Chapter 8 Potential Energy and Conservation of Energy

- Technically, **potential energy** is energy that can be associated with the configuration (arrangement) of a system of objects that exert forces on one another.

- We can account for a jumper's motion and increase in kinetic energy by defining a **gravitational potential energy** U.

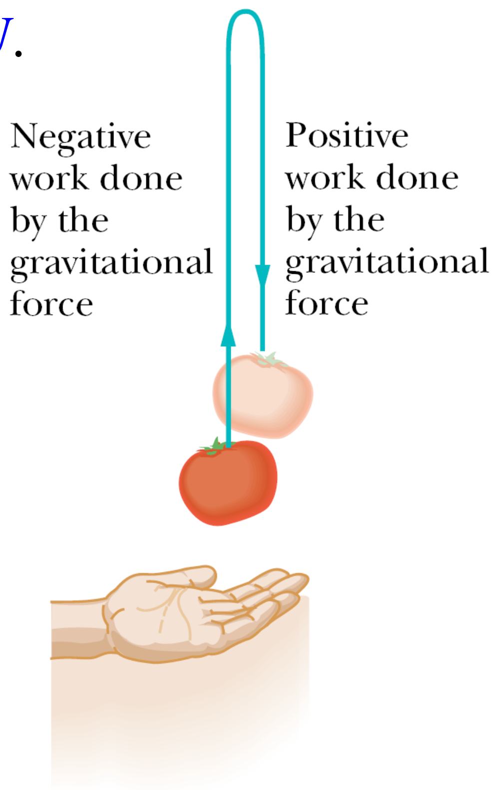
- This is the energy associated with the state of separation between 2 objects that attract each other by the gravitational force, here the jumper and Earth.

- We can account for the jumper's decrease in kinetic energy and the cord's increase in length by defining an **elastic potential energy** U.

- This is the energy associated with the state of compression or extension of an elastic object, here the bungee cord.

Work and Potential Energy

- When we throw an object upward, as the object rises, the work done on the object by the gravitational force is negative because the force transfers energy *from* the kinetic energy of the object *to* the gravitational potential energy of the object-Earth system.

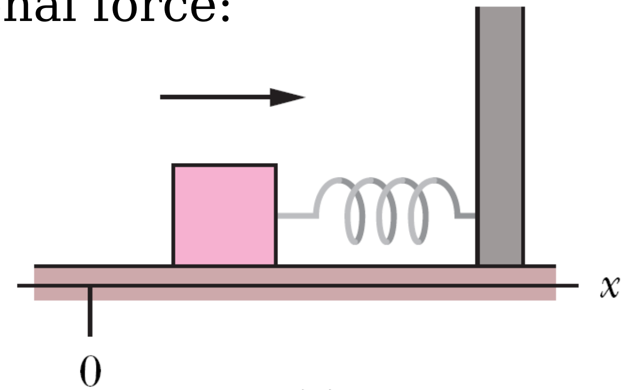


- During the fall the transfer is reversed: The work done on the object by the gravitational force is now positive — that force transfers energy *from* the gravitational potential energy of the object-Earth system *to* the kinetic energy of the object.

- The change in gravitational potential energy is defined as being equal to the negative of the work done on the object by the gravitational force:

$$\Delta U = -W$$

- The equation also applies to a block-spring system.

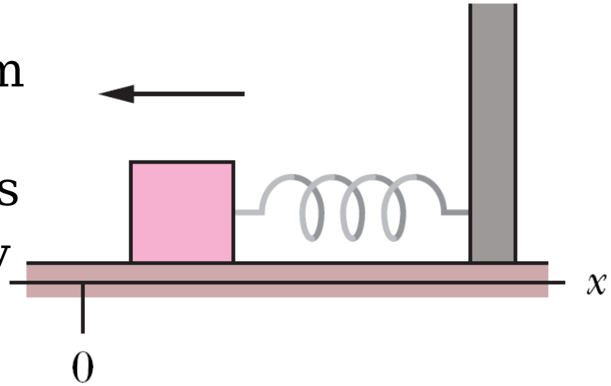


(a)

Conservative and Nonconservative Forces

- Key elements of the 2 situations:

1. The *system* consists of 2 or more objects.
2. A *force* acts between a particle-like object in the system and the rest of the system.
3. When the system configuration changes, the force does *work* W_1 on the particle-like object, transferring energy between the kinetic energy K of the object and some other type of energy of the system.
4. When the configuration change is reversed, the force reverses the energy transfer, doing work W_2 in the process.



(b)

- In a situation in which $W_1 = -W_2$ is always true, the other type of energy is a potential energy and the force is said to be a **conservative force**.
— gravitational force & spring force

- A force that is not conservative is called a **nonconservative force**.
— kinetic friction force & drag force (ie, *thermal energy*)

- *When only conservative forces act on a particle-like object, we can greatly simplify otherwise difficult problems involving motion of the object.*

Path Independence of Conservative Forces

- determining whether a force is conservative or nonconservative: Let the force act on a particle that moves along any *closed path*.

The net work done by a conservative force on a particle moving around any closed path is 0.

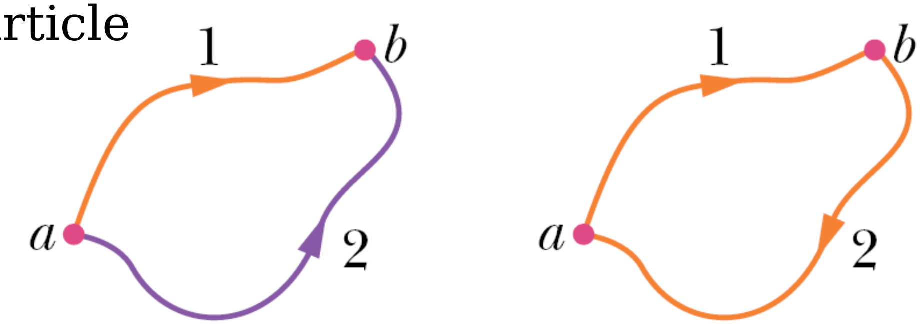
For example: the gravitational force

- An important result of the closed-path test is that:

The work done by a conservative force on a particle moving between 2 points does not depend on the path taken by the particle.

- If only a conservative force acts on the particle

$$W_{ab,1} = W_{ab,2}$$

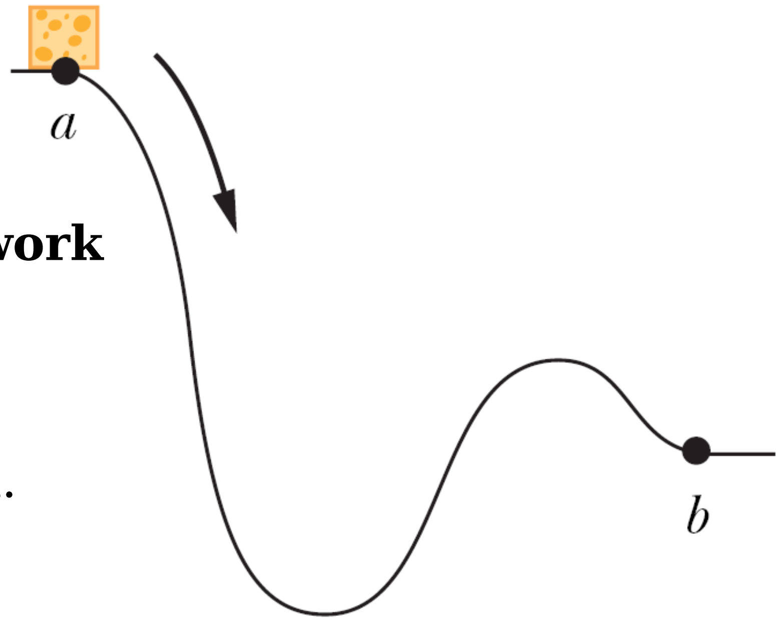


proof the above equation:

If the force is conservative, the net work done during the round trip must be 0:

