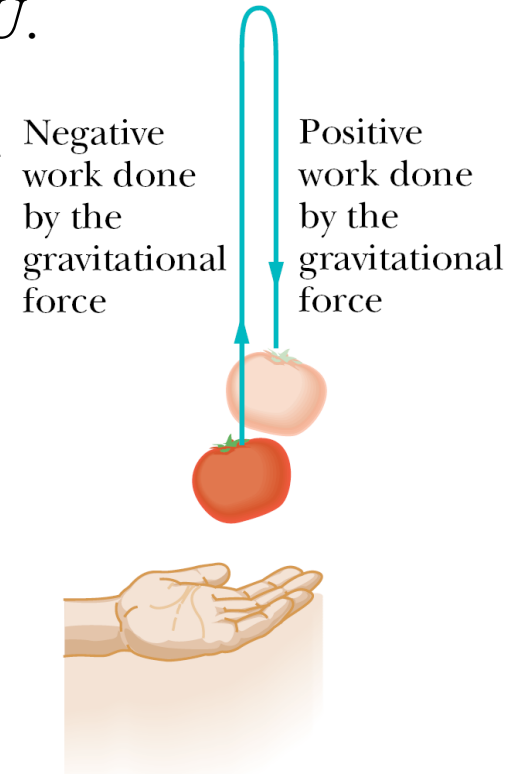


Chapter 8 Potential Energy and Conservation of Energy

- Technically, **potential energy** is energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another.
- We can account for a jumper's motion and increase in kinetic energy by defining a **gravitational potential energy** U .
- This is the energy associated with the state of separation between 2 objects that attract each other by the gravitational force, here the jumper and Earth.
- We can account for the jumper's decrease in kinetic energy and the cord's increase in length by defining an **elastic potential energy** U .
- This is the energy associated with the state of compression or extension of an elastic object, here the bungee cord.

Work and Potential Energy

- When we throw an object upward, as the object rises, the work done on the object by the gravitational force is negative because the force transfers energy *from* the kinetic energy of the object *to* the gravitational potential energy of the object-Earth system.



- During the fall the transfer is reversed: The work done on the object by the gravitational force is now positive — that force transfers energy *from* the gravitational potential energy of the object-Earth system *to* the kinetic energy of the object.

- The change in gravitational potential energy is defined as being equal to the negative of the work done on the object by the gravitational force:

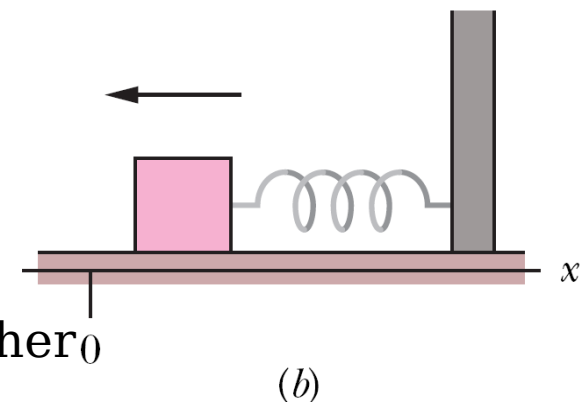
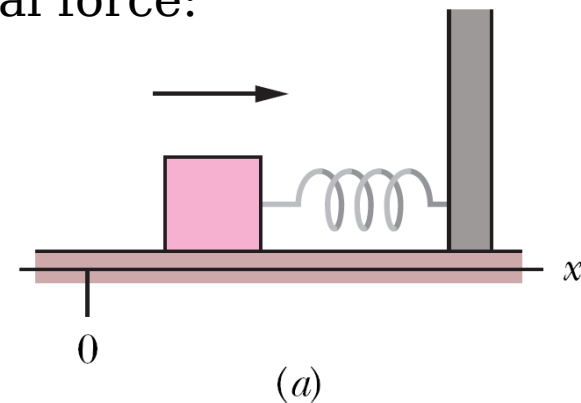
$$\Delta U = -W$$

- The equation also applies to a block-spring system.

Conservative and Nonconservative Forces

- Key elements of the 2 situations:

1. The *system* consists of 2 or more objects.
2. A *force* acts between a particle-like object in the system and the rest of the system.
3. When the system configuration changes, the force does *work* W_1 on the particle-like object, transferring energy between the kinetic energy K of the object and some other type of energy of the system.
4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2 in the process.



- In a situation in which $W_1 = -W_2$ is always true, the other type of energy is a potential energy and the force is said to be a **conservative force**.
 - gravitational force & spring force
- A force that is not conservative is called a **nonconservative force**.
 - kinetic friction force & drag force (ie, *thermal energy*)
- *When only conservative forces act on a particle-like object, we can greatly simplify otherwise difficult problems involving motion of the object.*

Path Independence of Conservative Forces

- determining whether a force is conservative or nonconservative:
 - Let the force act on a particle that moves along any *closed path*.

