Chapter 7 Kinetic Energy and Work

What is Energy?

- a (loose) definition: Energy is a number that we associate with a system. If a force changes the system, then the energy number changes.

- a technical definition: Energy is a scalar quantity associated with the state of a system.

- Its property: Energy can be transformed from one type to another and transform from one object to another, but the total amount is always the same (energy is *conserved*). (**principle of energy conservation**)

Kinetic Energy

- **Kinetic energy** K is energy associated with the *state of motion* of an object. For an object of mass m whose speed v is well below the speed of light,

$$\boxed{K = \frac{1}{2} m v^2 \quad \text{kinetic energy}}$$

The SI unit of energy is **joule**: 1 joule = 1 J = 1 kg m^2/s^2 problem 7-1

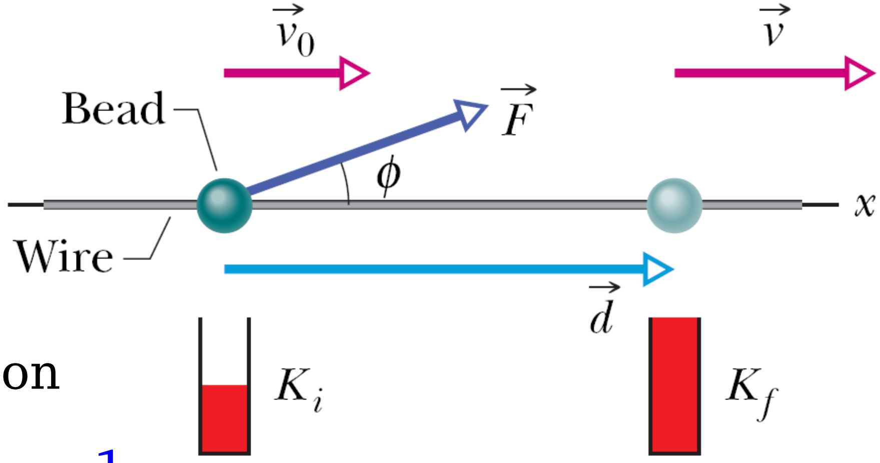
Work

- In a transfer of energy via force, **work** W is said to be *done on the object by the force*.

Work W is energy transferred to or from an object by means of a force acting on the object. Energy transferred to the object is positive work, and energy transferred from the object is negative work.

Work and Kinetic Energy

Finding an Expression for work



- A constant force, directed at an angle ϕ to the wire, accelerates the bean along the wire,

$$f_x = m\, a_x$$

- Because the force is constant, the acceleration is also constant.

$$v^2 = v_0^2 + 2\, a_x\, d \qquad \Rightarrow \qquad \frac{1}{2} m\, v^2 - \frac{1}{2} m\, v_0^2 = F_x\, d$$

$$K_f - K_i = F_x\, d \qquad \Rightarrow \qquad W = F_x\, d$$

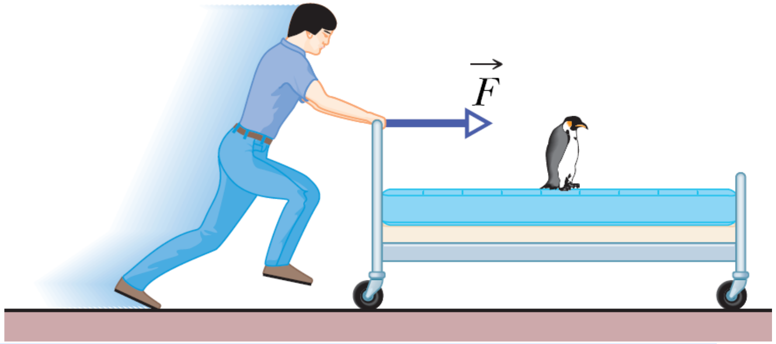
To calculate the work a force does on an object as the object moves through some displacement, we use only the force component along the object's displacement. The force component perpendicular to the displacement does zero work.

$$W = F_x\, d = F\, d \cos \phi$$

$$W = \vec{F} \cdot \vec{d} \qquad (\text{work done by a constant force})$$

$$W = \vec{F} \cdot \vec{d} = F_x\, d_x + F_y\, d_y + F_z\, d_z \qquad (\text{in general})$$

- *Cautions*: 1) the force must be a *constant force*;
  2) the object must be *particle-like*.



- *signs for work*:

> A force does positive work when it has a vector component in the same direction as the displacement, and it does negative work when it has a vector component in the opposite direction. It doest zero work when it has no such vector component.

- the SI *unit for work* is joule since 1 N m = 1 kg m^2/s^2 = 1 J.

