Chapter 14 Fluids

What is a Fluid?

- A **fluid** is a substance that can flow. A fluid is a substance that flows because it cannot withstand a shearing stress. It can, however, exert a force in the direction perpendicular to its surface.

- The usual fluids are liquids and gases.

Density and Pressure

- With fluids, we are interested in the extended substance and in properties that can vary from point to point in that substance. It is more useful to speak of **density** and **pressure** than of mass and force.

Density

- For a small volume element dV around a point and measure the mass dm of the fluid contained within that element. Then the density

$$\rho = \frac{\mathrm{d}\, m}{\mathrm{d}\, V}$$

- For the density being constant, $\rho = \frac{m}{V}$ (uniform density), where m and V are the mass & volume of the sample.

- Density is a scalar property; its SI unit is the kilogram per cubic meter.

- the density of a gas varies considerably with pressure, but the density of a liquid does not; that is, gases are readily *compressible* but liquids are not.

Pressure

- We define the **pressure** on the piston from the fluid as

$$p = \lim \frac{\Delta F}{\Delta A} = \frac{dF}{dA}$$

- if the force is uniform over a flat area A,

$$p = \frac{F}{A} \qquad \text{pressure of uniform force on flat area}$$

where F is the magnitude of the normal force on area A.

- Pressure is a scalar, having no directional properties (from definition). Thus the pressure p has the same value no matter how the pressure sensor is oriented.

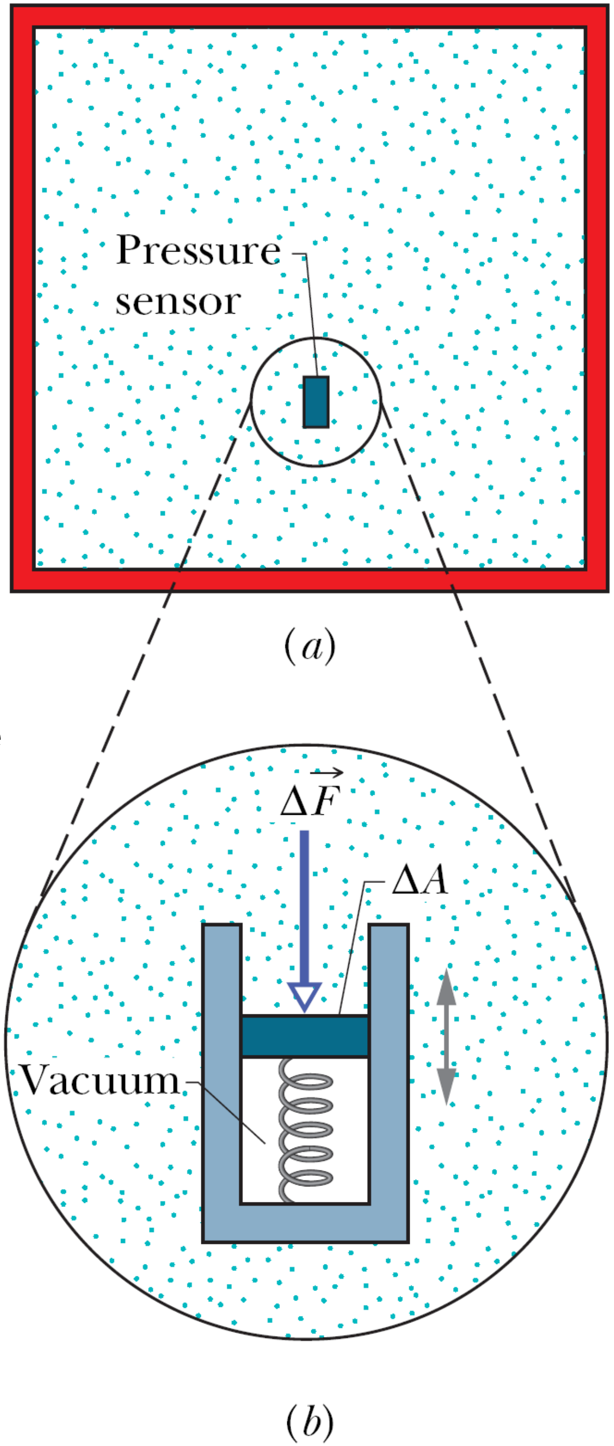
- The SI unit of pressure is the newton per square meter, which is given a special name, the **pascal** (Pa),

$$1 \text{ atm} = 1.01 \times 10^5 \text{ Pa} = 760 \text{ torr} = 14.7 \text{ lb/in}^2$$

atmosphere(**atm**): the approximate average pressure of the atmosphere at sea level.

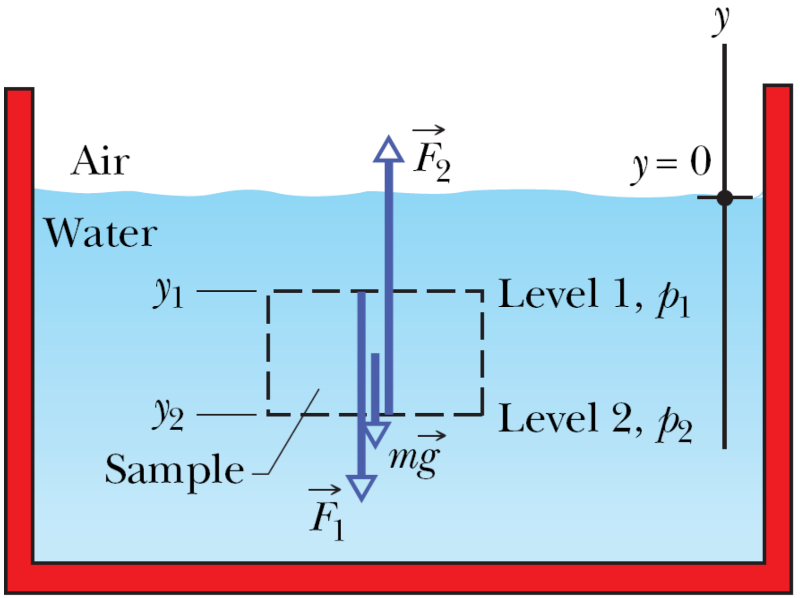
The **torr**: the *millimeter of mercury* (mm Hg).

The pound per square inch is often abbreviated **psi**.



problem 14-1

Fluids at Rest



(a)

- the pressure *increases* with depth below the air–water interface, and *decreases* with altitude as one ascends into the atmosphere.

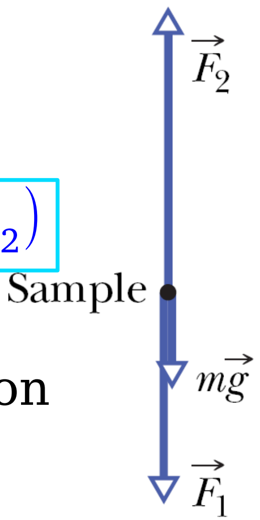
- The pressure is called *hydrostatic pressures*, due to fluids that are static (at rest).

- The water is in *static equilibrium*; ie, it is stationary and the forces on it balance.