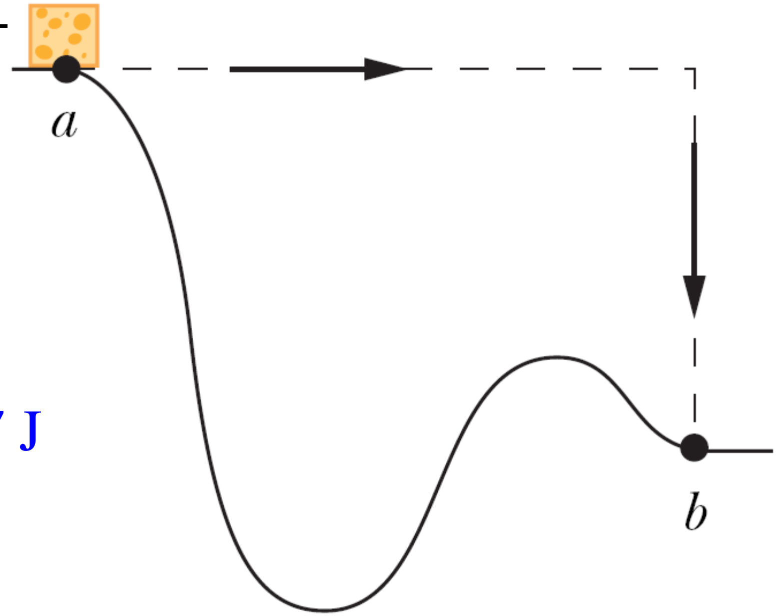
$$W_{ab,1} + W_{ba,2} = 0 \quad \Rightarrow \quad W_{ab,1} = -W_{ba,2}$$

If the force is conservative, then $W_{ab,2} = -W_{ba,2} \quad \Rightarrow \quad W_{ab,1} = W_{ab,2}$



<u>Sample 8.1.1</u>: **Equivalent paths for calculating work**

It shows a 2 kg block of slippery cheese that slides along a frictionless track from point a to point b. The cheese travels through a total distance of 2 m along the track, and a net vertical distance of 0.8 m. How much work is done on the cheese by the gravitational force during the slide?



$$\vec{F}_g = -m\,g\,\hat{\mathrm{j}}\,, \quad \vec{d} = \vec{d}_h + \vec{d}_v \quad \Leftarrow \quad \begin{array}{l} \vec{d}_h = d_h\,\hat{\mathrm{i}} \\ \vec{d}_v = -0.8\ \mathrm{m}\ \hat{\mathrm{j}} \end{array}$$

$$\Rightarrow \quad \begin{array}{l} W_h = \vec{F}_g \cdot \vec{d}_h = (-m\,g\,\hat{\mathrm{j}}) \cdot (d_h\,\hat{\mathrm{i}}) = 0 \\ W_v = \vec{F}_g \cdot \vec{d}_v = (-m\,g\,\hat{\mathrm{j}}) \cdot (d_v\,\hat{\mathrm{j}}) = -m\,g\,d_v = 15.7\ \mathrm{J} \end{array}$$

$$\Rightarrow \quad W = W_h + W_v = 15.7\ \mathrm{J}$$

Determining Potential Energy Values

- When a conservative force does work W on a object, the change ΔU in the potential energy associated with the system is the negative of the work done, ie, $\Delta U = - W$.

- For the most general case, the work

$$W = \int_{x_i}^{x_f} F(x)\, \mathrm{d}x \quad \Rightarrow \quad \Delta U = -\int_{x_i}^{x_f} F(x)\, \mathrm{d}x$$

Gravitational Potential Energy

- Consider a particle with mass m moving vertically along a y axis. As the particle moves from point y_i to point y_f, the gravitational force does work on it. And the corresponding change in the gravitational potential energy of the particle-Earth system is

$$\Delta U = -\int_{y_i}^{y_f} (-mg)\, \mathrm{d}y = mg \int_{y_i}^{y_f} \mathrm{d}y = mg\, y \Big|_{y_i}^{y_f} = mg(y_f - y_i) = mg\,\Delta y$$

- Only changes ΔU in gravitational potential energy are physically meaningful.

- For convenience, we sometimes associate a certain gravitational potential value U with a certain particle-Earth system when the particle is at a certain height y,

$$U - U_i = mg(y - y_i)$$

- U_i is the gravitational potential energy of the system when it is in a **reference configuration** in which the particle is at a **reference point** y_i. Usually we take $U_i=0$ and $y_i=0$, then $U(y) = m\,g\,y$ gravitational potential energy

The gravitational potential energy associated with a particle-Earth system depends only on the vertical position y (or height) of the particle relative to the reference position $y = 0$, not on the horizontal position.

Elastic Potential Energy

- Consider a block-spring system with a spring of spring constant k. As the block moves from point x_i to point x_f, the spring force $F_x = -k\,x$ does work on the block. And the corresponding change in the elastic potential energy of the block-spring system is

$$\Delta U = -\int_{x_i}^{x_f} (-k\,x)\,\mathrm{d}x = k\int_{x_i}^{x_f} x\,\mathrm{d}x = \frac{1}{2}k\,x^2\Big|_{x_i}^{x_f} = \frac{1}{2}k\,x_f^2 - \frac{1}{2}k\,x_i^2$$

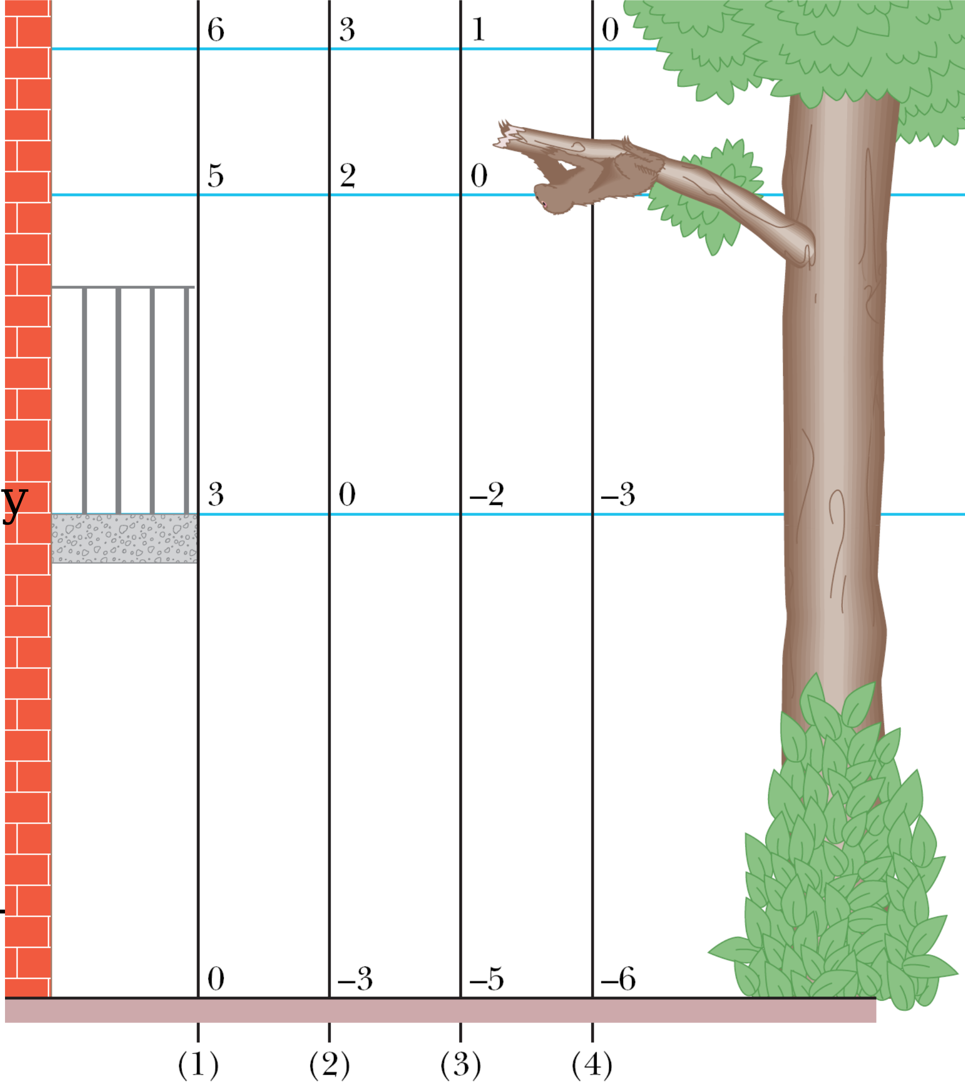
- To associate a potential energy value U with the block at position x, we choose the reference configuration to be when the spring is at its relaxed length and the block is at $x_i=0$, and $U_i=0$, thus,

$$U(x) = \frac{1}{2}k\,x^2 \quad \text{elastic potential energy}$$

Sample 8.1.2: **Choosing reference level for gravitational potential energy, sloth**

Here is an example with this lesson plan: Generally you can choose any level to be the reference level, but once chosen, be consistent. A 2 kg sloth hangs 5 m above the ground.