- *Net work done by several forces*: the **net work** can be calculated in 2 ways:
  1) find the work done by each force and then sum those works;
  2) find the net force of those forces and then $W = \vec{F}_{\text{net}} \cdot \vec{d}$

Work-Kinetic Energy Theorem

- Let ΔK be the change in the kinetic energy of the object, and let W be the net work done on it, then

$$\Delta K = K_f - K_i = W$$

which says that

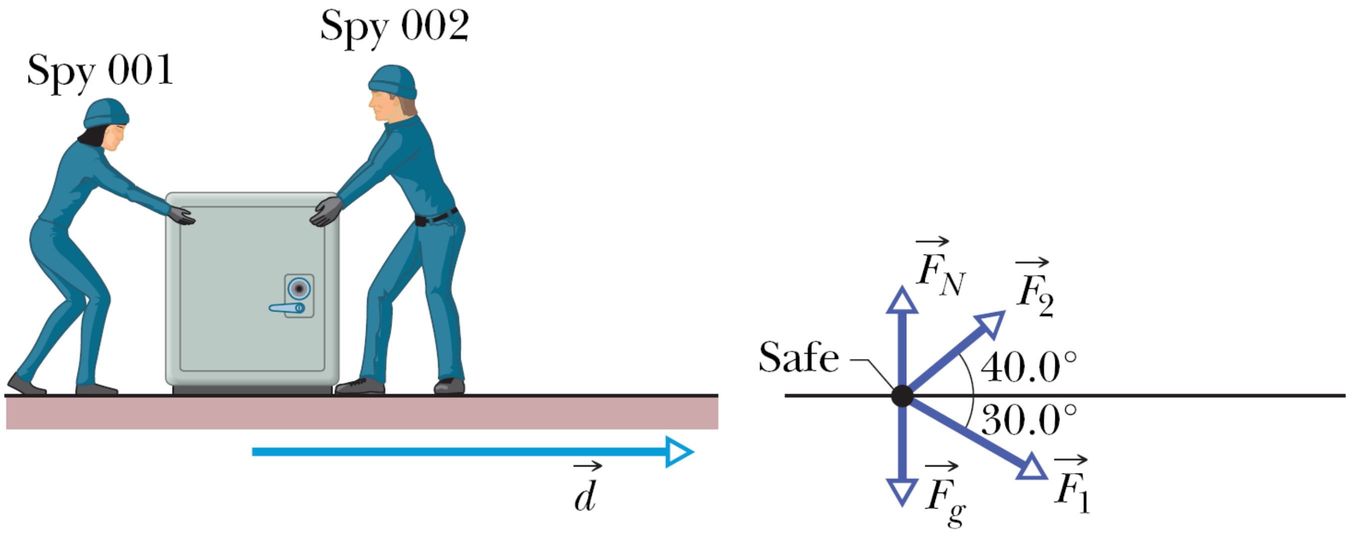
$$\begin{pmatrix}\text{change in the kinetic}\\ \text{energy of a particle}\end{pmatrix} = \begin{pmatrix}\text{net work done on}\\ \text{the particle}\end{pmatrix}$$

We can also write $K_f = K_i + W$

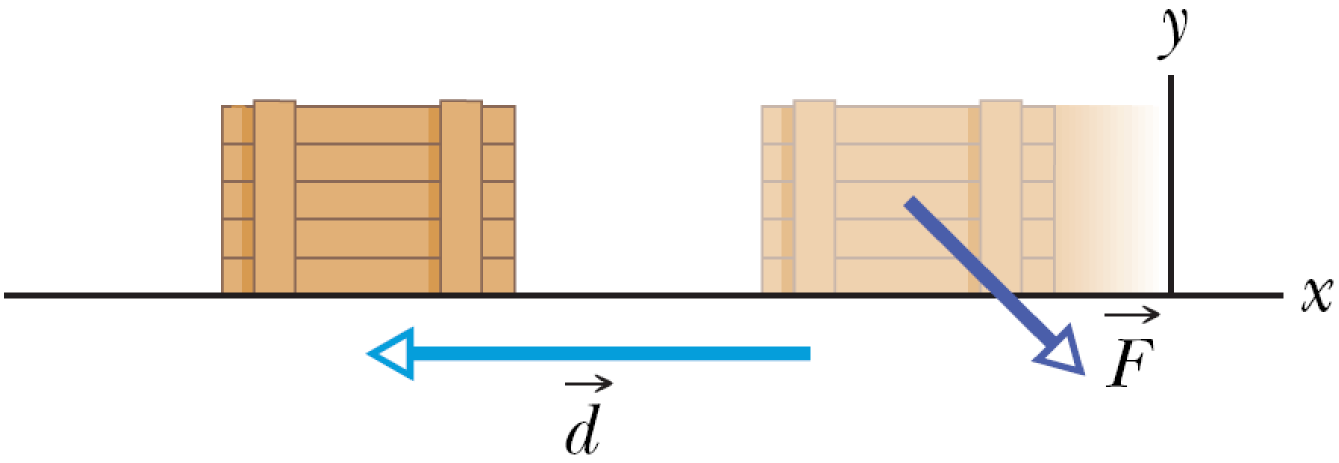
which says that $\begin{pmatrix}\text{kinetic energy after}\\ \text{the net work is done}\end{pmatrix} = \begin{pmatrix}\text{kinetic energy}\\ \text{before the net work}\end{pmatrix} + \begin{pmatrix}\text{the net}\\ \text{work done}\end{pmatrix}$

- These statements are the **work-kinetic energy theorem**.