- The balance of these forces is: $F_2 = F_1 + m g$

- $F_1 = p_1 A \,, \quad F_2 = p_2 A \,, \quad m = \rho V = \rho A (y_1 - y_2)$

  $\Rightarrow \quad p_2 A = p_1 A + \rho A g (y_1 - y_2) \quad \Rightarrow \quad \boxed{p_2 = p_1 + \rho g (y_1 - y_2)}$



(b)

- suppose we seek the pressure p at a depth h below the liquid surface, and p_0 to represent the atmospheric pressure on the surface,

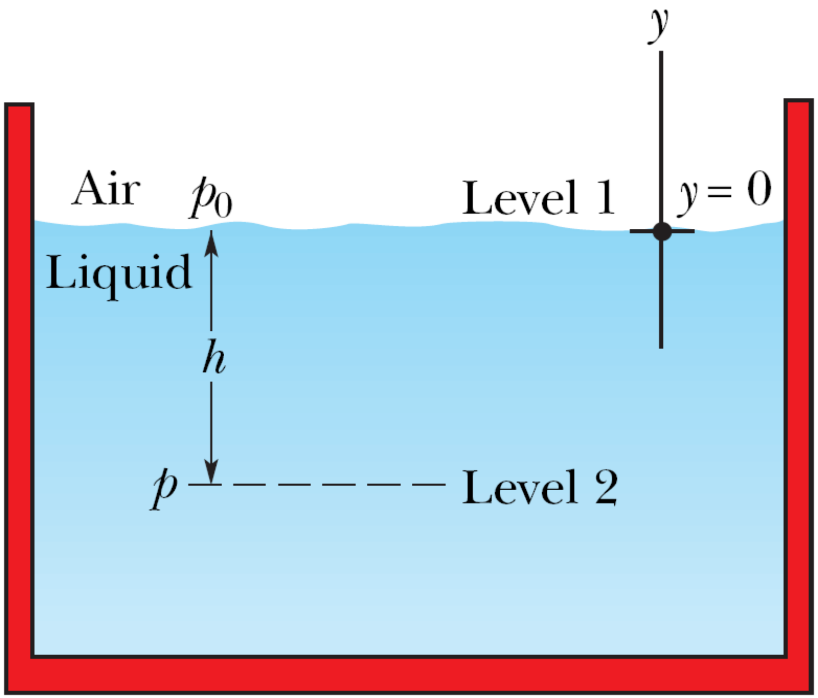
$$\boxed{p = p_0 + \rho g h \quad \text{pressure at depth } h}$$

The pressure at a point in a fluid in static equilibrium depends on the depth of that point but not on any horizontal dimension of the fluid or its container.

- p is said to be the total pressure, or **absolute pressure**, consisting of 2 contributions:

(1) p_0, the pressure due to the atmosphere, which bears down on the liquid, and

(2) ρgh, the pressure due to the liquid above level 2.



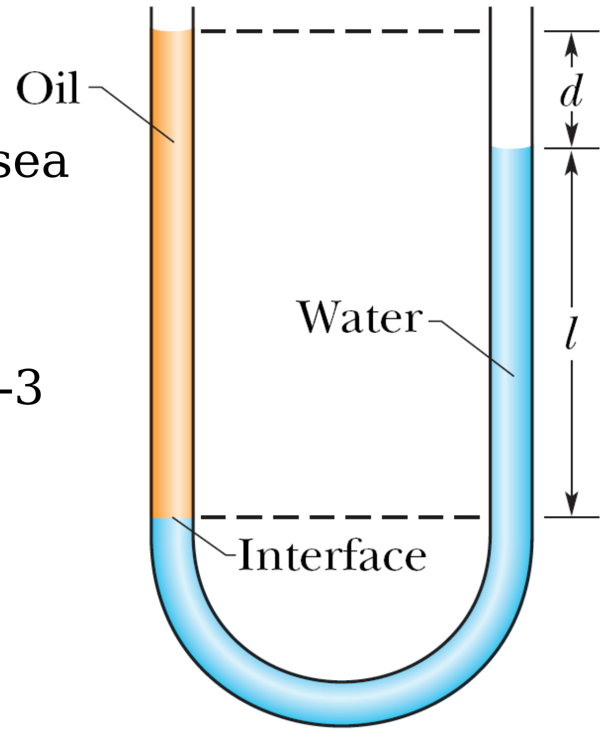
- **Gauge pressure**: the difference between an absolute pressure and an atmospheric pressure (in the case, ρgh).

- The relation also holds above the liquid surface: It gives the atmospheric pressure at a given distance above sea level in terms of the atmospheric pressure at sea level (*assuming* that the atmospheric density is uniform over that distance), eg, to find the atmospheric pressure at a distance d above sea level

$$p = p_0 - \rho_{air}\, g\, d$$

problem 14-2

problem 14-3



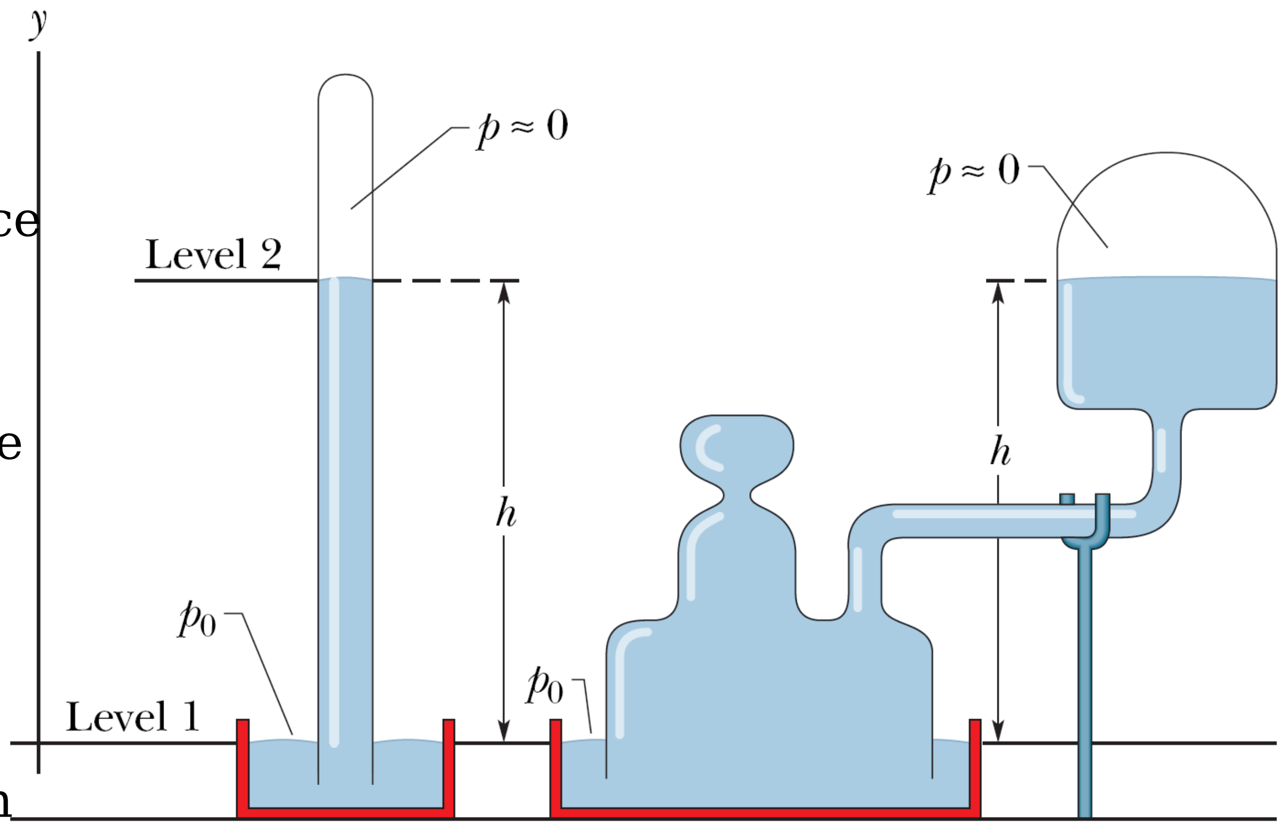
Measuring Pressure

The Mercury Barometer

- **mercury barometer**: a device to measure the pressure of the atmosphere.