(a) What is the gravitational potential energy U of the sloth–Earth system if we take the reference point y=0 to be (1) at the ground, (2) at a balcony floor that is 3 m above the ground, (3) at the limb, and (4) 1 m above the limb? Take the gravitational potential energy to be zero at y=0.



$$(1)\quad U_1 = m\,g\,y_1 = 2\times 9.8\times 5 = 98\text{ J}$$

$$(2)\quad U_2 = m\,g\,y_2 = 2\times 9.8\times (5-3) = 39\text{ J}$$

$$(3)\quad U_3 = m\,g\,y_3 = 2\times 9.8\times (5-5) = 0$$

$$(4)\quad U_4 = m\,g\,y_4 = 2\times 9.8\times (-1) = -19.6\text{ J}$$

(b) The sloth drops to the ground. For each choice of reference point, what is the change ΔU in the potential energy of the sloth–Earth system due to the fall?

$$\text{For all } 4 \text{ cases}, \ \Delta y = -5\text{ m} \ \Rightarrow \ \Delta U = m\,g\,\Delta y = 2\times 9.8\times(-5) = -98\text{ J}$$

Conservation of Mechanical Energy

- The **mechanical energy** E_{mec} of a system is the sum of its potential energy U and the kinetic energy K of the objects within it: $E_{\text{mec}} = K + U$ mechanical energy

- Examine the mechanical energy when only conservative forces cause energy transfers within the system. Also assume the system is isolated, ie, no *external force* form an object outside that system causes energy changes inside the system.

- When a conservative force does work W on an object within the system, the change ΔK in kinetic energy is $\Delta K = +W$
the change ΔU in potential energy is $\Delta U = -W$
then

$$\Delta K = -\Delta U \quad \Rightarrow \quad K_2 - K_1 = -(U_2 - U_1)$$

$$\Rightarrow \quad K_2 + U_2 = K_1 + U_1 \quad \text{conservation of mechanical energy}$$

$$\Rightarrow \left(\begin{array}{c}\text{the sum of } K \text{ and } U \text{ for} \\ \text{any state of a system}\end{array}\right) = \left(\begin{array}{c}\text{the sum of } K \text{ and } U \text{ for} \\ \text{any other state of the system}\end{array}\right)$$

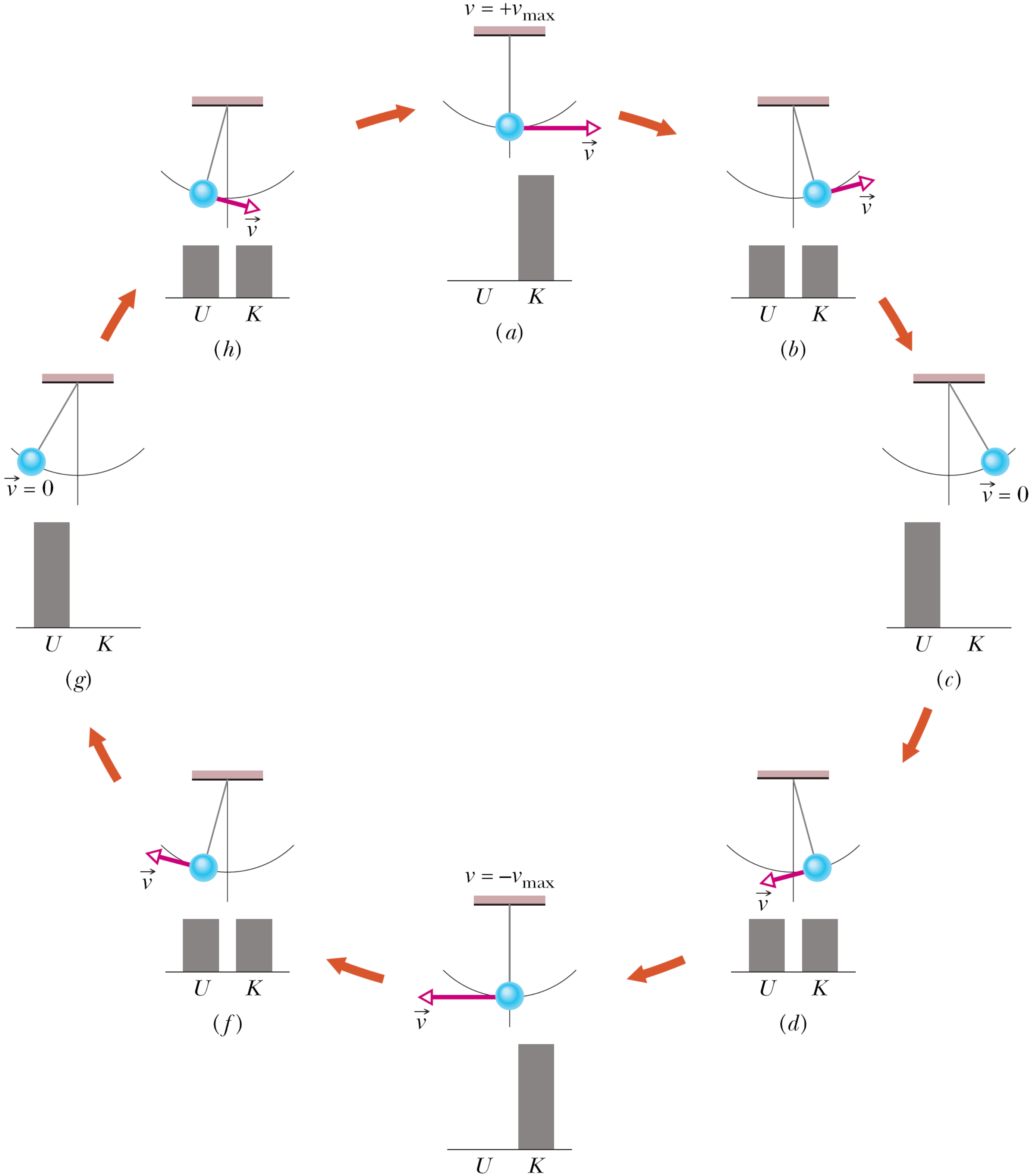
principle of conservation of mechanical energy

In an isolated system where only conservative forces cause energy changes, the kinetic energy and potential energy can change, but their sum, the mechanical energy E_{mec} of the system, cannot change.

- Write this principle in one more form $\Delta E_{\text{mec}} = \Delta K + \Delta U = 0$

When the mechanical energy of a system is conserved, we can relate the sum of kinetic energy and potential energy at one instant to that at another instant *without considering the intermediate motion* and *without finding the work done by the forces involved*.

- As a pendulum swings, the energy of the pendulum-Earth system is transferred back and forth between kinetic energy K and gravitational potential energy U, with the sum $K + U$ being constant.



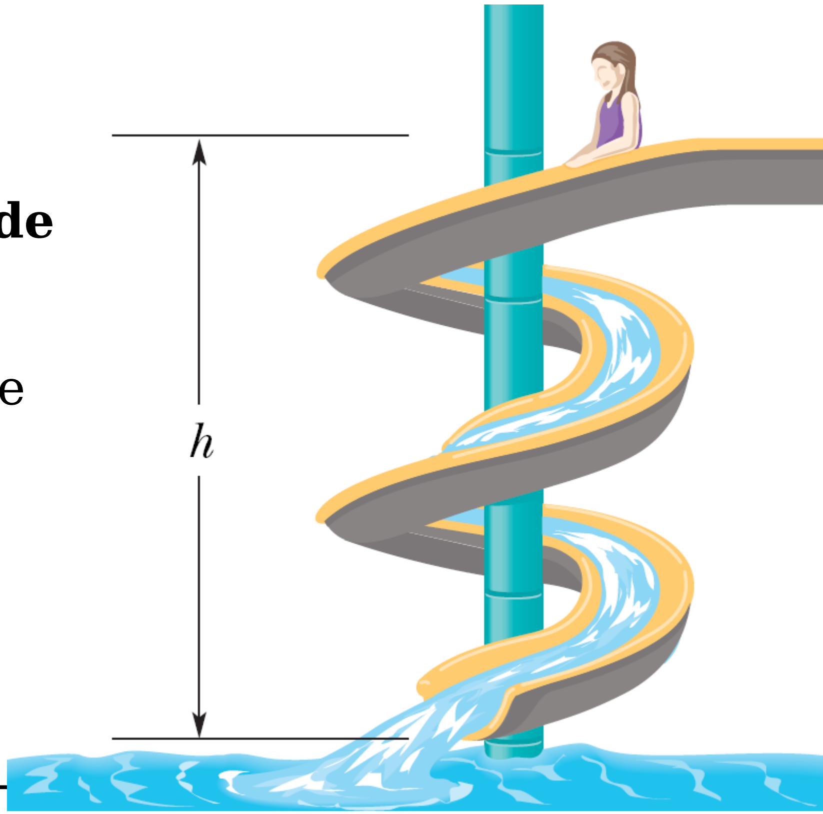
$v = +v_{max}$
$\vec{v}$
U K
(a)
$\vec{v}$
U K
(b)
$\vec{v} = 0$
U K
(c)
$\vec{v}$
U K
(d)
$v = -v_{max}$
$\vec{v}$
U K
$\vec{v}$
U K
(f)
$\vec{v} = 0$
U K
(g)
$\vec{v}$
U K
(h)