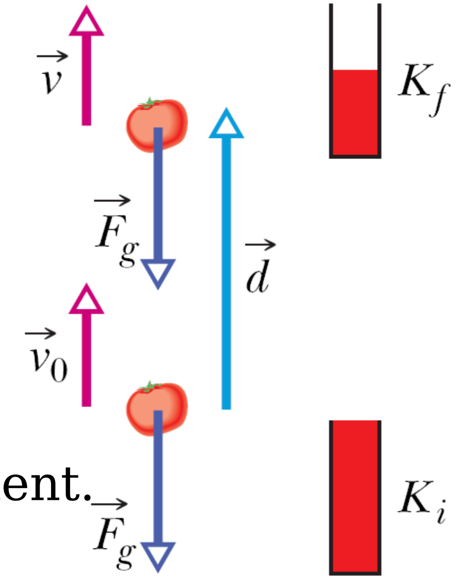
problem 7-2



problem 7-3



Work Done by the Gravitational Force



- As the particle rises, it is slowed by a gravitational force; that is, the particle's kinetic energy decreases because the gravitational force does work on the particle as it rises.

$$\boxed{W_g = \vec{F}_g \cdot \vec{d} = m\,g\,d\cos\phi \qquad \text{work done by gravitational force}}$$

For a rising object, the force is directed opposite the displacement. Thus, $\phi = 180°$ and $W_g = m\,g\,d\cos 180 \;= -\,m\,g\,d$

- The − sign tells us that during the object's rise, the gravitational force acting on the object transfers energy in the amount mgd from the kinetic energy of the object. This is consistent with the slowing of the object as it rises.

- After the object has reached its maximum height and is falling back down, then $\phi = 0$. Thus, $W_g = m\,g\,d\cos 0 = +\,m\,g\,d$

- The + sign shows that the gravitational force transfers energy in the amount mgd to the kinetic energy of the object. This is consistent with the speeding up of the object as it falls.

Work Done in Lifting and Lowering an Object

- Suppose we lift an object by applying a vertical force to it. The our applied force does positive work W_a on the object while the gravitational force does negative work W_g on it. The change in the kinetic energy of the object is

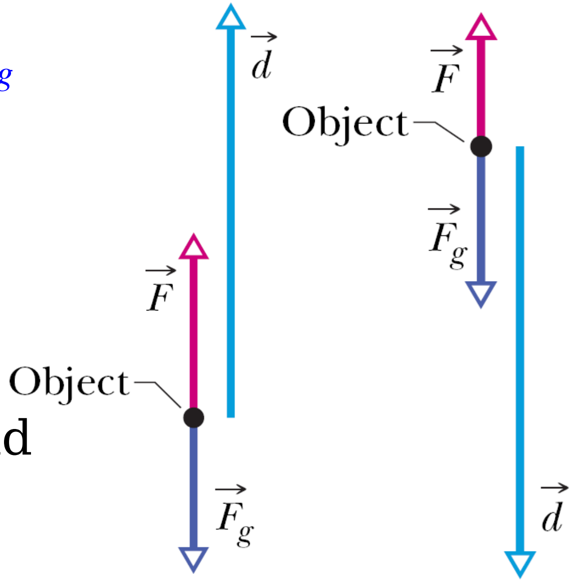
$$\boxed{\Delta K = K_f - K_i = W_a + W_g}$$

- In one common situation, the object is stationary before and after the lift. Then K_f and K_i are 0, thus, $W_a + W_g = 0$ or $W_a = -W_g$

- The result means that the applied force transfers the same amount of energy to the object as the gravitational force transfers from the object,

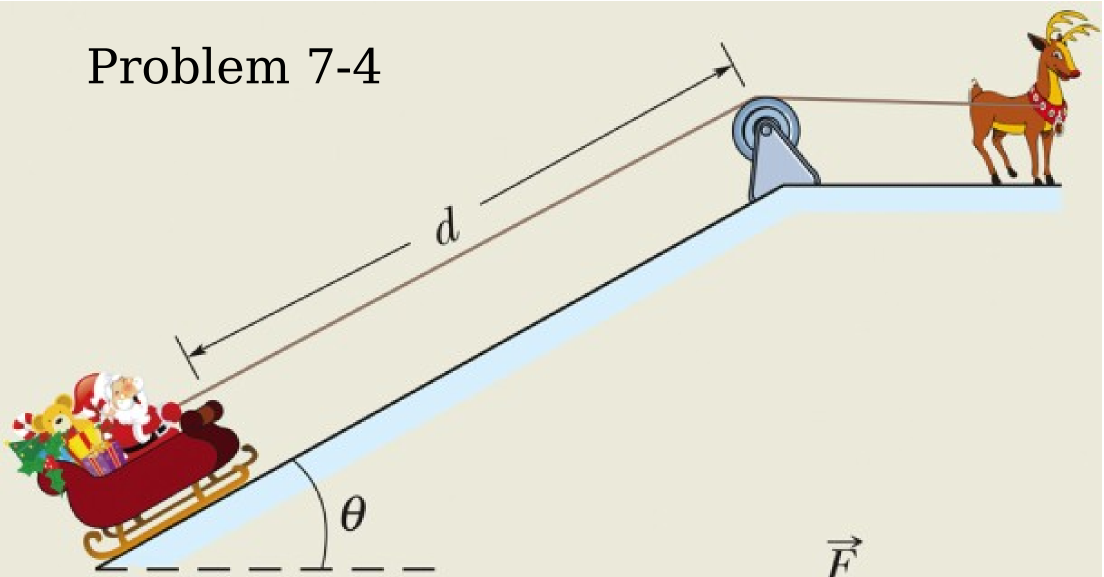
$$W_a = -m\,g\,d\cos\phi \quad \text{work done in lifting/lowering; } K_f = K_i$$