The net work done by a conservative force on a particle moving around any closed path is 0.

For example: the gravitational force

- An important result of the closed-path test is that:

The work done by a conservative force on a particle moving between 2 points does not depend on the path taken by the particle.

- If only a conservative force acts on the particle

$$W_{ab,1} = W_{ab,2}$$

proof the above equation:

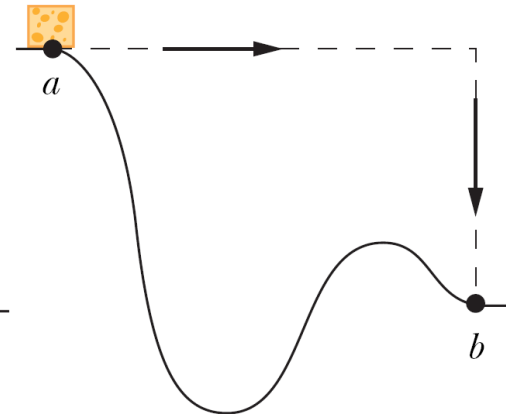
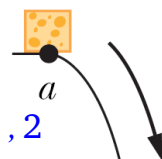
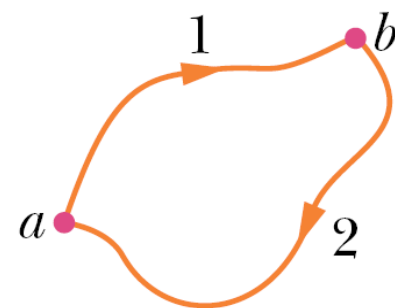
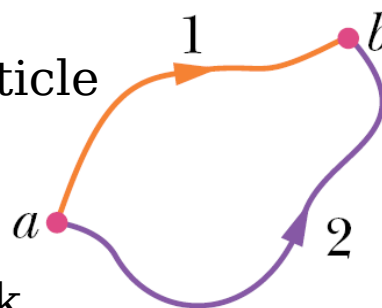
If the force is conservative, then the net work done during the round trip must be 0:

$$W_{ab,1} + W_{ba,2} = 0, \quad \Rightarrow \quad W_{ab,1} = -W_{ba,2}$$

If the force is conservative, then

$$W_{ab,2} = -W_{ba,2}, \quad \Rightarrow \quad W_{ab,1} = W_{ab,2}$$

problem 8-1



Determining Potential Energy Values

- When a conservative force does work W on a object, the change ΔU in the potential energy associated with the system is the negative of the work done, ie, $\Delta U = -W$.

- For the most general case, the work

$$W = \int_{x_i}^{x_f} F(x) dx \quad \Rightarrow \quad \Delta U = - \int_{x_i}^{x_f} F(x) dx$$



Gravitational Potential Energy

● consider a particle with mass m moving vertically along a y axis. As the particle moves from point y_i to point y_f , the gravitational force does work on it. And the corresponding change in the gravitational potential energy of the particle-Earth system is

$$\Delta U = - \int_{y_i}^{y_f} (-m g) d y = m g \int_{y_i}^{y_f} d y = m g [y]_{y_i}^{y_f} = m g (y_f - y_i) = m g \Delta y$$

● Only changes ΔU in gravitational potential energy are physically meaningful.

● For convenience, we sometimes associate a certain gravitational potential value U with a certain particle-Earth system when the particle is at a certain height y ,

$$U - U_i = m g (y - y_i)$$

● U_i is the gravitational potential energy of the system when it is in a **reference configuration** in which the particle is at a **reference point** y_i . Usually we take $U_i=0$ and $y_i=0$, then

$$U(y) = m g y \quad \text{gravitational potential energy}$$

The gravitational potential energy associated with a particle-Earth system depends only on the vertical position y (or height) of the particle relative to the reference position $y = 0$, not on the horizontal position.

Elastic Potential Energy

● consider a block-spring system with a spring of spring constant k . As the block moves from point x_i to point x_f , the spring force $F_x = -kx$ does work on the block. And the corresponding change in the elastic potential energy of the block-spring system is

$$\Delta U = - \int_{x_i}^{x_f} (-kx) dx = k \int_{x_i}^{x_f} x dx = \frac{1}{2} k [x^2]_{x_i}^{x_f} = \frac{1}{2} k x_f^2 - \frac{1}{2} k x_i^2$$

● To associate a potential energy value U with the block at position x , we choose the reference configuration to be when the spring is at its relaxed length and the block is at $x_i = 0$, and $U_i = 0$, thus,