Sample 8.2.1:

Conservation of mechanical energy, water slide

The huge advantage of using the conservation of energy instead of Newton's laws of motion is that we can jump from the initial state to the final state without considering all the intermediate motion.

A child of mass m is released from rest at the top of a water slide, at height m=8.5 m above the bottom of the slide. Assuming that the slide is frictionless because of the water on it, find the child's speed at the bottom of the slide.



$$E_{\text{bottom}} = E_{\text{top}} \quad \Rightarrow \quad K_{\text{bottom}} + U_{\text{bottom}} = K_{\text{top}} + U_{\text{top}}$$

$$\Rightarrow \quad \frac{1}{2} m\, v_b^2 + m\, g\, y_b = \frac{1}{2} m\, v_t^2 + m\, g\, y_t \quad \Rightarrow \quad v_b^2 + 2\, g\, y_b = v_t^2 + 2\, g\, y_t$$

$$\Rightarrow \quad v_b^2 = v_t^2 + 2\, g\, (y_t - y_b) = 2\, g\, h \quad \Leftarrow \quad v_t = 0\,, \quad y_t - y_b = h$$

$$\Rightarrow \quad v_b = \sqrt{2\, g\, h} = \sqrt{2 \times 9.8 \times 8.5} = 13\ \mathrm{m/s}$$

Reading a Potential Energy Curve

Finding the Force Analytically

- For 1-dim motion,

$$\Delta U(x) = -W = -F(x)\,\Delta x$$

Solving for $F(x)$ and passing to the differential limit yield

$$\boxed{F(x) = -\frac{dU(x)}{dx} \quad \text{1-d motion}}$$

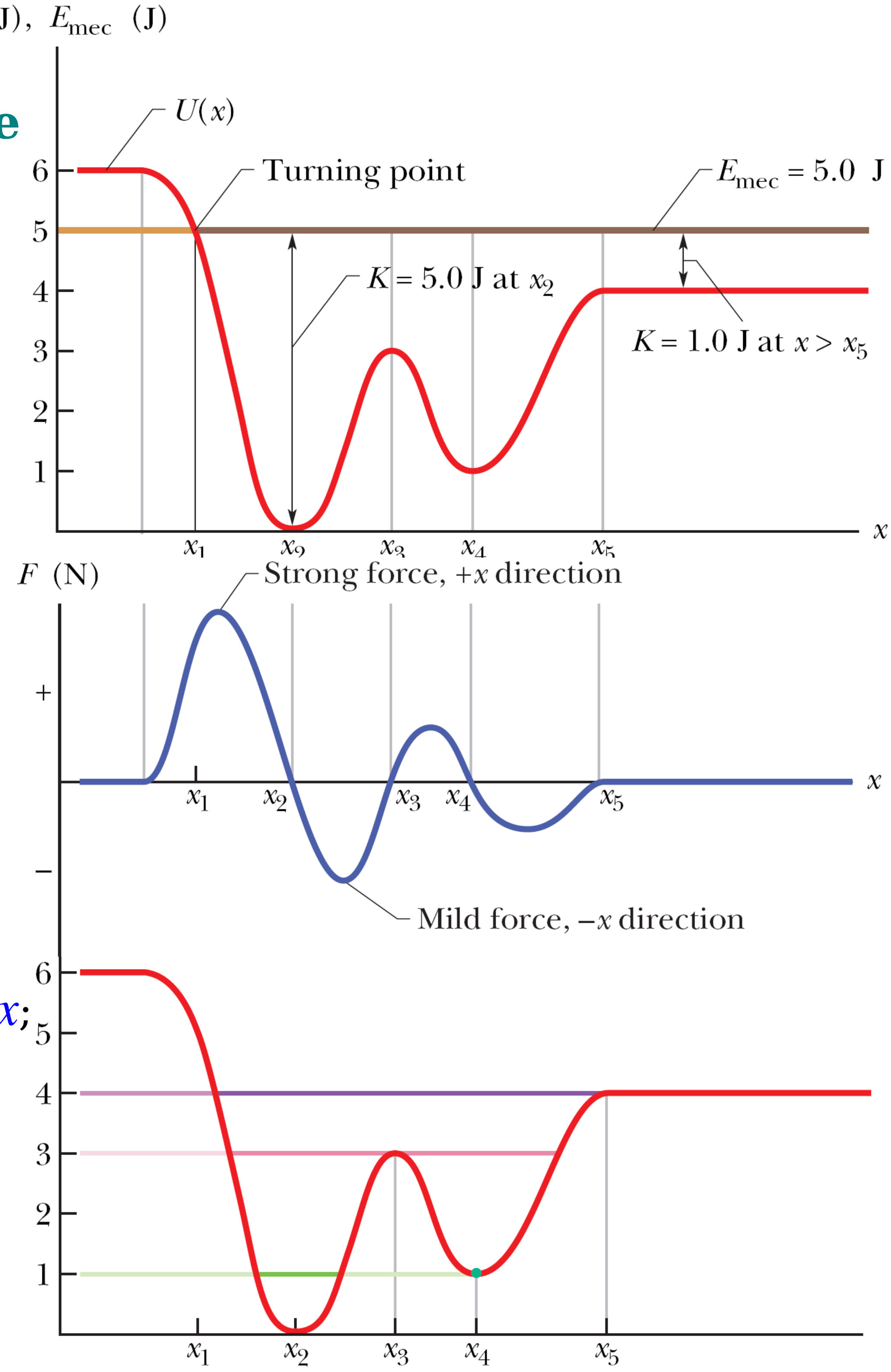
- $F(x)$ is the negative of the slope of the $U(x)$ curve.

- For example, the elastic potential

energy $U(x) = \dfrac{k}{2}x^2$, the force $F(x) = -k\,x$;

the gravitational potential energy

$U(x) = m\,g\,h$, and the force $F(x) = -m\,g$.



Turning points

- In the absence of a nonconservative force, the mechanical energy E of a system has a constant value $U(x)+K(x)=E_{\text{mec}} \Rightarrow K(x)=E_{\text{mec}}-U(x)$

- Determine the kinetic energy $K(x)$: On the $U(x)$ curve, find U for the location x and then subtract U from E_{mec}.

- Since K can never be negative, the particle will never move to the place (eg, x_1) where $E_{\text{mec}} - U$ is negative.

- A **turning point** is the place where $K = 0$ (because $U = E$) and the particle changes direction.

Equilibrium Points

- A particle at a position where the particle has no kinetic energy & no force acts on it (so it is stationary) is said to be in **neutral equilibrium**. For example, x_5.