- find the atmospheric pressure in terms of the height h of the mercury column:

$$p_0 = \rho g h$$



- For a given pressure, the height h of the mercury column does not depend on the cross-sectional area of the vertical tube.

- For a given pressure, the height depends on the value of g and on the density of mercury, varying with temperature. The height is equal to the pressure (in torr) only if the barometer is at a place where g = 9.80665 m/s^2 and T = 0°C. If these conditions do not prevail, small corrections must be made before the height of the mercury column can be transformed into a pressure.

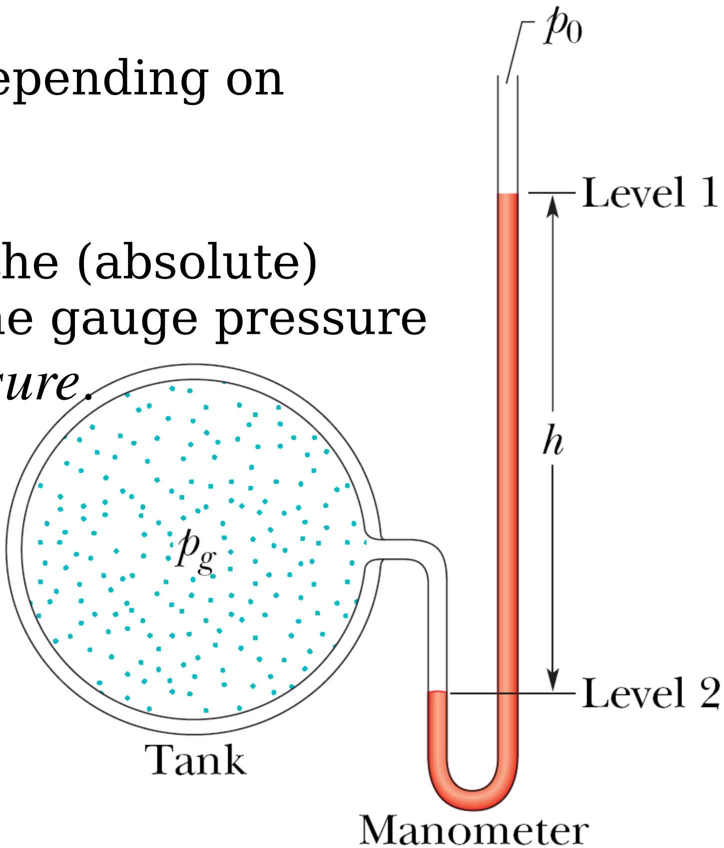
The Open-Tube Manometer

- An *open-tube manometer* measures the gauge pressure of a gas:

$$p_g = p - p_0 = \rho g h$$

- The gauge pressure can be positive or negative, depending on whether $p > p_0$ or $p < p_0$.

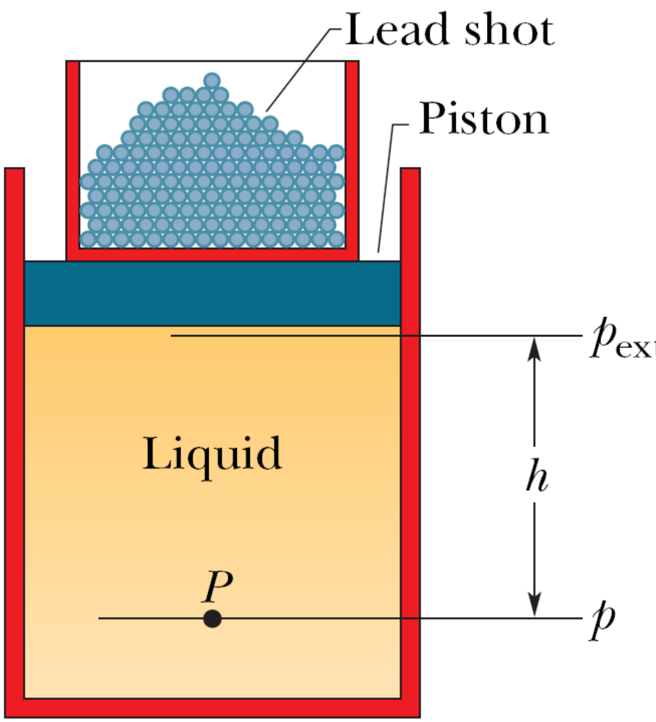
- In inflated tires or the human circulatory system, the (absolute) pressure is greater than atmospheric pressure, so the gauge pressure is a positive quantity, sometimes called the *overpressure*. If you suck on a straw to pull fluid up the straw, the (absolute) pressure in your lungs is actually less than atmospheric pressure. The gauge pressure in your lungs is then a negative quantity.



Pascal's Principle

- **Pascal's principle**:

> A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.