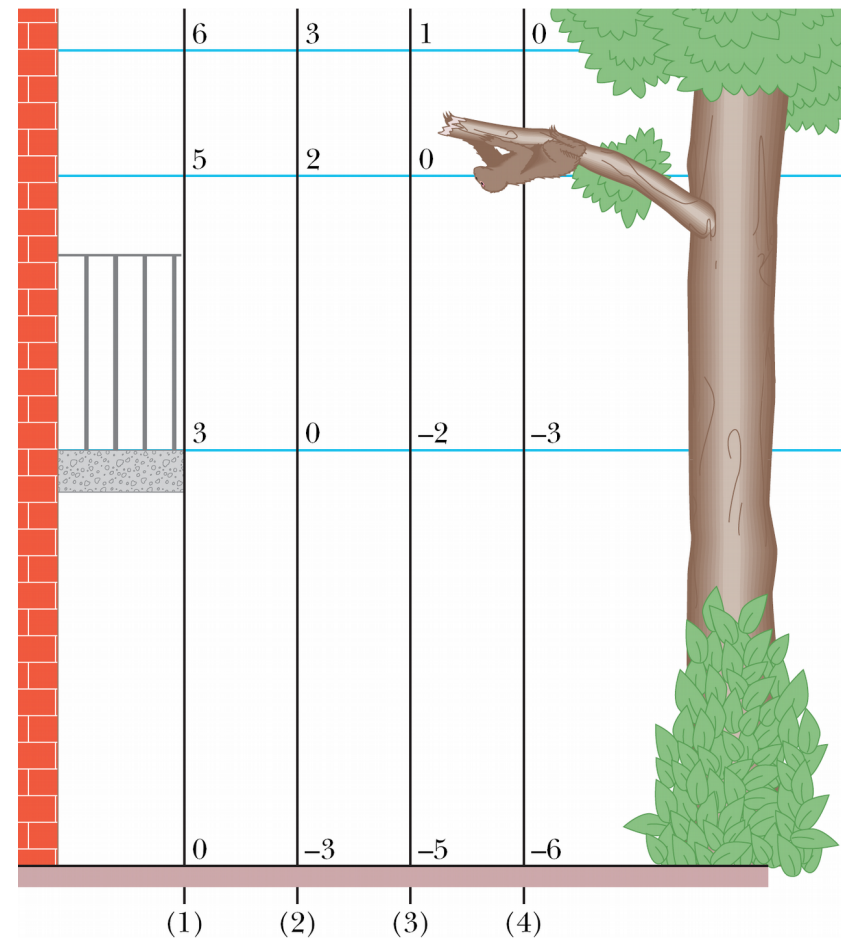
$$U(x) = \frac{1}{2} k x^2 \quad \text{elastic potential energy}$$

problem 8-2

Conservation of Mechanical Energy

● The **mechanical energy** E_{mec} of a system is the sum of its potential energy U and the kinetic energy K of the objects within it:

$$E_{\text{mec}} = K + U \quad \text{mechanical energy}$$



● examine the mechanical energy when only conservative forces cause energy transfers within the system. Also assume the system is isolated, ie, no *external force* from an object outside that system causes energy changes inside the system.

● When a conservative force does work W on an object within the system, the change ΔK in kinetic energy is $\Delta K = +W$

and the change ΔU in potential energy is $\Delta U = -W$

then $\Delta K = -\Delta U \Rightarrow K_2 - K_1 = -(U_2 - U_1)$

thus,

$$K_2 + U_2 = K_1 + U_1 \quad \text{conservation of mechanical energy}$$

ie, $\left(\begin{array}{l} \text{the sum of } K \text{ and } U \text{ for} \\ \text{any state of a system} \end{array} \right) = \left(\begin{array}{l} \text{the sum of } K \text{ and } U \text{ for} \\ \text{any other state of the system} \end{array} \right)$

