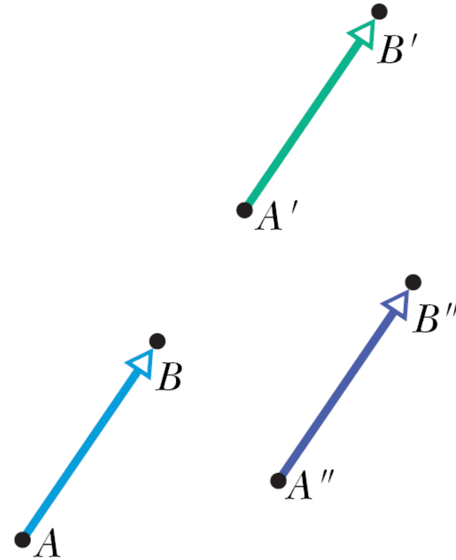
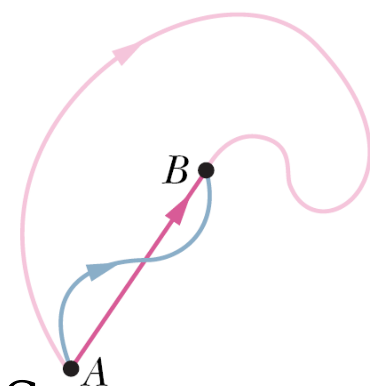
Chapter 3 Vectors

Vectors and Scalars



(a)

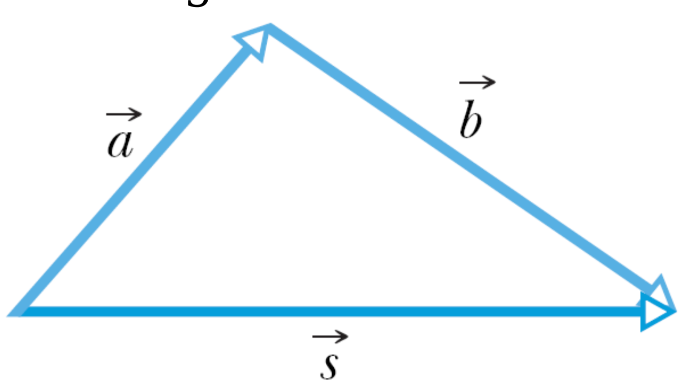
- A **vector** has magnitude as well as direction.
- A **vector quantity** is a quantity that has both a magnitude and a direction and thus can be represented with a vector, eg, displacement, velocity, acceleration.
- A **scalar** has magnitude but no direction, eg, temperature, energy, mass, time.
- **Displacement vectors** AB, A'B', A"B" represent the same *change of position*.
- The displacement vector tell us nothing about the actual path that the particle take, but the overall effect of the motion.

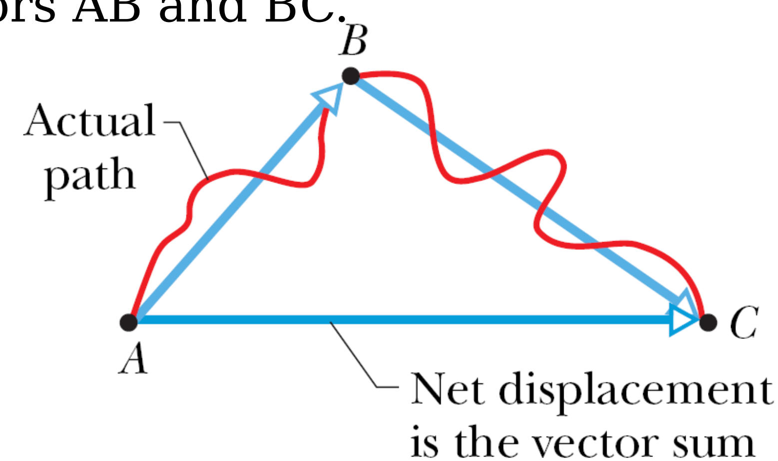


Adding Vectors Geometrically

- AC is the **vector sum** (or **resultant**) of the vectors AB and BC. This sum is not the usual algebraic sum.
- Vector equation