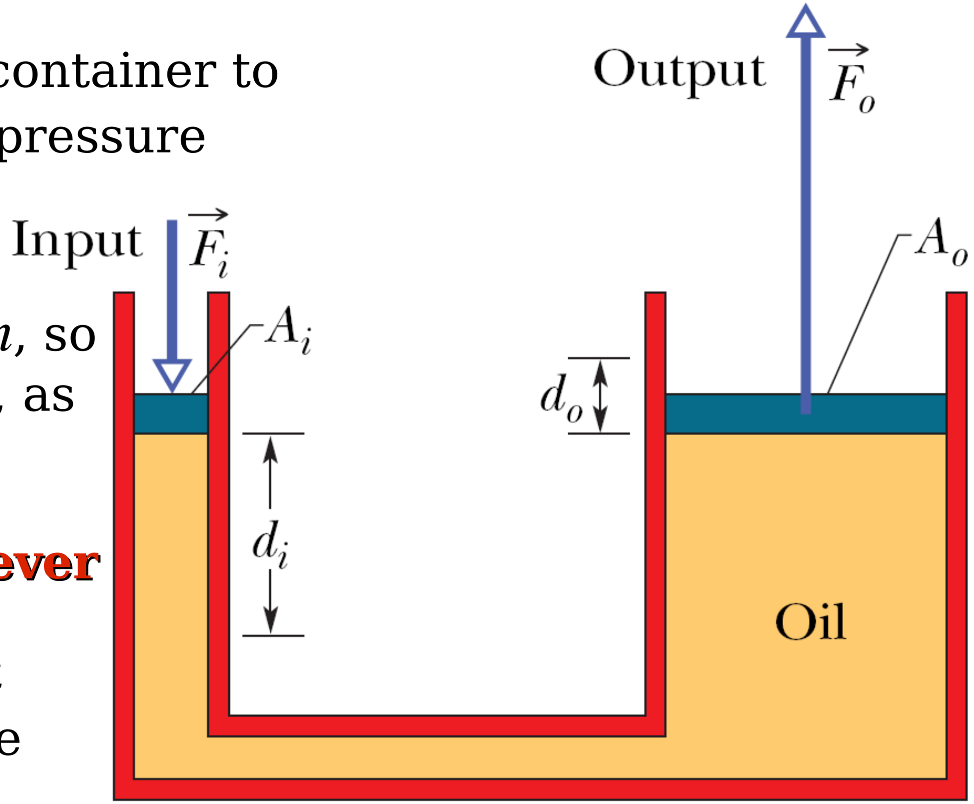
Demonstrating Pascal's Principle

- Let p_{ext} is the pressure coming from the atmosphere, the container, the lead shots, and the piston, then the pressure p at any point P in the liquid is

$$p = p_{ext} + \rho g h$$

- Let us add a little more lead shot to the container to increase p_{ext} by an amount Δp_{ext}. Then the pressure change at P is $\Delta p = \Delta p_{ext}$

- This pressure change is independent of h, so it must hold for all points within the liquid, as Pascal's principle states.



Pascal's Principle and the Hydraulic Lever

- The force applied on the left and the downward force from the load on the right produce a change Δp in the pressure of the liquid

$$\Delta p = \frac{F_i}{A_i} = \frac{F_o}{A_o} \quad \Rightarrow \quad F_o = F_i \frac{A_o}{A_i} \quad \Rightarrow \quad F_o > F_i \quad \text{if} \quad A_o > A_i$$

- Require that the same volume V of the incompressible liquid is displaced at both pistons. Then

$$V = d_i A_i = d_o A_o \quad \Rightarrow \quad d_o = d_i \frac{A_i}{A_o}$$

- If $A_o > A_i$, the output piston moves a smaller distance than the input piston moves.

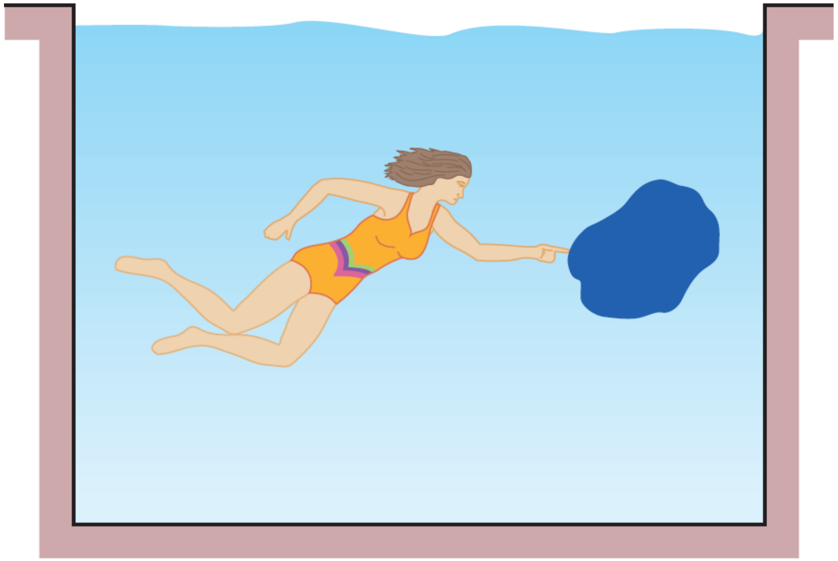
- The output work is: $W = F_o d_o = F_i \frac{A_o}{A_i} d_i \frac{A_i}{A_o} = F_i d_i$

- The work done on the input piston by the applied force is equal to the work done by the output piston in lifting the load placed on it.

- The advantage of a hydraulic lever is:

> With a hydraulic lever, a given force applied over a given distance can be transformed to a greater force applied over a smaller distance.

- For example, we can lift an automobile with a hydraulic jack.



Archimedes' Principle

- the sack and its contained water are in static equilibrium, tending neither to rise nor to sink. The gravitational force on the contained water must be balanced by a net upward force from the water surrounding the sack.

- This net upward force is a **buoyant force**. It exists because the pressure in the surrounding water increases with depth below the surface

$$p_{\text{bottom}} > p_{\text{top}} \quad \Rightarrow \quad F_{\text{bottom}} > F_{\text{top}}$$

- If we vectorially add all the forces on the sack from the water, the horizontal components cancel and the vertical components add to yield the upward buoyant force on the sack.

- the sack of water is in static equilibrium $\Rightarrow \quad F_b = m_f\, g$

The magnitude of the buoyant force is equal to the weight of the water in the sack.

- **Archimedes' principle**:

> When a body is fully or partially submerged in a fluid, a buoyant force $\vec{F}_b$ from the surrounding fluid acts on the body. The force is directed upward and has a magnitude equal to the weight $m_f g$ of the fluid that has been displaced by the body.

- The buoyant force on a body in a fluid

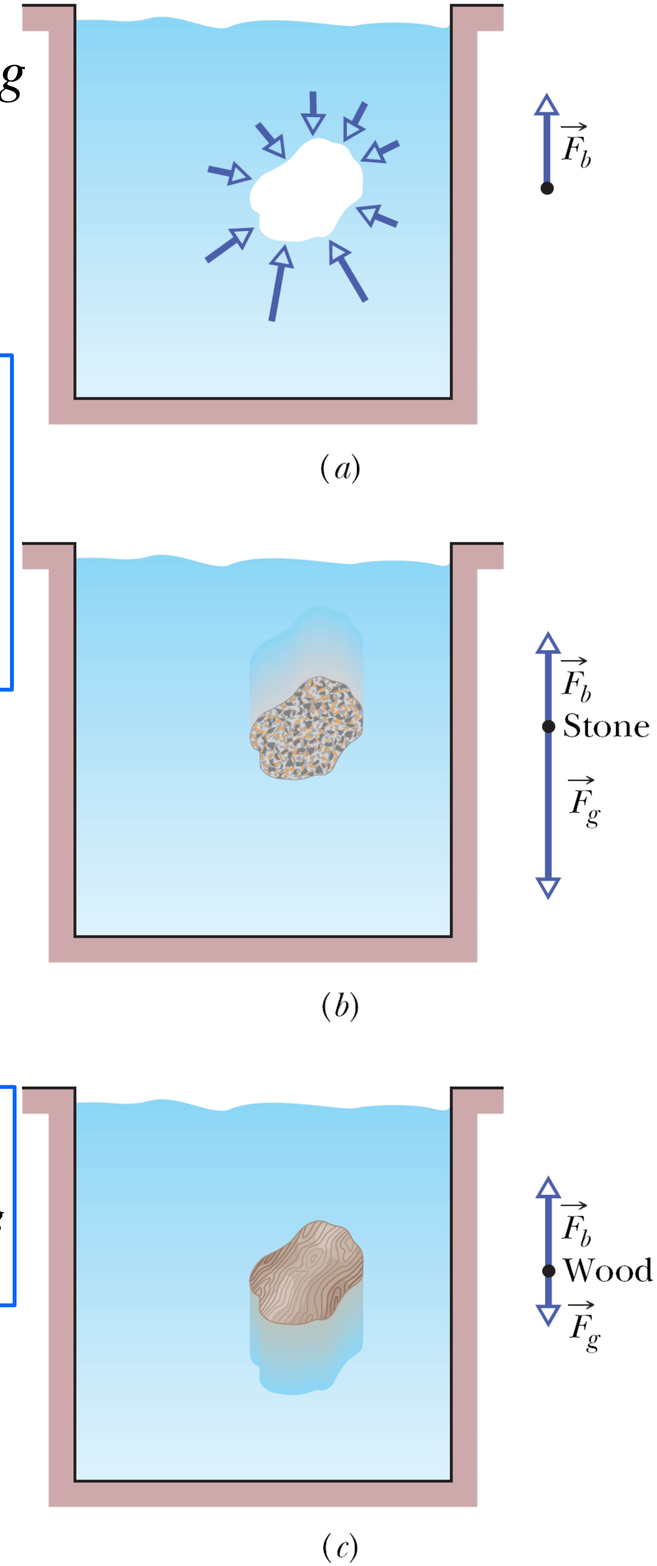
$$\vec{F}_b = -m_f\, \vec{g} \quad \text{buoyant force}$$