principle of conservation of mechanical energy

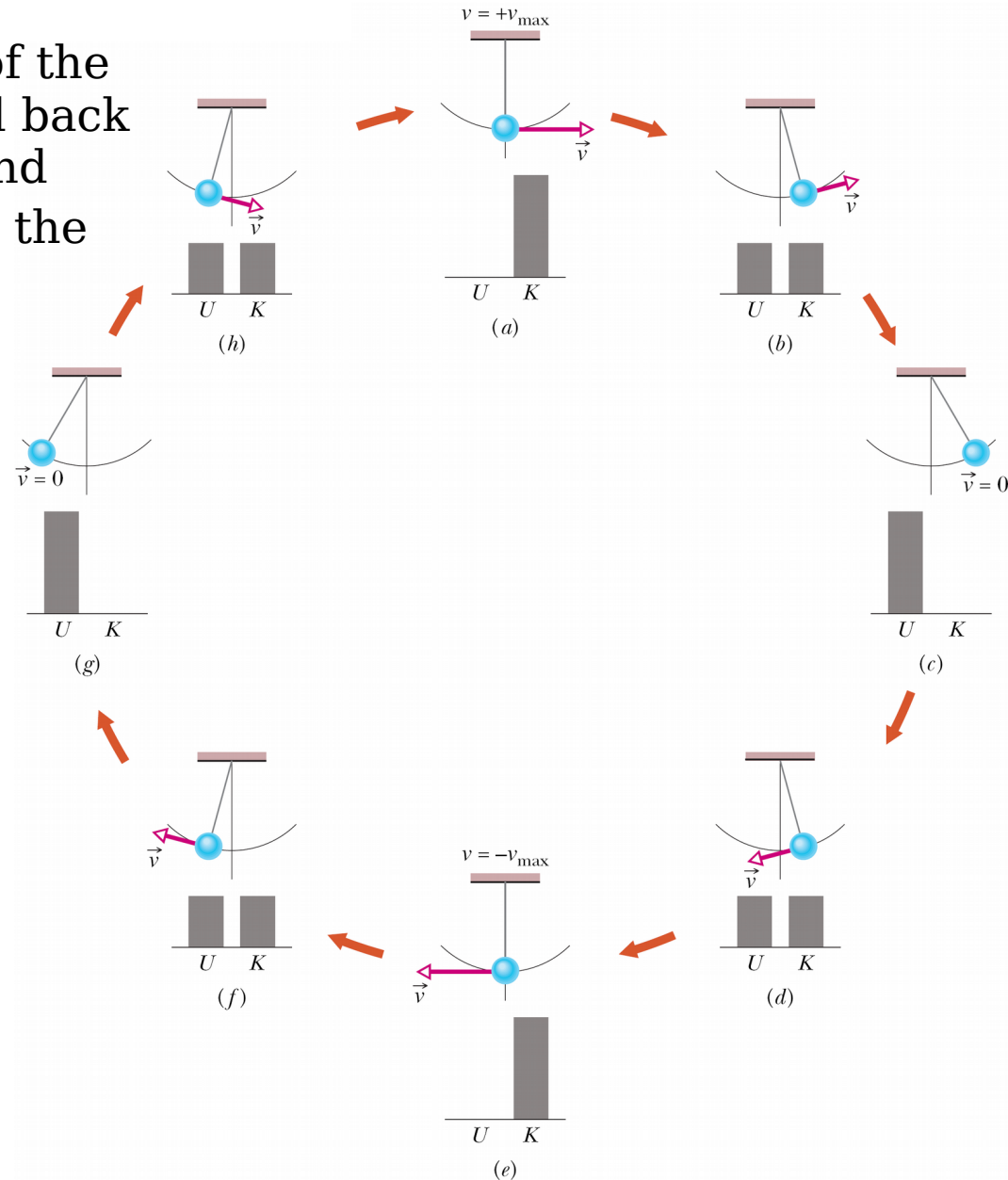
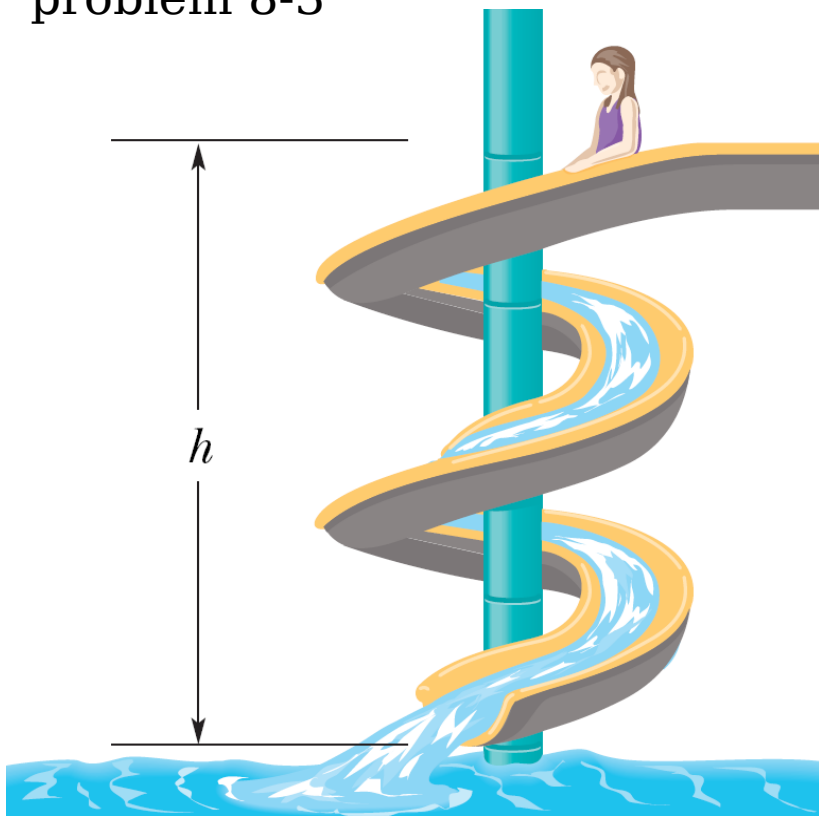
In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

● write this principle in one more form $\Delta E_{\text{mec}} = \Delta K + \Delta U = 0$

When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant *without considering the intermediate motion and without finding the work done by the forces involved.*