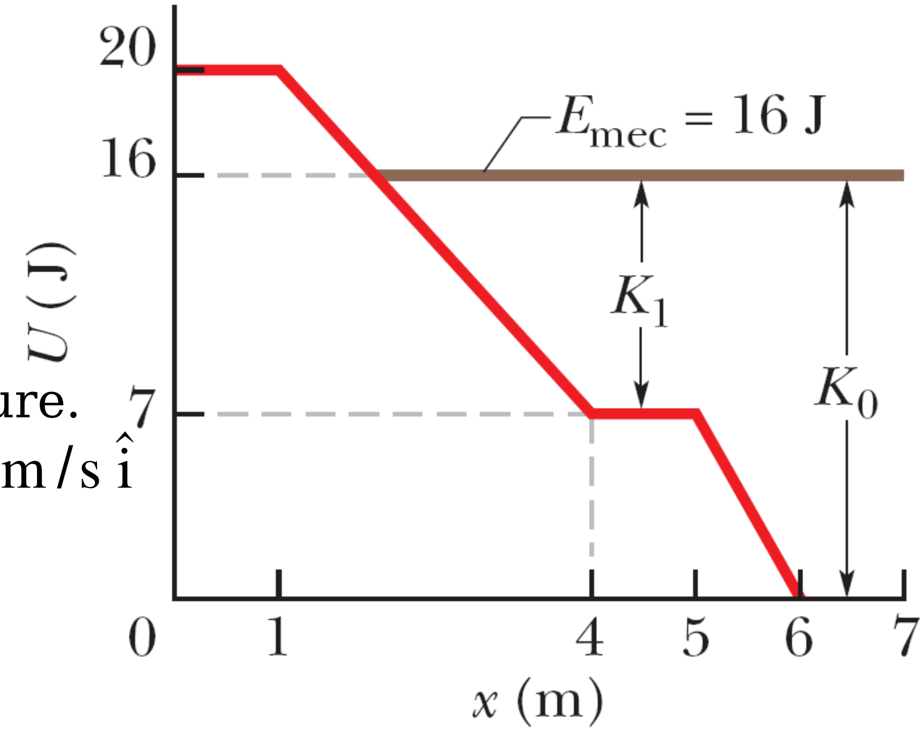
- A particle at a position where a small perturbation in either direction will cause a nonzero force pushing it farther in the same direction (and the particle continues to move) is said to be in **unstable equilibrium**. For example x_3.

- A particle at a position where the restoring force appear will move the particle back to this position if a slight push is applied to it in either direction is said to be in **stable equilibrium**. For example x_4.

Sample 8.3.1:

Reading a potential energy graph

A 2 kg particle moves along an x axis in 1d motion while a conservative force along that axis acts on it. The potential energy $U(x)$ associated with the force is plotted in the figure. At x=6.5 m, the particle has velocity $\vec{v}_0 = -4\ \mathrm{m/s}\ \hat{\mathrm{i}}$

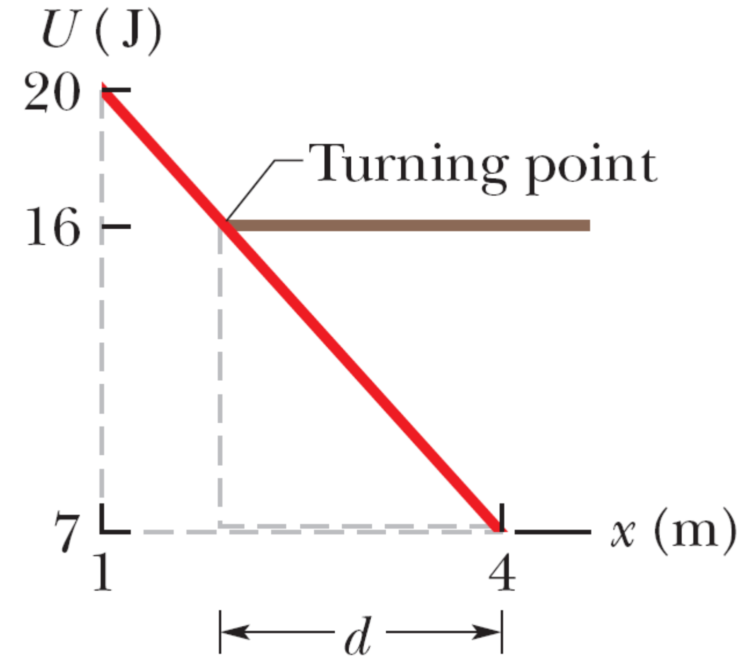


(a) From the figure, determine the particle's speed at x_1=4.5 m.

$$E = K_0 + U_0 = \frac{1}{2} m v_0^2 + 0 = 16\ \mathrm{J}$$

$$= K_1 + U_1 = \frac{1}{2} m v_1^2 + 7 = v_1^2 + 7 \quad \Rightarrow \quad v_1 = 3\ \mathrm{m/s}$$

(b) Where is the particle's turning point located?



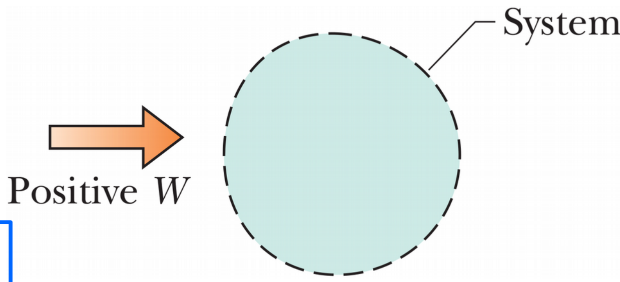
$$\frac{7-20}{4-1} = \frac{7-16}{4-x} \quad \Rightarrow \quad x = 1.92\ \mathrm{m}$$

(c) Evaluate the force acting on the particle when it is in the region 1.9 m < x < 4 m.

$$F = -\frac{\mathrm{d}\,U}{\mathrm{d}\,x} = -\frac{7-20}{4-1} = 4.3\ \mathrm{N}$$

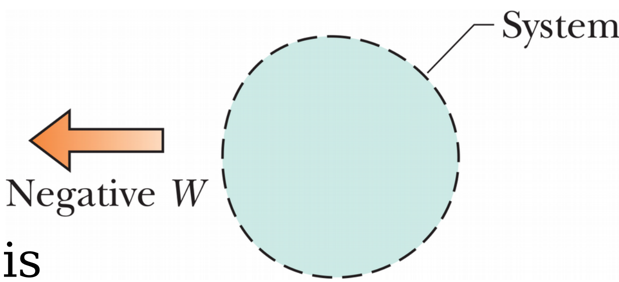
Work Done on a System by an External Force

- Extend the definition of work to an external force acting on a system



> Work is energy transferred to or from a system by means of an external force acting on the system.

- External forces can transfer energy into different types of energy of a system.

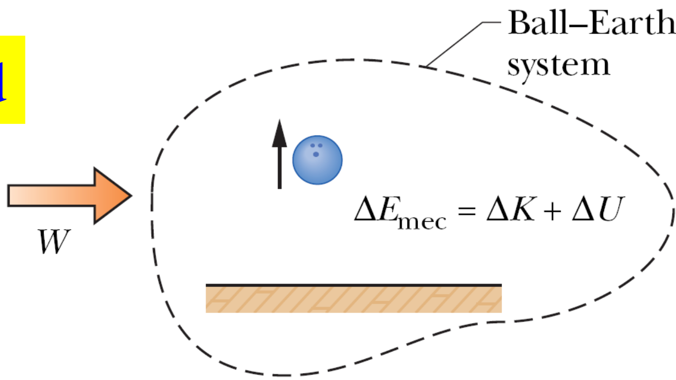


No Friction Involved

- For an external force doing work on a system the work is

$$W = \Delta K + \Delta U$$

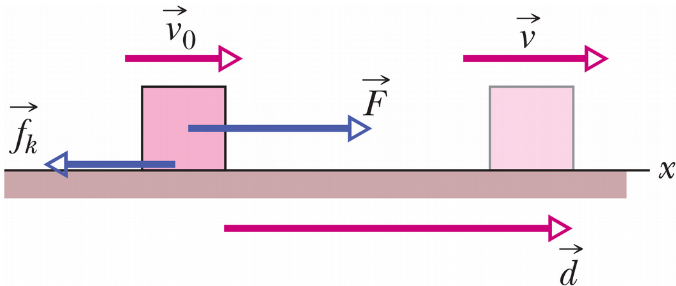
$$\Rightarrow \quad W = \Delta E_{mec} \quad \text{work done on system, no friction involved}$$



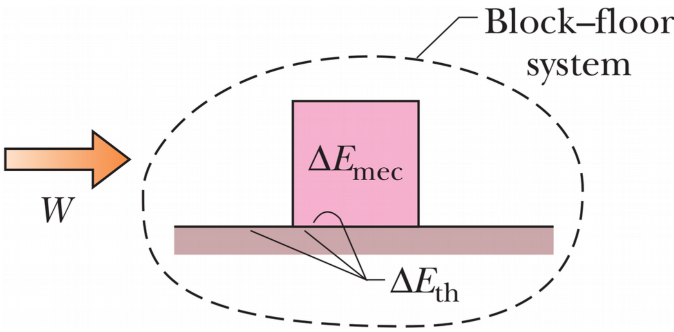
Friction Involved

- Consider a constant horizontal force pulls a block along an x axis and through a displacement of magnitude d, also increasing the block's velocity. During the motion, a constant kinetic frictional force from the floor acts on the block.