m_f :the mass of the fluid that is displaced by the body.

Floating

> When a body floats in a fluid, the magnitude F_b of the buoyant force on the body is equal to the magnitude F_g of the gravitational force on the body.

- write this statement as $\quad F_b = F_g \quad$ floating



Since $F_b = m_f g$

When a body floats in a fluid, the magnitude F_g of the gravitational force on the body is equal to the weight $m_f g$ of the fluid that has been displaced by the body.

- write this statement as $F_g = m_f g$ floating
- A floating body displaces its own weight of fluid.

Apparent Weight in a Fluid

- An **apparent weight** is related to the actual weight of a body and the buoyant force on the body by

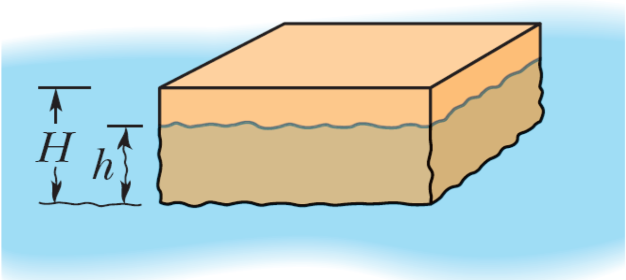
$$\begin{pmatrix}\text{apparent}\\ \text{weight}\end{pmatrix} = \begin{pmatrix}\text{actual}\\ \text{weight}\end{pmatrix} + \begin{pmatrix}\text{magnitude of}\\ \text{buoyant force}\end{pmatrix}$$

or

$$\text{weight}_{\text{app}} = \text{weight} - F_b \quad (\text{apparent weight})$$

- A floating body has an apparent weight of 0.

Problem 14-4



Ideal Fluids in Motion

- The motion of *real fluids* is very complicated and not yet fully understood. An **ideal fluid** is simpler to handle mathematically and yet provides useful results.

- four assumptions about our ideal fluid:

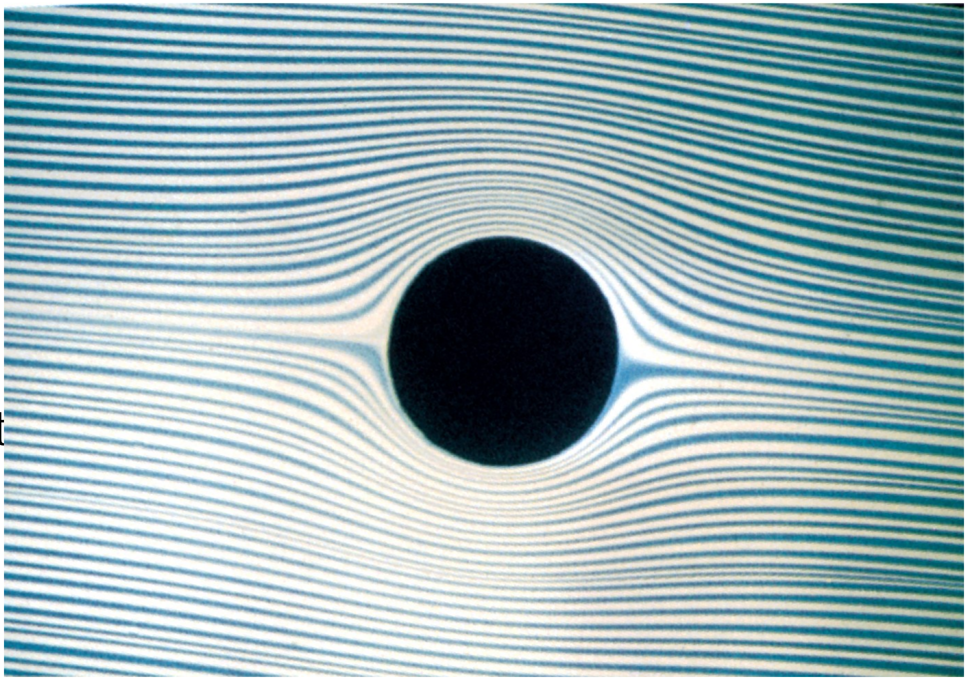
1. ***Steady flow***: In *steady* (or *laminar*) *flow*, the velocity of the moving fluid at any fixed point does not change with time, either in magnitude or in direction.

2. ***Incompressible flow***: The ideal fluid is incompressible, ie, its density has a constant, uniform value.

3. ***nonviscous flow***: The viscosity of a fluid is a measure of how resistive the fluid is to flow. Viscosity is the fluid analog of friction between solids. An object moving through a nonviscous fluid would experience no *viscous drag force*.

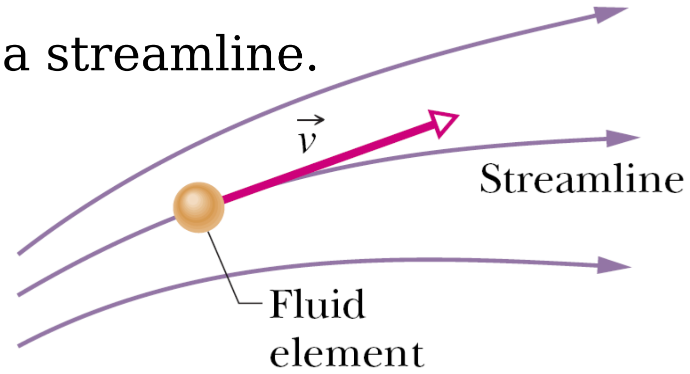
4. ***Irrotational flow***: In irrotational flow a test body will not rotate about an axis through its own center of mass, although this test body may (may not) move in a circular path.