- As a pendulum swings, the energy of the pendulum-Earth system is transferred back and forth between kinetic energy K and gravitational potential energy U , with the sum $K + U$ being constant.

problem 8-3



Reading a Potential Energy Curve

Finding the Force Analytically

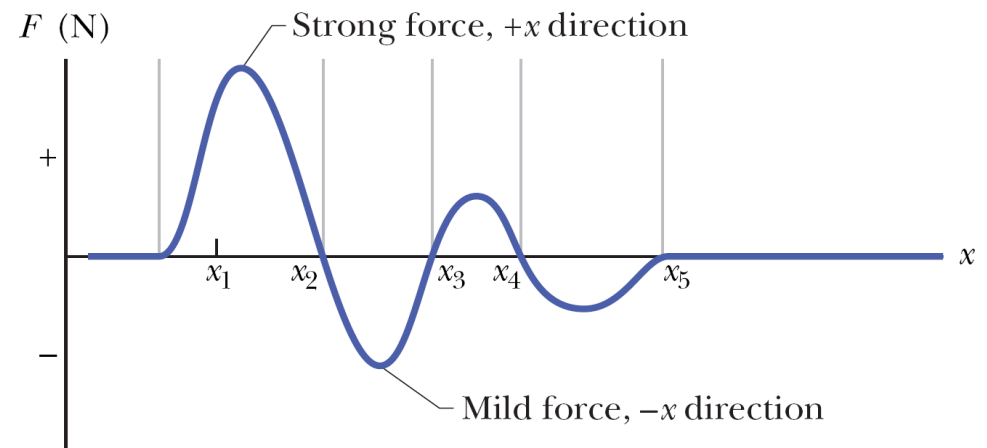
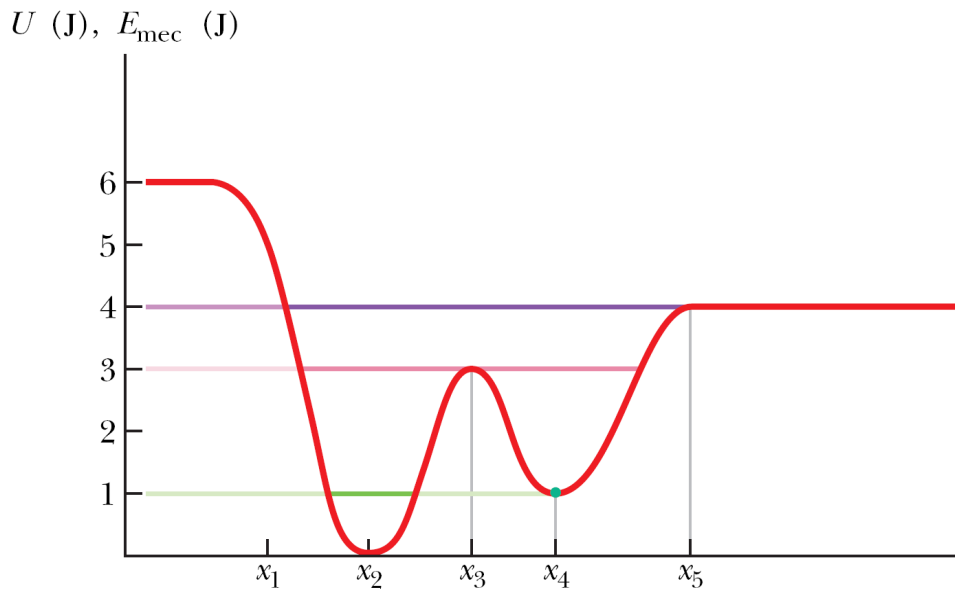
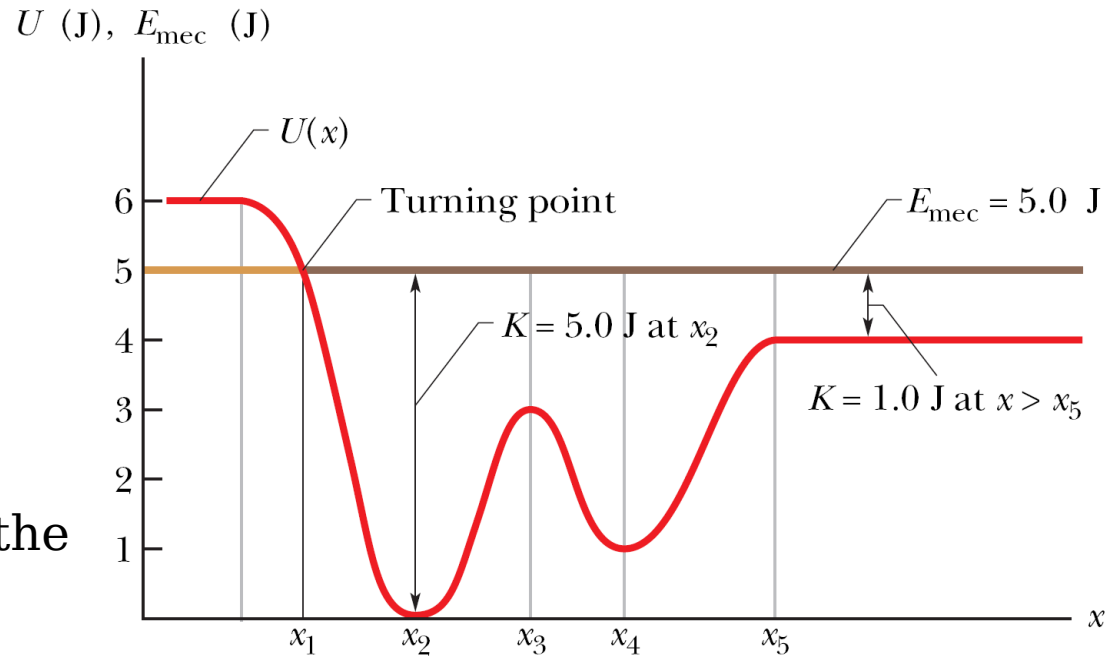
- For 1-dim motion,