$$\vec{s}=\vec{a}+\vec{b}$$





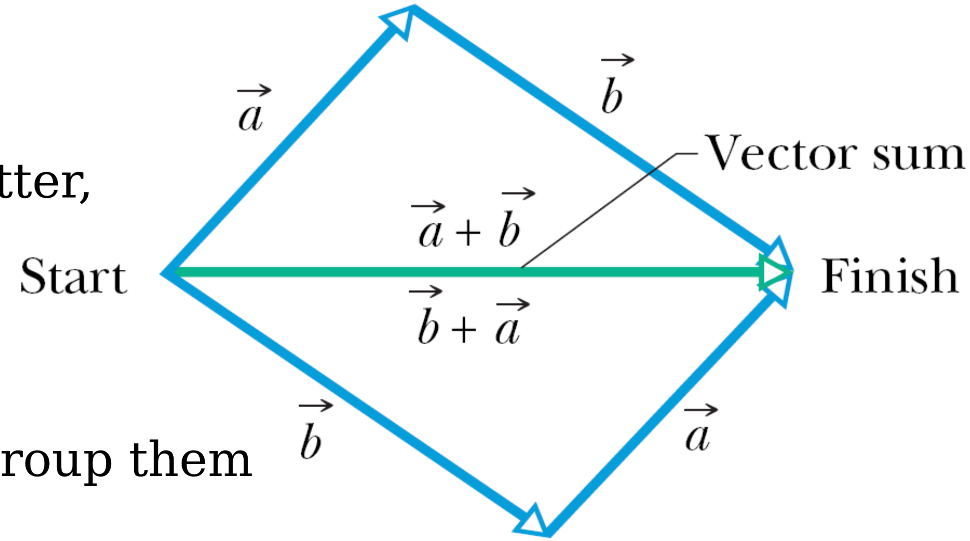
- The symbol **+** has different meanings for vectors than it does in the usual algebra because it involves both magnitude *and* direction.

- 2 important properties:

  (1) the order of addition does not matter,

$$\vec{a}+\vec{b}=\vec{b}+\vec{a}$$

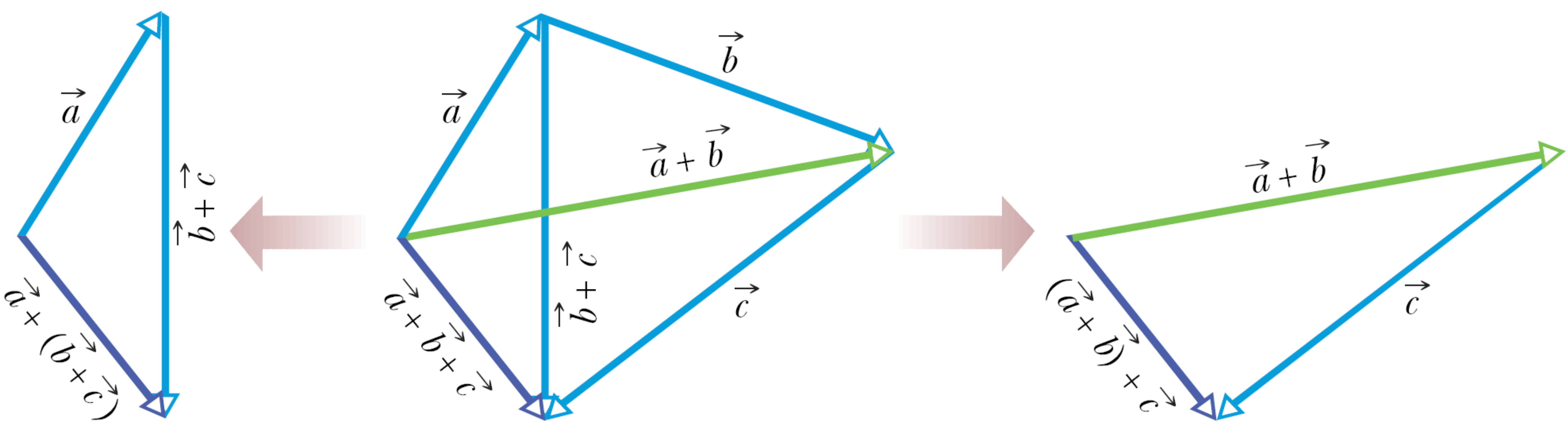
(commutative law)



  (2) for more than 2 vectors, we can group them in any order as we add them,

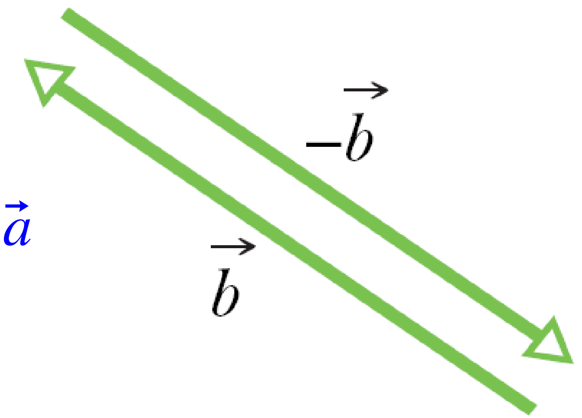
$$(\vec{a}+\vec{b})+\vec{c}=\vec{a}+(\vec{b}+\vec{c})$$

(associative law)



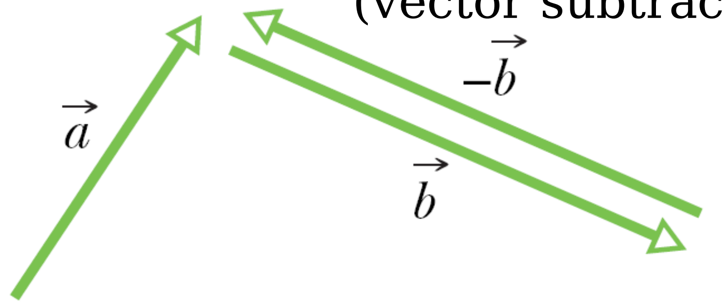
- The vector $-\vec{b}$ is a vector with the same magnitude as $\vec{b}$ but the opposite direction.