- A *streamline* is the path that a tiny element of the fluid would take as the fluid flows.



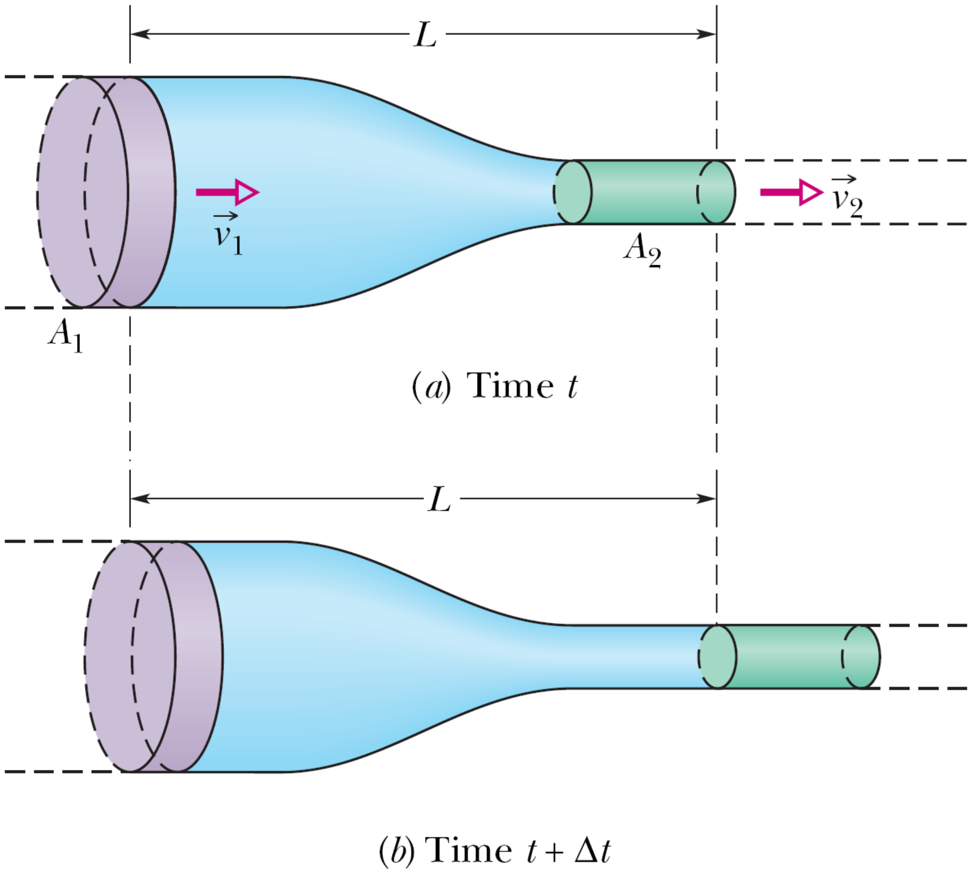
- The velocity of a fluid element is always tangent to a streamline.

- 2 streamlines can never intersect; if they did, then an element arriving at their intersection would have 2 different velocities simultaneously — an impossibility.



The Equation of Continuity

- Suppose that in a time interval Δt a volume ΔV of fluid enters the tube segment at its left end. Because the fluid is incompressible, an identical volume ΔV must emerge from the right end of the segment.

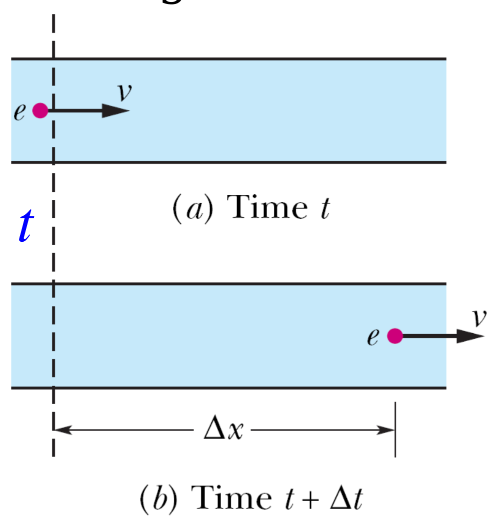


$$\Delta V = A \Delta x = Av \Delta t$$

$$\Rightarrow \quad A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

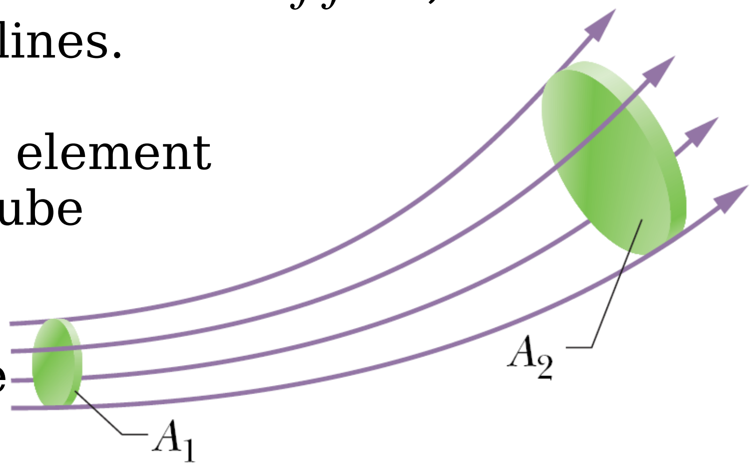
$$\Rightarrow \quad \boxed{A_1 v_1 = A_2 v_2}$$

equation of continuity



- The **equation of continuity** for the flow of an ideal fluid tells us that the flow speed increases when we decrease the cross-sectional area through which the fluid flows.

- The equation of continuity also applies to any so-called *tube of flow*, or imaginary tube whose boundary consists of streamlines.



- Such a tube acts like a real tube because no fluid element can cross a streamline; thus, all the fluid within a tube of flow must remain within its boundary.

- If the streamlines in a cross-section is denser, the velocity of the fluid is greater.

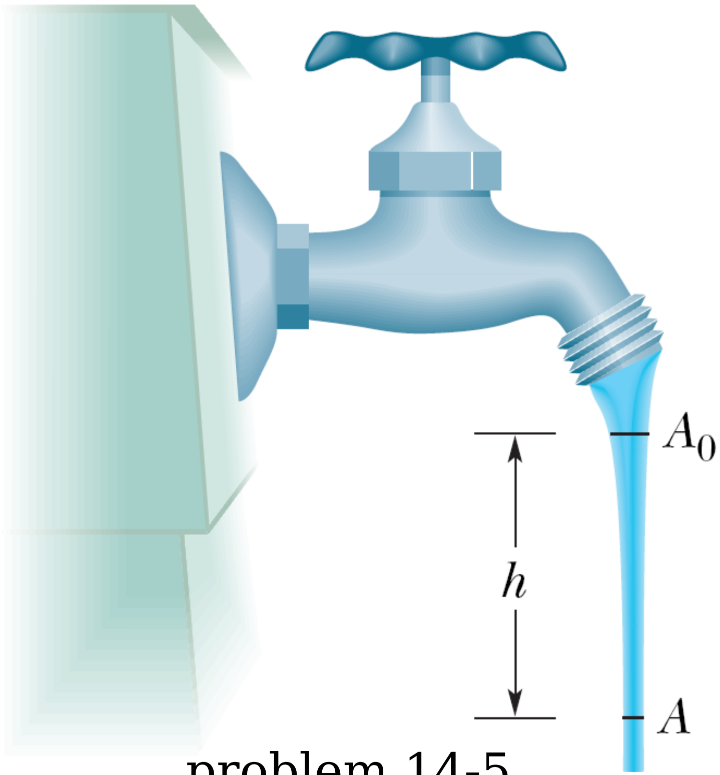
- Define R_V as the **volume flow rate** of the fluid, ie, volume past a given point per unit time [the SI unit is the cubic meter per second (m^3/s)], then

$$R_V = A\,v = \text{a constant} \quad (\text{volume flow rate, equation of continuity})$$