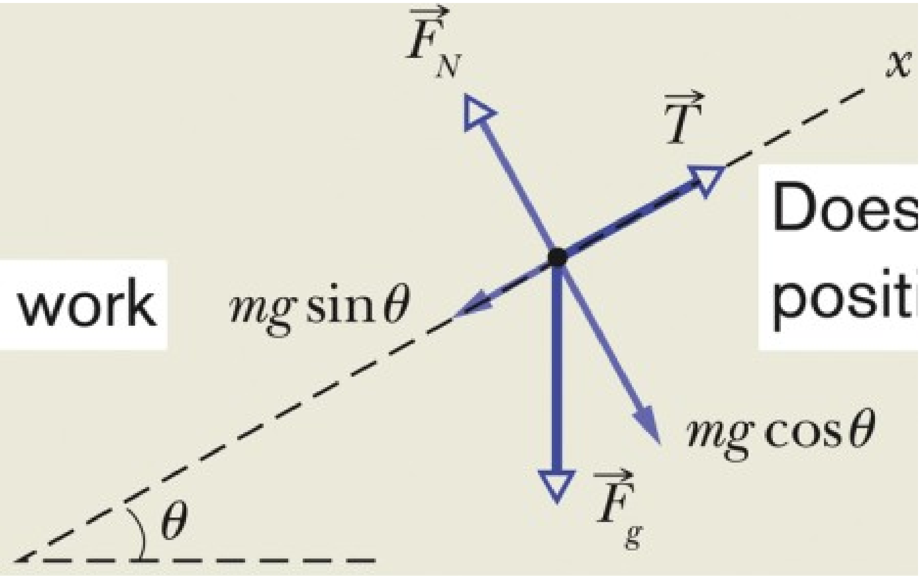
with ϕ being the angle between the gravitational force and the displacement.



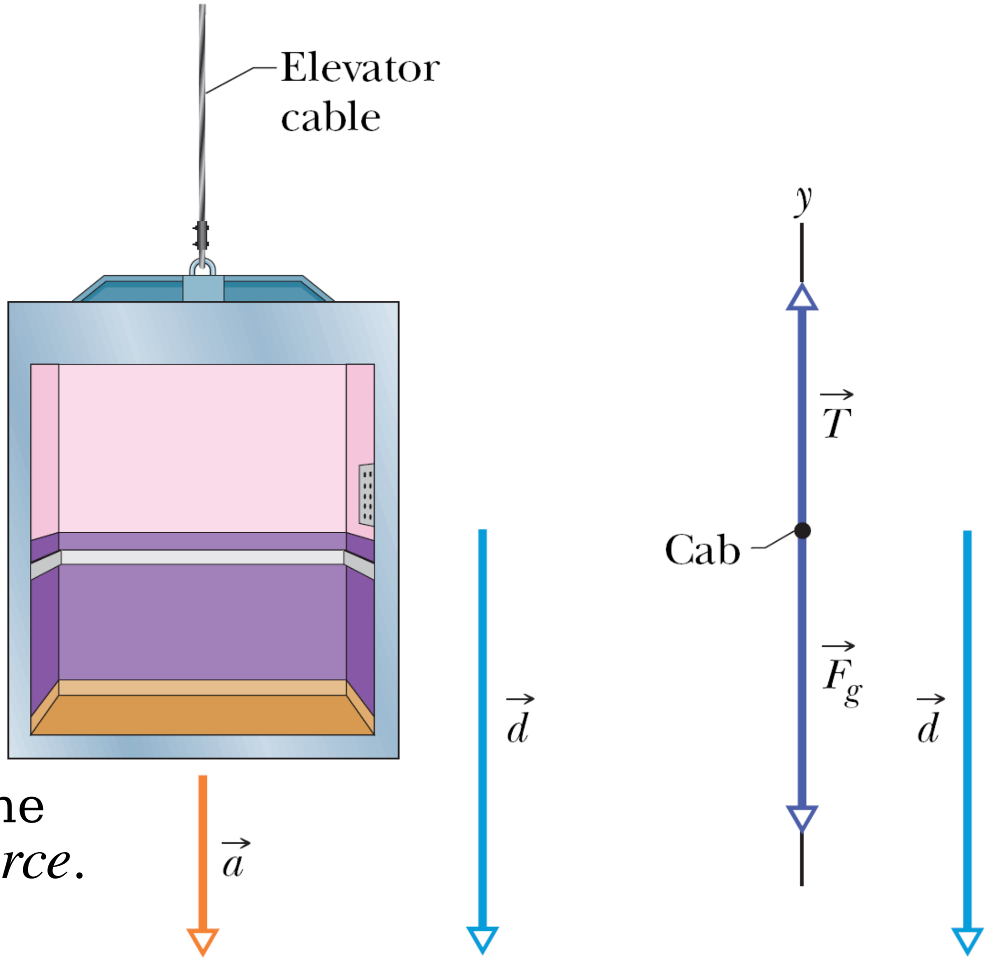
- The equations apply to any situation in which an object is lifted or lowered , with the object stationary before and after the lift. They are independent of the magnitude of the force used.

