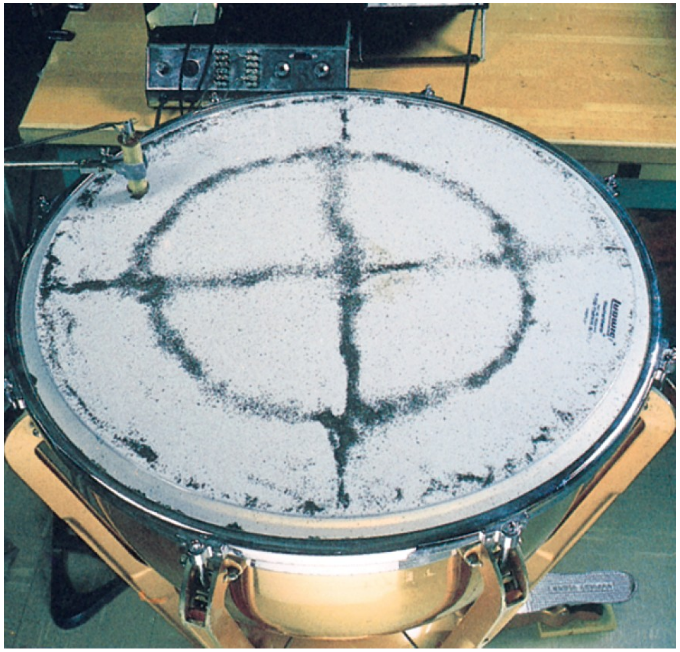
- The resonant frequencies

$$f = \frac{v}{\lambda} = \frac{nv}{2L} \quad \text{for } n = 1, 2, 3, \cdots$$

- The resonant frequencies are integer multiples of the lowest resonant frequency,

$$f = \frac{v}{2L}$$

- We call the *fundamental mode* or the 1st *harmonic* for $n=1$, the 2nd *harmonic* for $n=2$, the 3rd *harmonic* for $n=3$, … . The collection of all possible oscillation modes is called the **harmonic series**, and n is called the **harmonic number** of the n^{th} harmonic.



- The phenomenon of resonance is common to all oscillating systems and can occur in 2 and 3 dim.

Footbridges and Mosh Pits

- The footsteps of the pedestrians produces forces that sets up the 2nd harmonic on the bridge.

- Once the force exceeds a critical value, the 2nd harmonic becomes noticeable and walking becomes difficult, even dangerous. Need a damping device to counterbalance the swaying.

- In a mosh pit on a floor, the crowded dancers could set up resonance in the floor, with a typical resonant frequency of 2Hz, which could make the floor collapse.

- To avoid that possibility, modern building codes require that suspended dance floors be built with resonant frequencies > 5Hz.

The chosen problems: 21, 27, 36, 60.

Problem 16-6