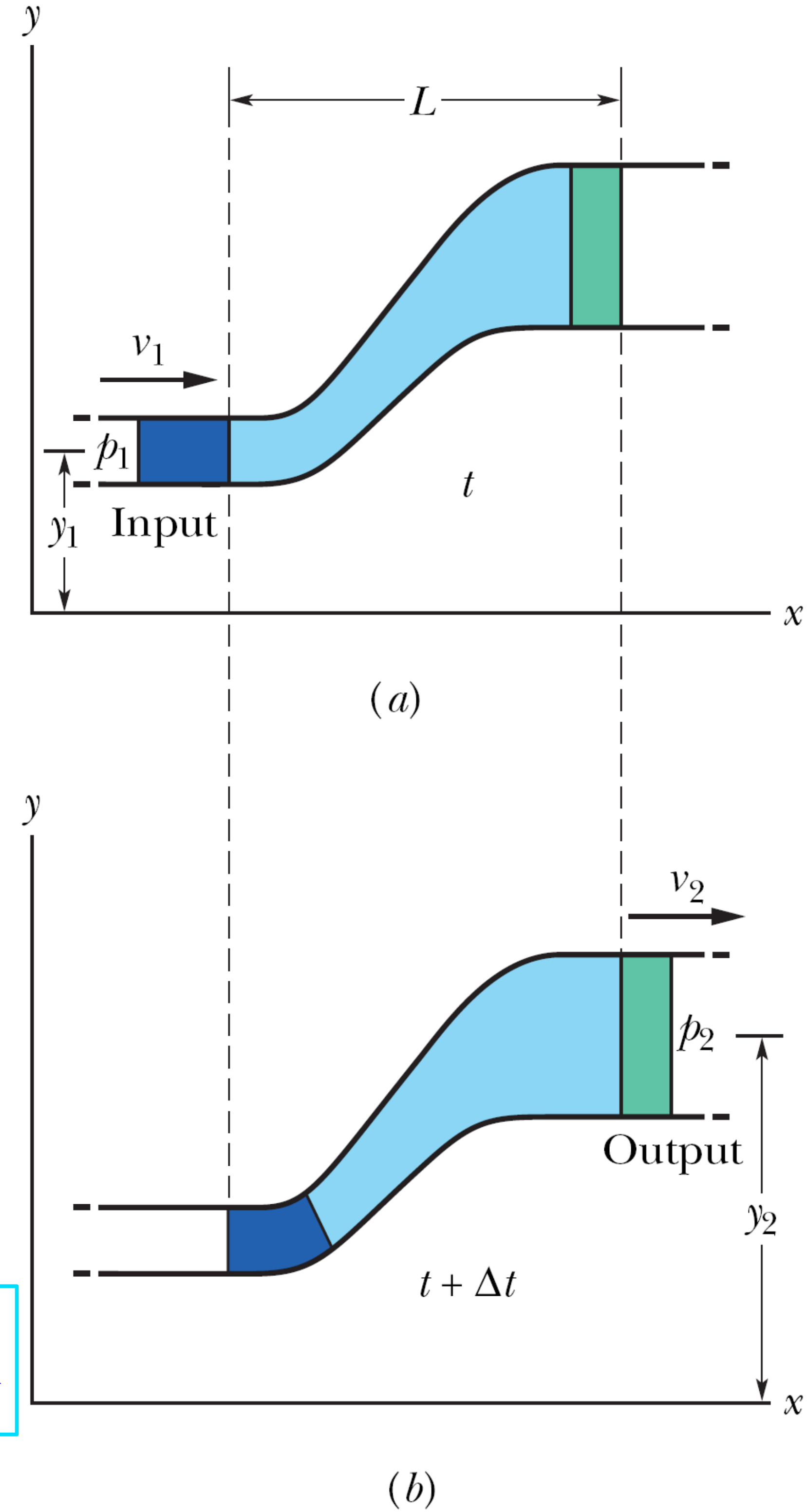
- If the density of the fluid is uniform, we can multiply the above equation by that density to get the **mass flow rate** R_m (mass per unit time) [the SI unit of mass flow rate is the kilogram per second (kg/s)]:

$$R_m = \rho\, R_V = \rho\, A\, v = \text{a constant} \quad (\text{mass flow rate})$$

- The mass that flows into the tube segment each second must be equal to the mass that flows out of that segment each second.



problem 14-5



Bernoulli's Equation

- By applying the principle of conservation of energy to the fluid, we shall show that these quantities are related by

$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

- Rewrite this equation as

$$p + \frac{1}{2}\rho v^2 + \rho g y = \text{a constant} \quad \text{Bernoulli's equation}$$

Proof of Bernoulli's Equation

For energy conservation in the form of the work-kinetic energy theorem, $W = \Delta K$

$$\Delta K = \frac{1}{2}\Delta m\, v_2^2 - \frac{1}{2}\Delta m\, v_1^2 = \frac{1}{2}\rho\,\Delta V\left(v_2^2 - v_1^2\right) \quad \Leftarrow \quad \Delta m = \rho\,\Delta V$$

$$W_g = -\Delta m\, g\left(y_2 - y_1\right) = -\rho\, g\,\Delta V\left(y_2 - y_1\right) \quad \text{work done by the gravitational force}$$

$$W_p = F_1\,\Delta x_1 + F_2\left(-\Delta x_2\right) = \left(p_1 - p_2\right)\Delta V \quad \Leftarrow \quad F\,\Delta x = p\,A\;\Delta x = p\;\,A\,\Delta x = p\,\Delta V$$