Problem 7-4





problem 7-5



Work Done by a Spring Force

- A **spring force** is a *variable force,* symbolized as $F(x)$.

- Many forces in nature have the same mathematical form as the spring force.

- the **relaxed state**: neither compressed nor extended.

- Because a spring force acts to restore the relaxed state, it is said to be a *restoring force*.

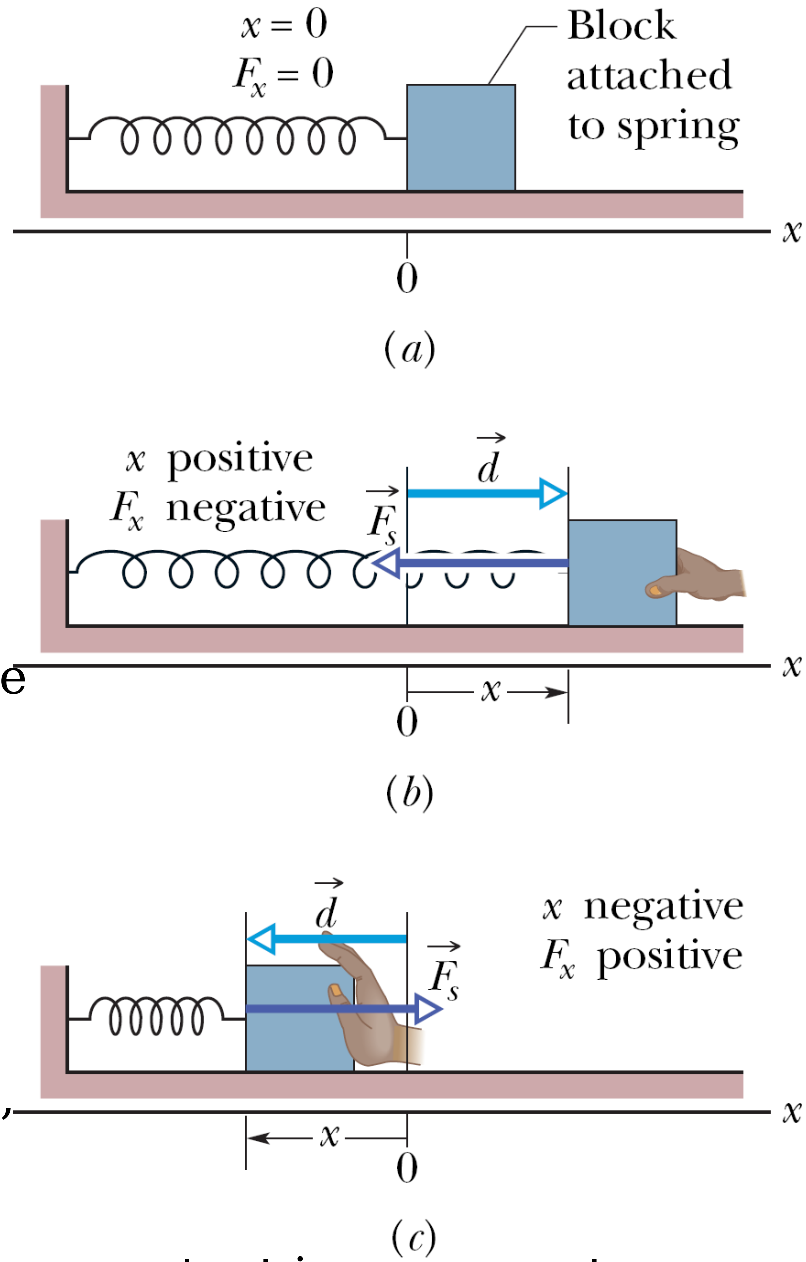
- the spring force form a spring is *linearly* proportional to the displacement of the free end from its position when the spring is in the relaxed state,

$$\boxed{\vec{F}_s = -k\,\vec{d} \quad \text{or} \quad F_x = -k\,x \qquad \text{Hooke's law}}$$

where the constant is called the **spring constant**(or **force constant**) and is a measure of the stiffness of the spring, and its SI unit is N/m.

- The − sign indicates that the direction of the spring force is always opposite the direction of the displacement of the spring's free end.

The Work Done by a Spring Force



- 2 simplifying assumptions about the spring:
  1) *massless*, ie, its mass is negligible relative to the block's mass;
  2) *ideal spring*, ie, it obeys Hook's law exactly.

- also assume it's frictionless between the block and the surface and the block is particle-like.

- *cannot* use $W=Fd\cos\phi$ to find the work because the spring force is a variable force. We need calculus to find the work.

- Let the block's initial and later positions be x_i and x_f. Then divide the distance between those 2 positions into many segments, each of tiny length Δx. Label these segments, starting from x_i, as segment 1, 2 … .