$$\Delta U(x) = -W = -F(x) \Delta x$$

Solving for $F(x)$ and passing to the differential limit yield

$$F(x) = -\frac{dU(x)}{dx} \quad \text{1-d motion}$$

- $F(x)$ is the negative of the slope of the $U(x)$ curve.



For example, the elastic potential energy $U(x) = kx^2/2$, and the force $F(x) = -kx$; the gravitational potential energy $U(x) = mgh$, and the force $F(x) = -mg$.

Turning points

● In the absence of a nonconservative force, the mechanical energy E of a system has a constant value

$$U(x) + K(x) = E_{\text{mec}} \Rightarrow K(x) = E_{\text{mec}} - U(x)$$

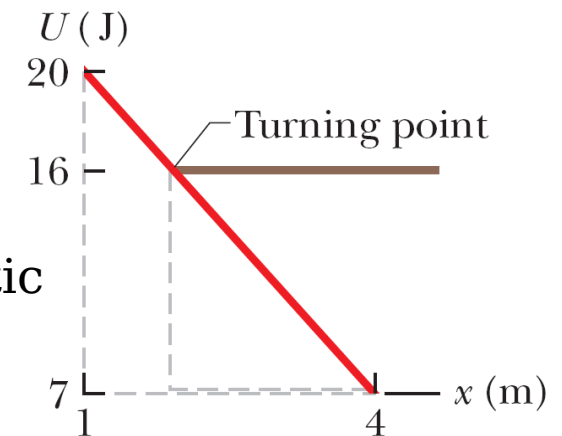
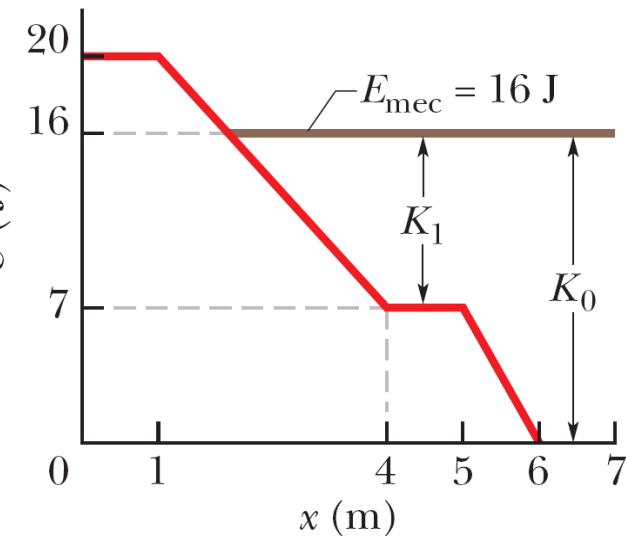
● determine the kinetic energy $K(x)$: On the $U(x)$ curve, find U for the location x and then subtract U from E_{mec} .

● Since K can never be negative, the particle will never move to the place (eg, x_1) where $E_{\text{mec}} - U$ is negative.

● A **turning point** is the place where $K = 0$ (because $U = E$) and the particle changes direction.

Equilibrium Points

● A particle at a position where the particle has no kinetic energy and no force acts on it (so it is stationary) is said to be in **neutral equilibrium**. For example, x_5 .



Problem 8-4

- A particle at a position where a small perturbation in either direction will cause a nonzero force pushing it farther in the same direction (and the particle continues to move) is said to be in **unstable equilibrium**. For example x_3 .

- A particle at a position where the restoring force appear will move the particle back to this position if a slight push is applied to it in either direction is said to be in **stable equilibrium**. For example x_4 .

Work Done on a System by an External Force

- extend the definition of work to an external force acting on a system

Work is energy transferred to or from a system by means of an external force acting on the system.