- Using Newton's 2nd law: $F - f_k = m\,a$



- Since the acceleration is constant, $v^2 = v_0^2 + 2ad$



and $Fd = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + f_k d \quad \Rightarrow \quad Fd = \Delta K + f_k d$

- In a more general situation, there can be a changes in potential energy, thus

$$Fd = \Delta E_{mec} + f_k d$$

- Thermal energy: the energy associated with the random motion of the atoms and molecules in the object.

- The thermal energy of the block and floor increases because (1) there is friction between them and (2) there is sliding.

- The increase ΔE_{th} in thermal energy is equal to the product of the magnitudes f_k and d:

$$\Delta E_{th} = f_k d \quad \text{increase in thermal energy by sliding}$$

$$\Rightarrow \quad Fd = \Delta E_{mec} + \Delta E_{th}$$

$$\Rightarrow \quad W = \Delta E_{mec} + \Delta E_{th} \quad \text{work done on system, friction involved}$$

Sample 8.4.1: **Easter Island**

People most likely cradled each statue in a wooden sled and then pulled the sled over a "runway" consisting of almost identical logs acting as rollers. In a modern reenactment of this technique, 25 men were able to move a 9000 kg Easter Island–type statue 45 m over level ground in 2 min.

(a) Estimate the work the net force $\vec{F}$ from the men did during the 45 m displacement of the statue, and determine the system on which that force did work.

Assume a man's mass is 80 kg, and each man can pull with a force magnitude equal to twice his weight

$$W = F\,d = 25 \times 2 \times m\,g\,d = 25 \times 2 \times 80 \times 9.8 \times 45 = 1.8 \times 10^6\ \text{J} = 1.8\ \text{MJ}$$

(b) What was the increase ΔE_{th} in the thermal energy of the system during the 45m displacement?

$$\Delta E_{mech} = 0 \quad \Rightarrow \quad W = \Delta E_{mech} + \Delta E_{th} = \Delta E_{th} \quad \Rightarrow \quad \Delta E_{th} = 1.8\ \text{MJ}$$

(c) Estimate the work that would have been done by the 25 men if they had moved the statue 10 km across level ground on Easter Island. Also estimate the total change ΔE_{th} that would have occurred in the statue–sled–logs–ground system.

$$\Delta E_{th} = W = 25 \times 2 \times 80 \times 9.8 \times 10000 = 392\ \text{MJ}$$

Conservation of Energy

- In all the cases, we assumed that energy obeys a law called the **law of conservation of energy**, which is concerned with the **total energy** of a system.

- The total is the sum of the system's mechanical energy, thermal energy, and any type of *internal energy* in addition to thermal energy.

The total energy E of a system can change only by amounts of energy that are transferred to or from the system.

Thus $W = \Delta E = \Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}}$

- The law of conservation of energy is based on the results of countless experiments, not derived form basic physics principle.

Isolated System

- If a system is isolated from its environment, there can be no energy transfers to or from it.

The total energy E of an isolated system cannot change.

$$\Delta E_{\text{mec}} + \Delta E_{\text{th}} + \Delta E_{\text{int}} = 0 \quad \text{isolated system}$$

$$\Delta E_{\text{mec}} = E_{\text{mec},2} - E_{\text{mec},1} \quad \Rightarrow \quad E_{\text{mec},2} = E_{\text{mec},1} - \Delta E_{\text{th}} - \Delta E_{\text{int}}$$

In an isolated system, we can relate the total energy at one instant to the total energy at another instant *without considering the energies at intermediate times*.

External Forces and Internal Energy Transfers

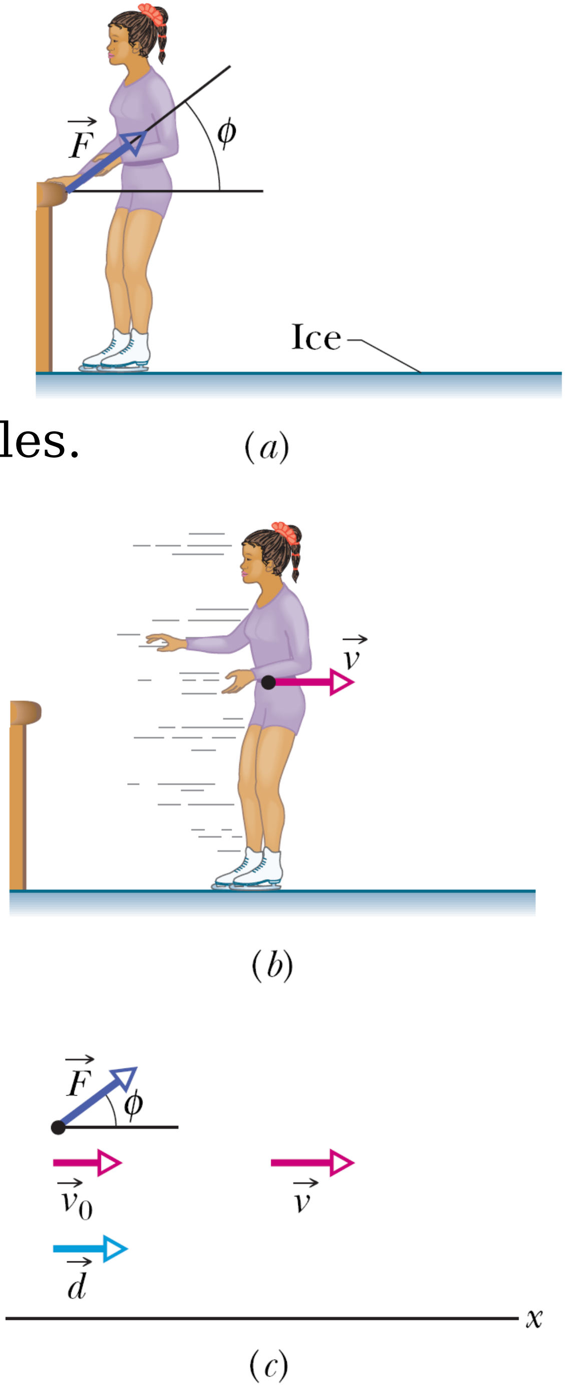
- An external force can change the kinetic energy or potential energy of an object without doing work on the object — that is, without transferring energy to the object. Instead, the force is responsible for transfers of energy from one type to another inside the object.

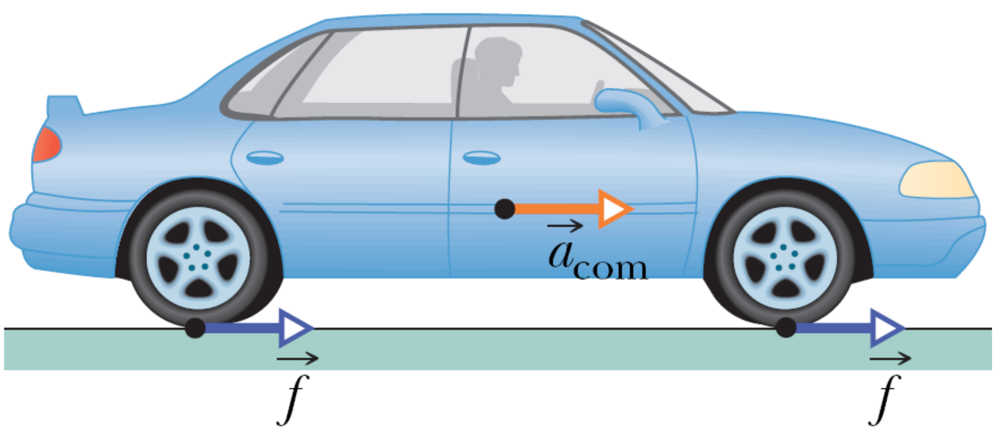
- The ice skater changes her kinetic energy as a result of internal transfers from the biochemical energy in her muscles.