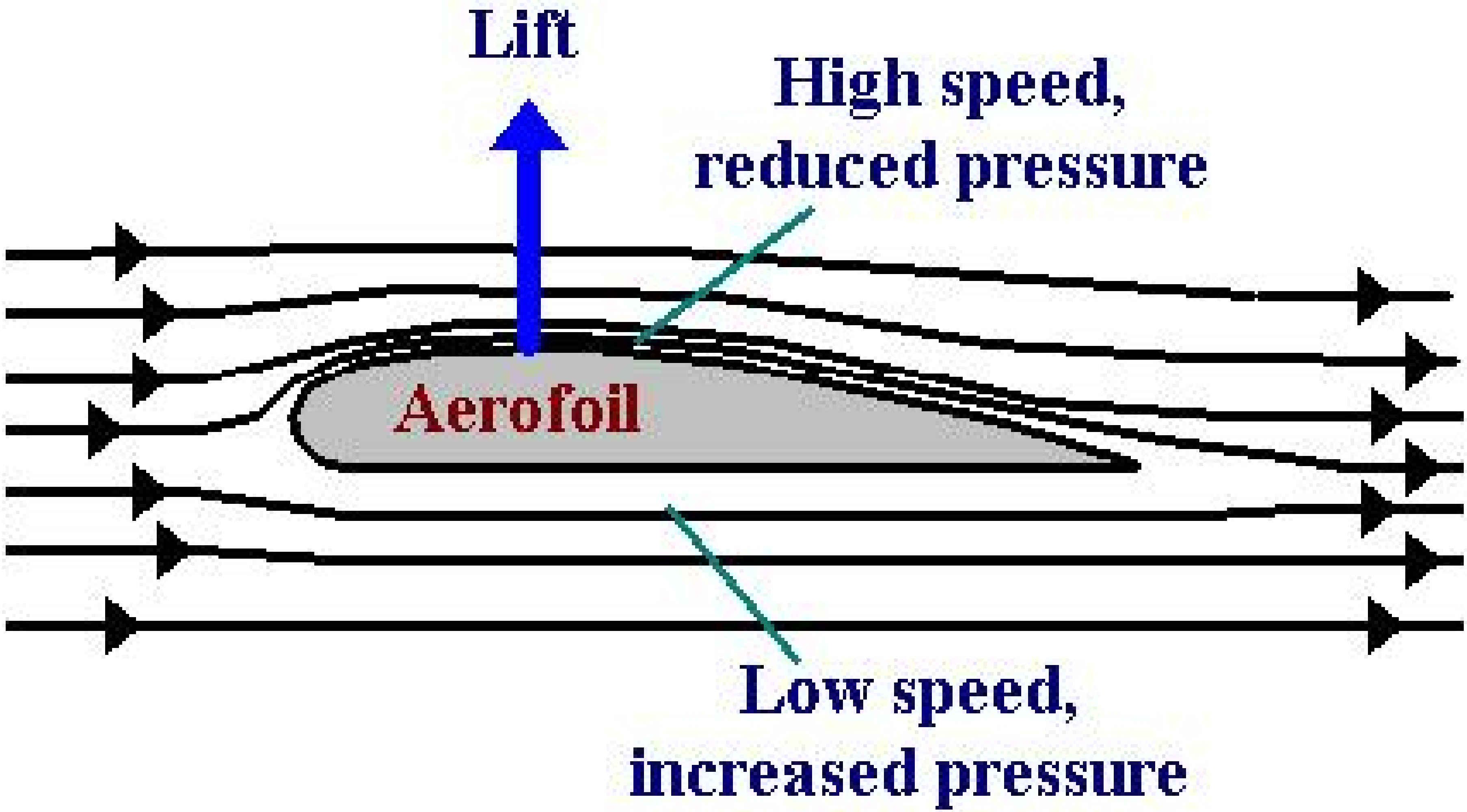
work done on the system

$$\Rightarrow \quad -\rho\, g\,\Delta V\left(y_2 - y_1\right) + \left(p_1 - p_2\right)\Delta V = \frac{1}{2}\rho\,\Delta V\left(v_2^2 - v_1^2\right) \quad \Leftarrow \quad W = W_g + W_p = \Delta K$$

$$\Rightarrow \quad p_1 + \frac{1}{2}\rho\, v_1^2 + \rho\, g\, y_1 = p_2 + \frac{1}{2}\rho\, v_2^2 + \rho\, g\, y_2 \quad \Rightarrow \quad p + \frac{1}{2}\rho\, v^2 + \rho\, gy = \text{a constant}$$

- Bernoulli's equation is not a new principle but simply the reformulation of a familiar principle in a form suitable to fluid mechanics.
- For a fluid at rest (v_1=v_2=0): $p_2 = p_1 + \rho\, g\left(y_1 - y_2\right)$
- If we take y to be a constant: $p_1 + \frac{1}{2}\rho\, v_1^2 = p_2 + \frac{1}{2}\rho\, v_2^2$ which means

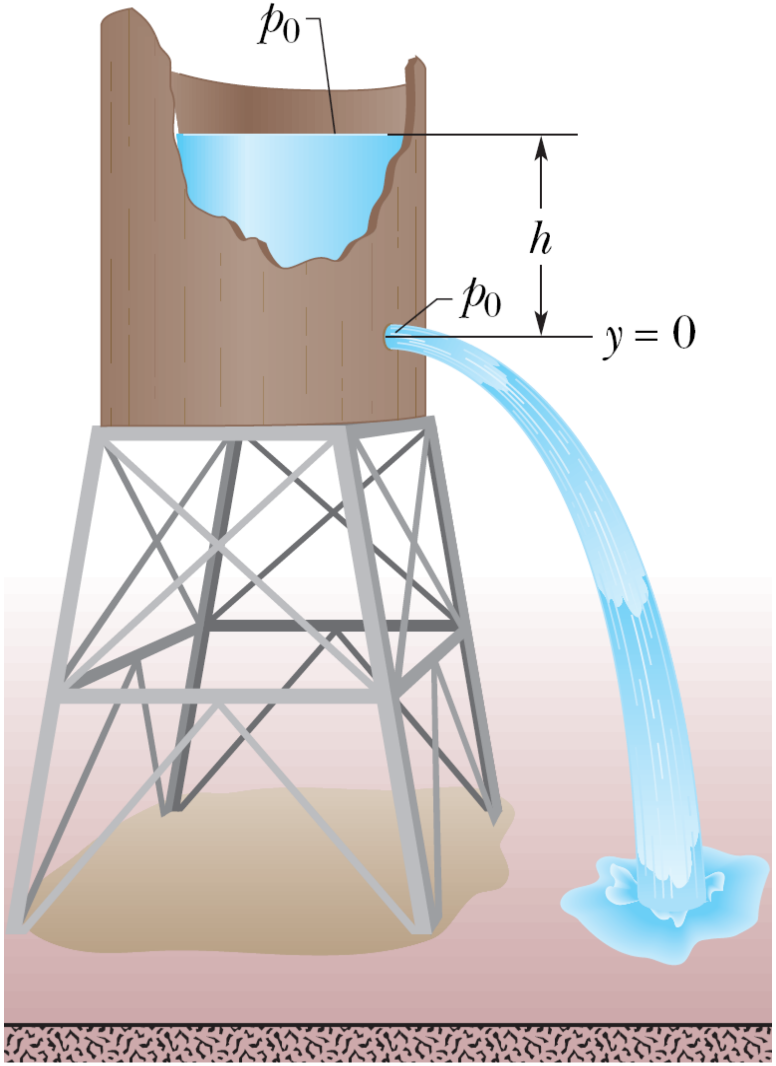
If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.


Lift
High speed,
reduced pressure
Aerofoil
Low speed,
increased pressure

- In the other word, where the streamlines are relatively close together (where the velocity is relatively great), the pressure is relatively low, and conversely.

- Bernoulli s equation is strictly valid only to the extent that the fluid is ideal. If viscous forces are present, thermal energy will be involved.

problem 14-6



Problem 14-7

The chosen problems: 3, 30, 46, 71.