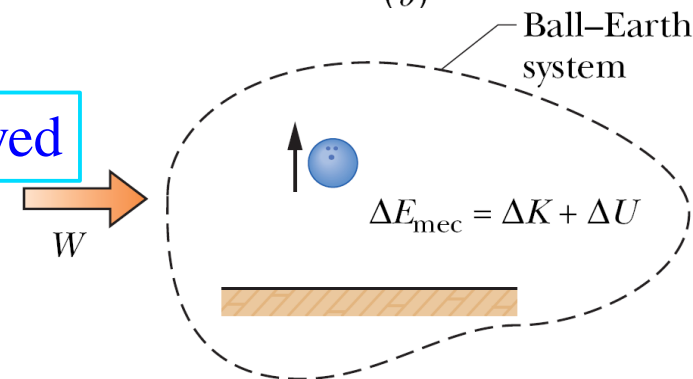
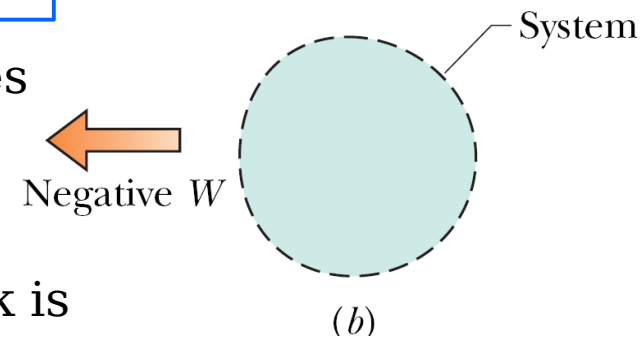
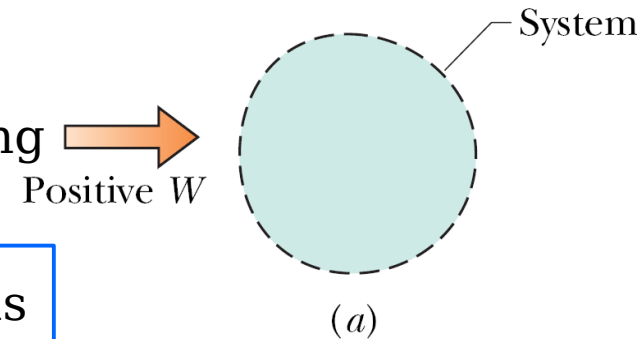
- External forces can transfer energy into different types of energy of a system.

No Friction Involved

- For an external force doing work on a system the work is

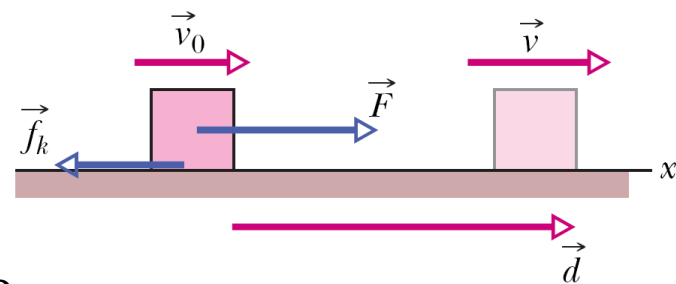
$$W = \Delta K + \Delta U$$

or $W = \Delta E_{\text{mec}}$ work done on system, no friction involved



Friction Involved

- consider a constant horizontal force pulls a block along an x axis and through a displacement of magnitude d , also increasing the block's velocity. During the motion, a constant kinetic frictional force from the floor acts on the block.

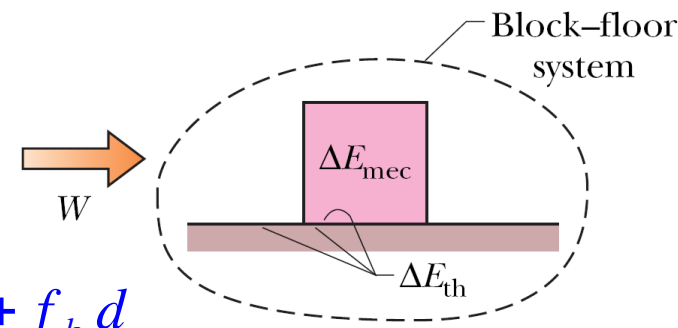


(a)

- Using Newton's 2nd law: $F - f_k = m a$

- since the acceleration is constant, $v^2 = v_0^2 + 2 a d$

and $F d = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2 + f_k d \Rightarrow F d = \Delta K + f_k d$



(b)

- In a more general situation, there can be a changes in potential energy, thus

$$F d = \Delta E_{\text{mec}} + f_k d$$

- thermal energy: the energy associated with the random motion of the atoms and molecules in the object.

- The thermal energy of the block and floor increases because (1) there is friction between them and (2) there is sliding.