- approximate the spring force magnitude as being constant in a segment because the segment is so short that x hardly varies. Label these magnitudes as F_{x1} in segment 1, F_{x2} in segment 2, … .

- With the force constant in each segment, we can find the work done within each segment. The work done is $F_{x1}\Delta x$ in segment 1, $F_{x2}\Delta x$ in segment 2, … .

- The net work W_s done by the spring, form x_i to x_f is the sum of all these works:

$$W_s = \sum_j F_{xj}\,\Delta x$$

In the limit as Δx goes to 0,

$$W_s = \int_{x_i}^{x_f} F_x\,\mathrm{d}x$$

substitute F_x with $-kx$,

$$W_s = \int_{x_i}^{x_f} (-k\,x\,\mathrm{d}x) = -k\int_{x_i}^{x_f} x\,\mathrm{d}x$$

$$= -\frac{1}{2}k\left[x^2\right]_{x_i}^{x_f} = -\frac{1}{2}k\left(x_f^2 - x_i^2\right)$$

thus,

$$\boxed{W_s = \frac{1}{2}k\,x_i^2 - \frac{1}{2}k\,x_f^2 \qquad \text{work by a spring force}}$$

Work W_s is positive if the block ends up closer to the relaxed position ($x = 0$) than it was initially. It is negative if the block ends up farther away from $x = 0$. It is 0 if the block ends up at the same distance from $x = 0$.

- If x_i =0 and if we call the final position x, then

$$W_s = -\frac{1}{2} k x^2 \quad \text{work by a spring force}$$

The Work Done by an Applied Force

- displace the block along the x-axis by continuing to apply a force on it. During the displacement, the applied force does work W_a on the block while the spring force does work W_s, then

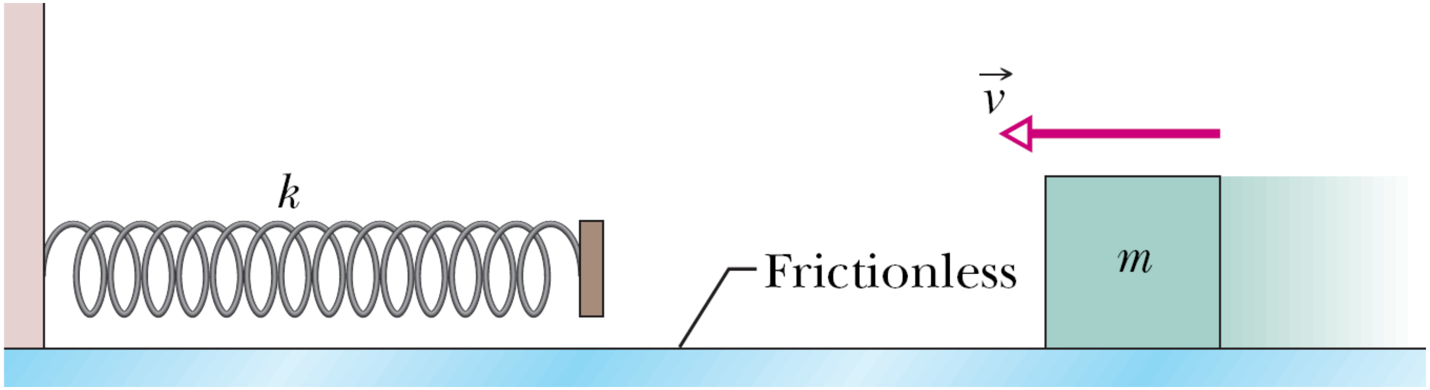
$$\Delta K = K_f - K_i = W_a + W_s$$

If the block is stationary before and after the displacement, then $W_a = -W_s$

> If a block that is attached to a spring is stationary before and after a displacement, then the work done on it by the applied force displacing it is the negative of the work done on it by the spring force.

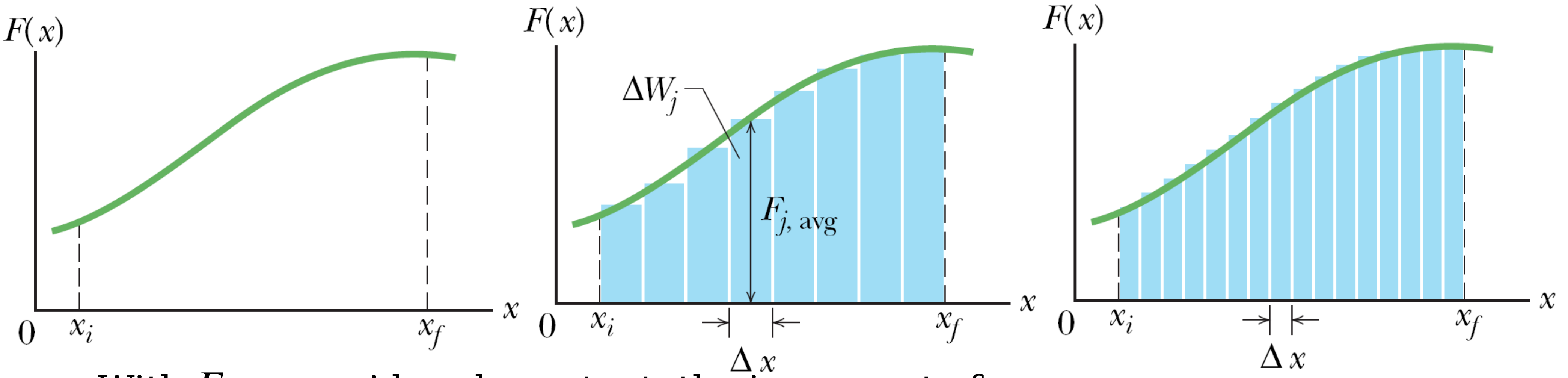
Caution: If the block is not stationary before and after the displacement, then the statement is *not* true.