- After the push, $\Delta K = K - K_0 = (F \cos\phi)\, d = F\, d \cos\phi$

- If the situation also involves a change in the elevation of an object, we can include the change ΔU in gravitational potential energy by $\Delta U + \Delta K = F\, d \cos\phi$

- The car's kinetic energy increases as a result of internal transfers from the energy stored in the fuel.





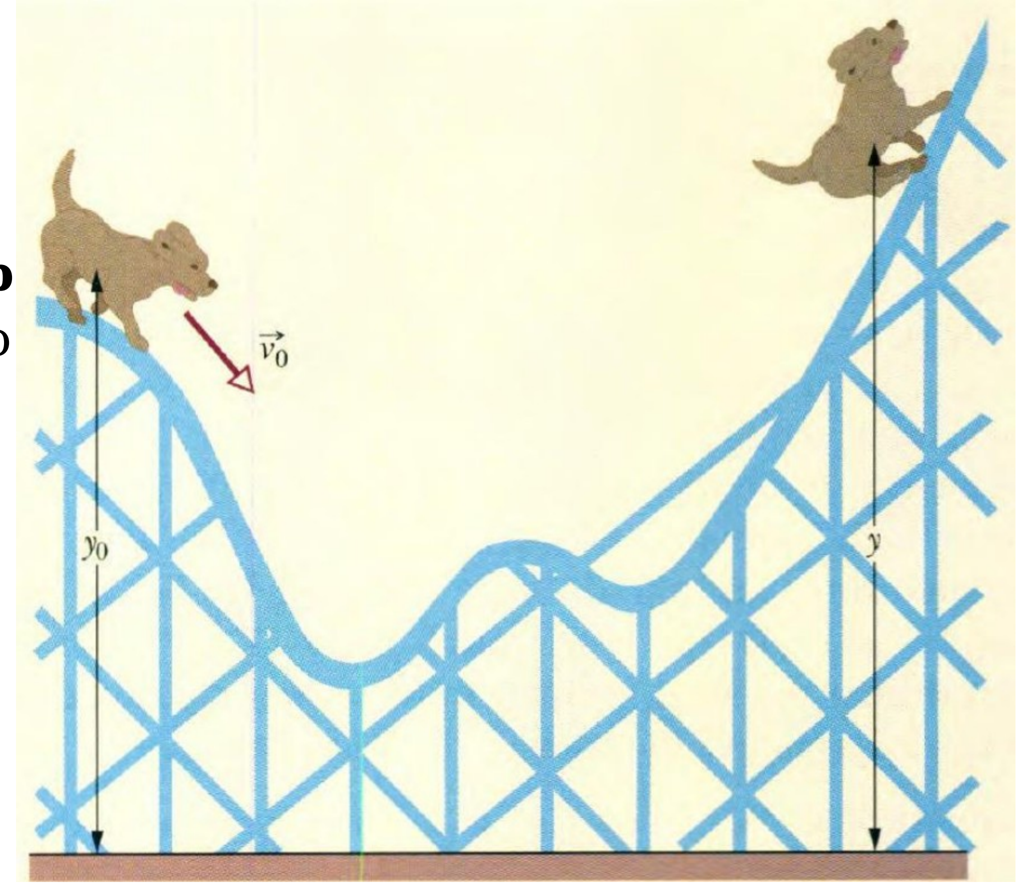
Power

- The rate at which energy is transferred by a force from one type to another.
- If an amount of energy ΔE is transferred in an amount of time Δt, the **average power** due to the force is $P_{\text{avg}} = \dfrac{\Delta E}{\Delta t}$
- The **instantaneous power** due to the force is $\boxed{P = \dfrac{dE}{dt}}$

The chosen problem: 13, 21, 31, 43

<u>Sample 8.5.1</u>: Dog sliding along a ramp

A circus beagle of mass m=6 kg runs onto the left end of a curved ramp with initial speed v_0= 7.8 m/s and at height y_0=8.5 m above the floor. It then slides to the right and comes to a momentary stop when it reaches height y=1.1 m above the floor. The ramp is not frictionless. What is the increase ΔE_{th} in the thermal energy of the beagle and ramp because of the sliding?



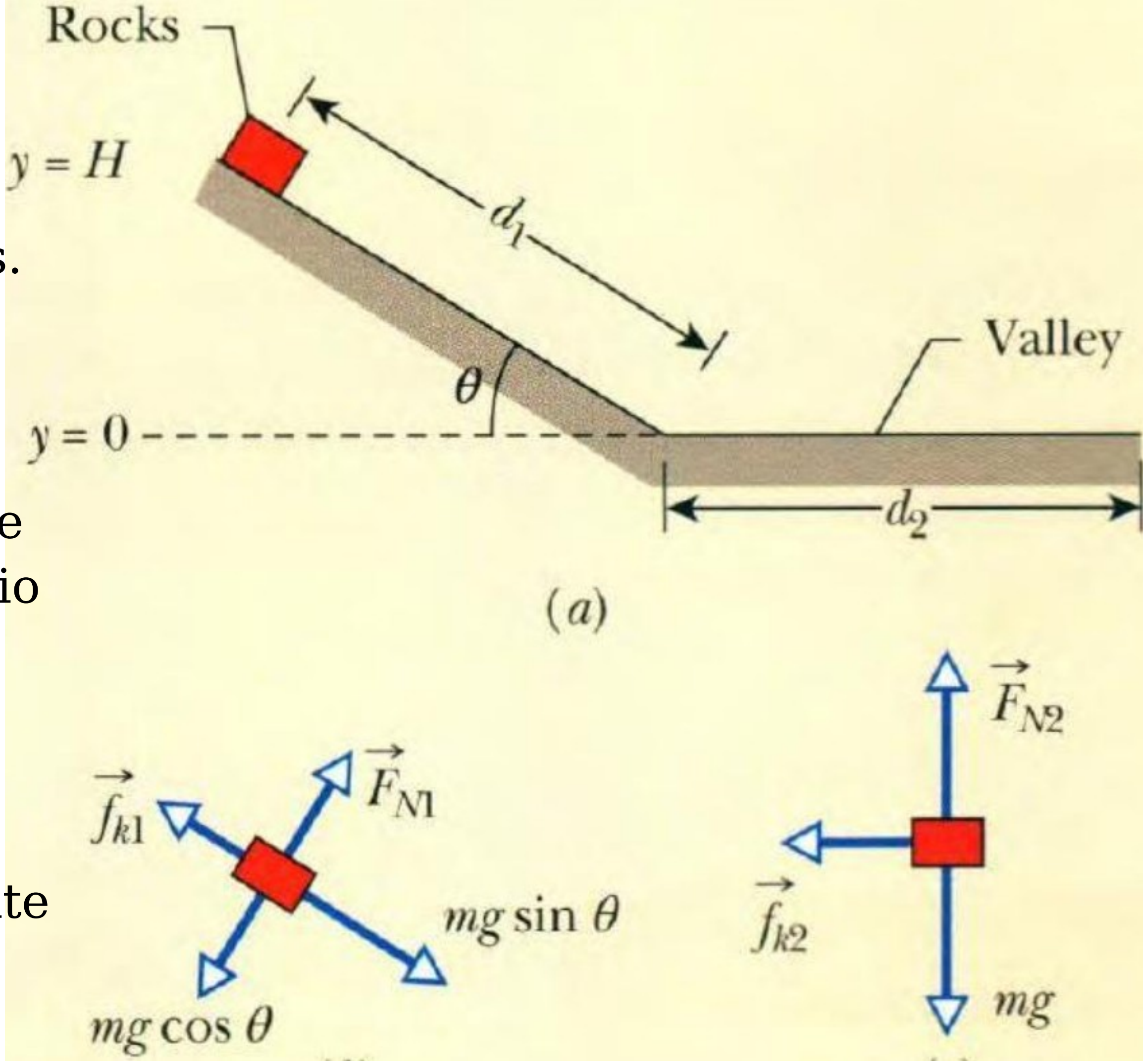
$$\Delta E_{mech} + \Delta E_{th} = 0$$

$$\Delta E_{mech} = \Delta K + \Delta U = K_f + U_f - K_i - U_i = \frac{1}{2} m v^2 + m g y - \frac{1}{2} m v_0^2 - m g y_0$$

$$= 0 - \frac{1}{2} \times 6 \times (7.8)^2 + 6 \times 9.8 \times (11.1 - 8.5) \approx -29.6 \text{ J}$$

$$\Rightarrow \quad \Delta E_{th} \approx 29.6 \text{ J}$$

<u>Sample 8.5.2</u>: **Avalanche runout**
It shows a simplified arrangement of a mountain slope and the valley along which a rock avalanche moves. The rocks have a total mass m, fall from a height $y=H$, move a distance d_1 along a slope of angle $\theta=45°$, and then move through a runout distance d_2 along a flat valley. What is the ratio d_2/H of the runout to the fall height if the coefficient of kinetic friction μ_k has the reasonable value of 0.6? By calculating this runout ratio, geologists and engineers can estimate the damage that might be done by a potential rock avalanche spotted at a weak spot on a mountainside.