- the increase ΔE_{th} in thermal energy is equal to the product of the magnitudes f_k and d :

$$\Delta E_{\text{th}} = f_k d \quad \text{increase in thermal energy by sliding}$$

thus, $F d = \Delta E_{\text{mec}} + \Delta E_{\text{th}}$

$$W = \Delta E_{\text{mec}} + \Delta E_{\text{th}} \quad \text{work done on system, friction involved}$$

problem 8-5

Conservation of Energy

- In all the cases considered, we assumed that energy obeys a law called the **law of conservation of energy**, which is concerned with the **total energy** E of a system.
- The total is the sum of the system's mechanical energy, thermal energy, and any type of *internal energy* in addition to thermal energy.

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

Thus $W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}$

- The law of conservation of energy is based on the results of countless experiments, not derived from basic physics principle.

The chosen problem: 6, 23, 34, 62.

Isolated System

- If a system is isolated from its environment, there can be no energy transfers to or from it.

The total energy E of an isolated system cannot change.

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0 \quad \text{isolated system}$$

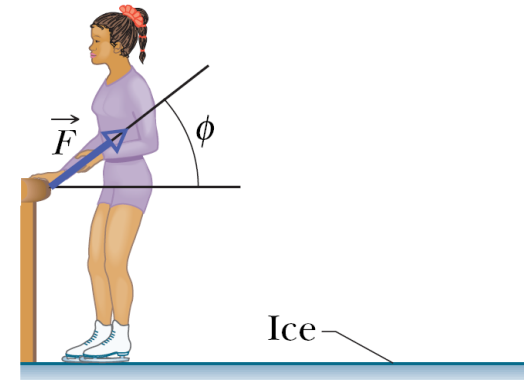
$$\Delta E_{\text{mec}} = E_{\text{mec}, 2} - E_{\text{mec}, 1} \Rightarrow E_{\text{mec}, 2} = E_{\text{mec}, 1} - \Delta E_{\text{th}} - \Delta E_{\text{int}}$$

In an isolated system, we can relate the total energy at one instant to the total energy at another instant *without considering the energies at intermediate times*.

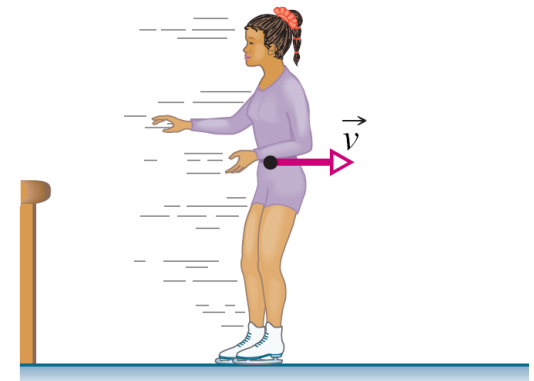
External Forces and Internal Energy Transfers

- An external force can change the kinetic energy or potential energy of an object without doing work on the object — that is, without transferring energy to the object. Instead, the force is responsible for transfers of energy from one type to another inside the object.

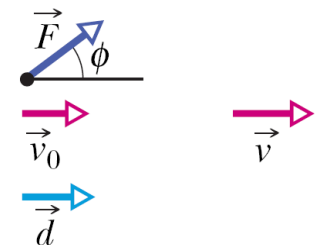
- The ice skater changes her kinetic energy as a result of internal transfers from the biochemical energy in her muscles.



(a)



(b)

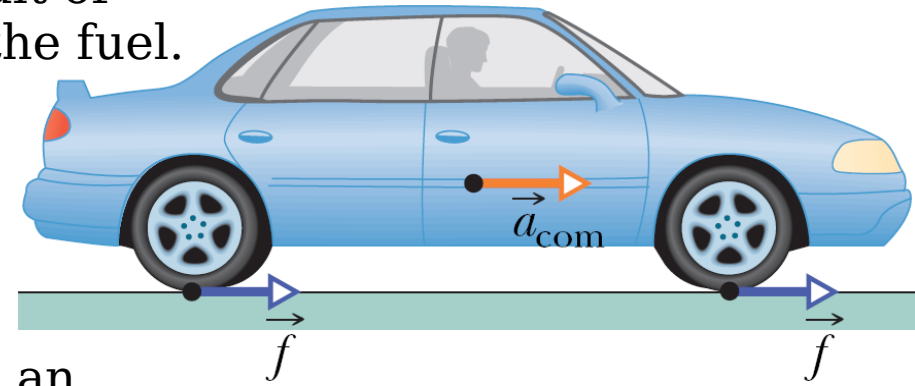


(c)

- The car's kinetic energy increases as a result of internal transfers from the energy stored in the fuel.

Power

the rate at which energy is transferred by a force from one type to another.



- If an amount of energy ΔE is transferred in an amount of time Δt , the **average power** due to the force is $P_{\text{avg}} = \frac{\Delta E}{\Delta t}$

- the **instantaneous power** due to the force is

$$P = \frac{dE}{dt}$$

problem 8-6