problem 7-6



Work Done by a General Variable Force

One-Dimensional Analysis



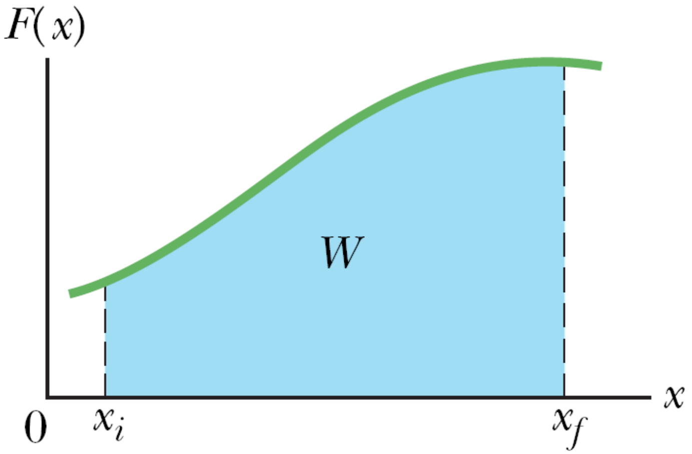
- With $F_{j,\text{avg}}$ considered constant, the increment of work ΔW_j done by the force in the j^{th} interval is approximately given by $\Delta W_j = F_{j,\text{avg}}\,\Delta x$

and the total work W is approximately

$$W = \sum \Delta W_j = \sum F_{j,\text{avg}}\,\Delta x$$

In the limit, the strip width approaches 0,

$$W = \lim_{\Delta x \to 0} \sum F_{j,\text{avg}}\,\Delta x$$



The limit is exactly what we mean by the integral of the function $F(x)$ between the limits x_i and x_f,

$$\boxed{W = \int_{x_i}^{x_f} F(x)\,dx \qquad \text{work: variable force}}$$

Three-Dimensional Analysis

- A 3-dimensional force $\vec{F} = F_x \hat{\mathrm{i}} + F_y \hat{\mathrm{j}} + F_z \hat{\mathrm{k}}$

in which the component F_x, F_y and F_z can depend on the position of the particle.

- Here we assume $F_x = F_x(x)$, $F_y = F_y(y)$, $F_z = F_z(z)$
- Let the particle move through an incremental displacement

$$\mathrm{d}\vec{r} = \mathrm{d}x\,\hat{\mathrm{i}} + \mathrm{d}y\,\hat{\mathrm{j}} + \mathrm{d}z\,\hat{\mathrm{k}}$$

- The increment of work $\mathrm{d}W$ done on the particle by the force during the displacement is $\mathrm{d}W = \vec{F}\cdot\mathrm{d}\vec{r} = F_x\,\mathrm{d}x + F_y\,\mathrm{d}y + F_z\,\mathrm{d}z$
- The work W done by the force while the particle moves from an initial position r_i having coordinates (x_i, y_i, z_i) to a final position r_f having coordinates (x_f, y_f, z_f) is

$$\boxed{W = \int_{x_i}^{x_f} F_x\,\mathrm{d}x + \int_{y_i}^{y_f} F_y\,\mathrm{d}y + \int_{z_i}^{z_f} F_z\,\mathrm{d}z}$$

Work-Kinetic Energy Theorem with a Variable Force

- Consider a particle of mass m, moving along an x-axis and acted on by a net force $F(x)$ that is directed along the axis. The work done on the particle by this force as the particle moves from an initial position x_i to a final position x_f is

$$W = \int_{x_i}^{x_f} F(x)\,\mathrm{d}x = \int_{x_i}^{x_f} m\,a\,\mathrm{d}x$$

$$m\,a\,\mathrm{d}x = m\frac{\mathrm{d}v}{\mathrm{d}t}\,\mathrm{d}x \quad \Rightarrow \quad m\,a\,\mathrm{d}x = m\frac{\mathrm{d}v}{\mathrm{d}x}\,v\,\mathrm{d}x = mv\,\mathrm{d}v \quad \Leftarrow \quad \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}x}\,\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}x}\,v$$

$$\Rightarrow \quad W = \int_{v_i}^{v_f} m\,v\,\mathrm{d}v = m\int_{v_i}^{v_f} v\,\mathrm{d}v = \frac{1}{2}\,m\,v_f^2 - \frac{1}{2}\,m\,v_i^2$$