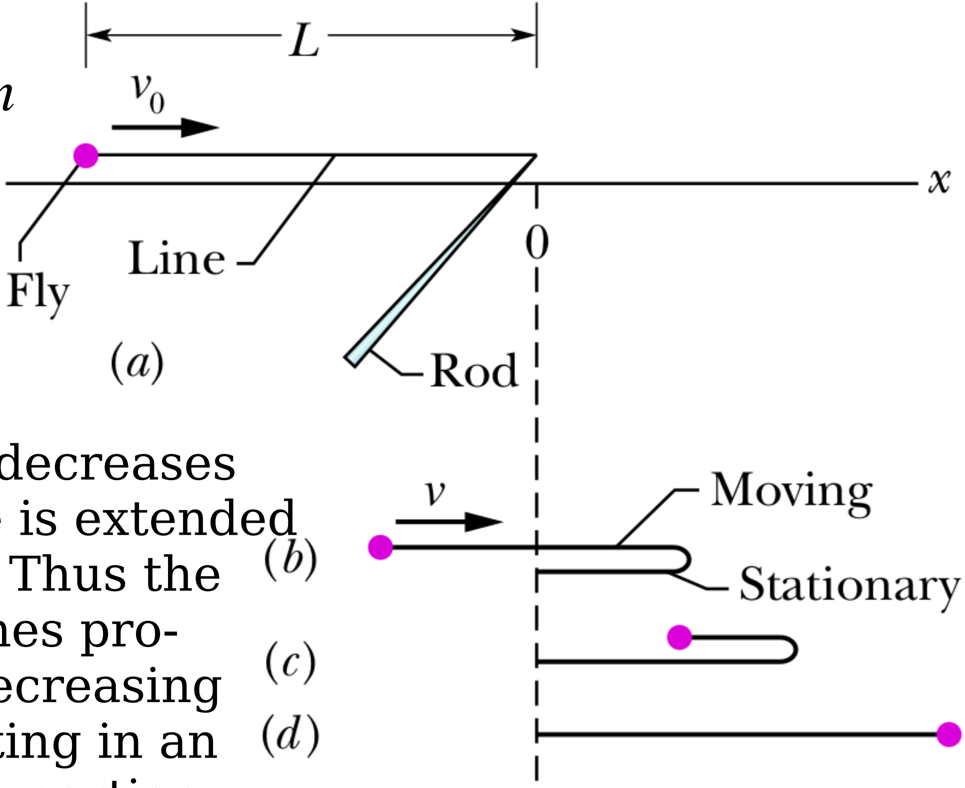
$$\Delta E_{\text{mech}} + \Delta E_{\text{th}} = 0 \quad \Rightarrow \quad \Delta E_{\text{th}} = -\Delta E_{\text{mech}} = E_{\text{mech},i} - E_{\text{mech},f}$$

$$K_i = 0, \quad U_i = mgH, \quad K_f = 0, \quad U_f = 0 \quad \Rightarrow \quad \Delta E_{\text{th}} = mgH \quad \Leftarrow \quad H = d_1 \sin\theta$$

$$\Delta E_{\text{th}} = f_{k1} d_1 + f_{k2} d_2 = \mu_k (mg\cos\theta) d_1 + \mu_k mg d_2 = \mu_k mg (d_1\cos\theta + d_2)$$

$$\Rightarrow \quad H = \mu_k (d_1\cos\theta + d_2) = \mu_k (H\cot\theta + d_2) \quad \Rightarrow \quad \frac{d_2}{H} = \frac{1 - \mu_k \cot\theta}{\mu_k} = \frac{2}{3} \quad \Leftarrow \begin{matrix} \theta = 45° \\ \mu_k = 0.6 \end{matrix}$$

Problem: *Fly-fishing and speed amplification*



Initially the line of length L is extended horizontally leftward and moving rightward with speed v_0. As the fly at the end of the line moves forward, the line doubles over, with the upper section still moving and the lower section stationary. The upper section decreases as the lower section increases, until the line is extended rightward and there is only a lower section. Thus the initial kinetic energy of the line in (a) becomes progressively concentrated in the fly and the decreasing portion of the line that is still moving, resulting in an amplification in the speed of the fly and that portion.

(1) Show that when the fly position is x, the length of the still-moving (upper) section of line is $(L - x)/2$.

$$x = \begin{bmatrix} -L \\ 0 \\ L \end{bmatrix} \Rightarrow \ell_{up} = \begin{bmatrix} L \\ L/2 \\ 0 \end{bmatrix} \Rightarrow \ell_{up}(x) = \frac{L-x}{2}$$

(b) If the linear density ρ is uniform, what is the mass of the still-moving section?

$$m_{up}(x) = \rho\,\ell_{up}(x) = \rho\,\frac{L-x}{2}$$

Let m_f is the mass of the fly and assume that the kinetic energy of the moving section does not change from its initial value even though the length of the moving section is decreasing during the cast.

(c) Find an expression for the speed of the still-moving section and the fly.

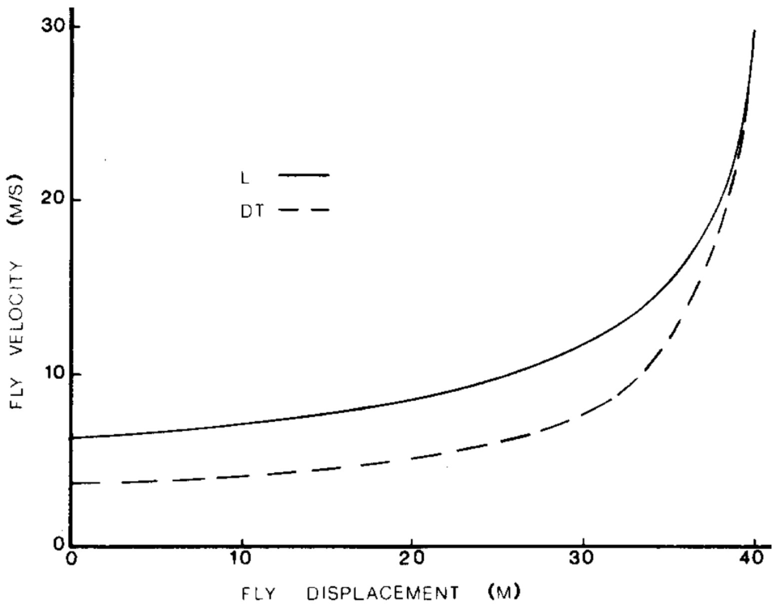
$$K_0 = \frac{1}{2} m_f v_0^2 + \frac{1}{2} m_{\mathrm{up}}(-L) v_0^2 = \frac{v_0^2}{2}(m_f + L\rho)$$

$$K = \frac{1}{2} m_f v^2 + \frac{1}{2} m_{\mathrm{up}}(x) v^2 = \frac{v^2}{2}\left(m_f + \rho \frac{L-x}{2}\right) \quad + \quad K_0 = K$$

$$\Rightarrow \quad v(x) = v_0 \sqrt{\frac{2m_f + 2L\rho}{2m_f + \rho(L-x)}}$$

Assume that initial speed v_0=6 m/s, line length L=20 m, fly mass m_f= 0.8 g, and linear density ρ=1.3 g/m.

(d) Plot the fly's speed v versus its position x.



(e) What is the fly's speed just as the line approaches its final horizontal orientation and the fly is about to flip over and stop?

$$x = L \quad \Rightarrow \quad v_{\max} = v_0\sqrt{1 + \frac{L\rho}{m_f}} = 34.7\ \mathrm{m/s} \approx 125\ \mathrm{km/h} \quad \text{vs} \quad v_0 = 21.6\ \mathrm{km/h}$$

(The fly then pulls out more line from the reel. In more realistic calculations, air drag reduces this final speed.) Speed amplification can also be produced with a bullwhip and even a rolled-up wet towel that is popped against a victim in a common locker-room prank.