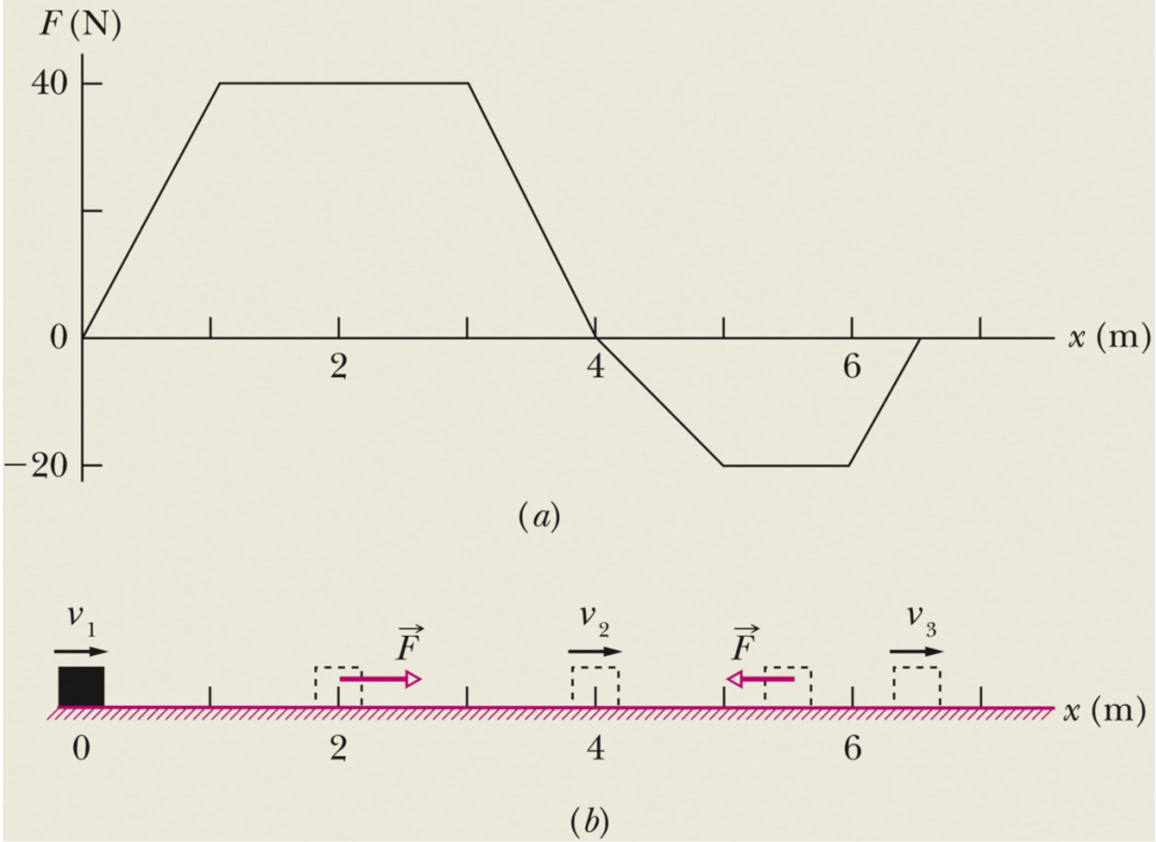
- When we change the variable from x to v we are required to express the limits on the integral in terms of the new variable.

- Recognizing the terms in the eqn allows us to write the equation as

$$W = K_f - K_i = \Delta K$$

the work-kinetic energy theorem.

Problem 7-8 problem 7-7



Power

- The time rate at which work is done by a force is said to be the **power** due to the force.

- The **average power** due to a force during an amount of time Δ*t* is

$$\boxed{P_{\text{avg}} = \frac{\Delta W}{\Delta t} \qquad \text{average power}}$$

- The **instantaneous power** P is the instantaneous time rate of doing work

$$\boxed{P=\frac{\mathrm{d}\,W}{\mathrm{d}\,t}\qquad\text{instananeous power}}$$

- The SI unit of power is **watt** (W), the joule per second,

$$1\ \text{watt}=1\ \text{W}=1\ \text{J/s}=0.738\ \text{ft}\cdot\text{lb/s}$$

$$1\ \text{horsepower}=1\ \text{hp}=550\ \text{ft}\cdot\text{lb/s}=746\ \text{W}$$

- Work can be expressed as power multiplied by time, as in the common unit kilowatt-hour;

$$\begin{aligned}1\ \text{kilowatt-hour} &= 1\ \text{kW}\cdot\text{h}=\left(10^{3}\ \text{W}\right)\left(3600\ \text{s}\right)\\ &=3.60\times10^{6}\ \text{J}=3.60\ \text{MJ}\end{aligned}$$

- We can express the rate at which a force does work on a particle in terms of that force and the particle's velocity.

- For a particle that is moving along a straight line and is acted on by a constant force directed at some angle ϕ to the line,

$$P=\frac{\mathrm{d}\,W}{\mathrm{d}\,t}=\frac{F\cos\phi\,\mathrm{d}\,x}{\mathrm{d}\,t}=F\cos\phi\frac{\mathrm{d}\,x}{\mathrm{d}\,t}=F\,v\cos\phi\quad\Rightarrow\quad\boxed{P=\vec{F}\cdot\vec{v}\quad\text{instantaneous power}}$$

problem 7-9

The chosen problem:
14, 25, 42.