- adding $-\vec{b}$ has the effect of subtracting $\vec{b}$.

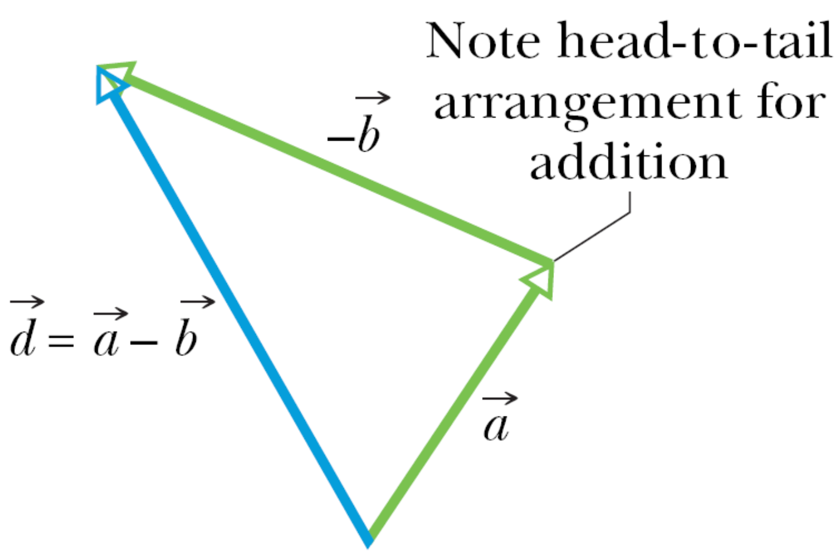


$$\boxed{\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b})} \quad \Rightarrow \quad \vec{d} + \vec{b} = \vec{a}$$

(vector subtraction)

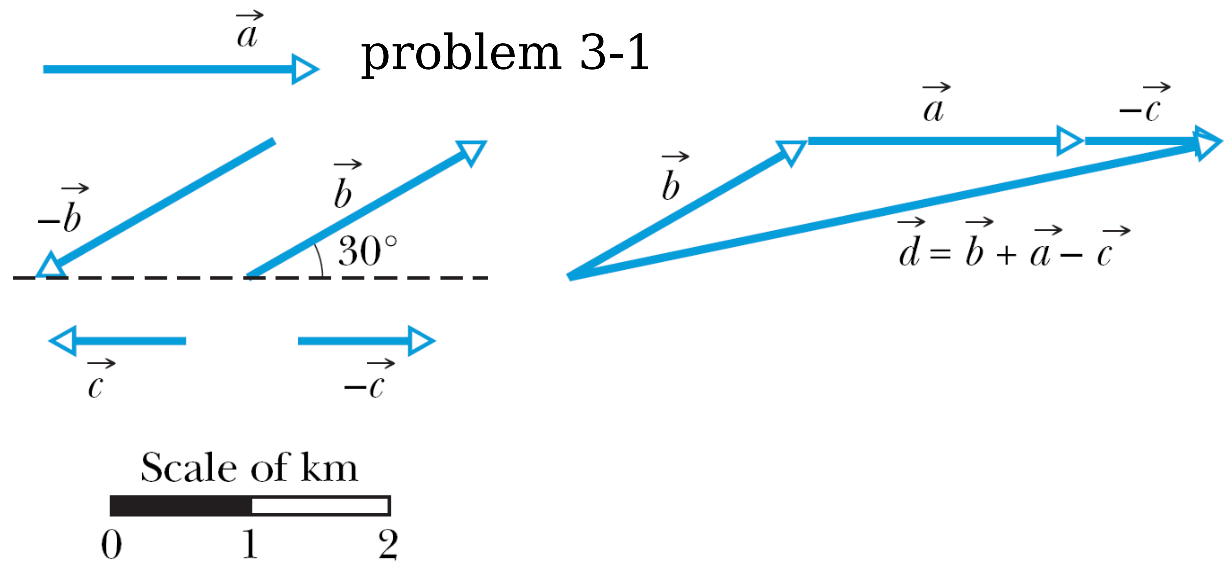


(a)



(b)

problem 3-1

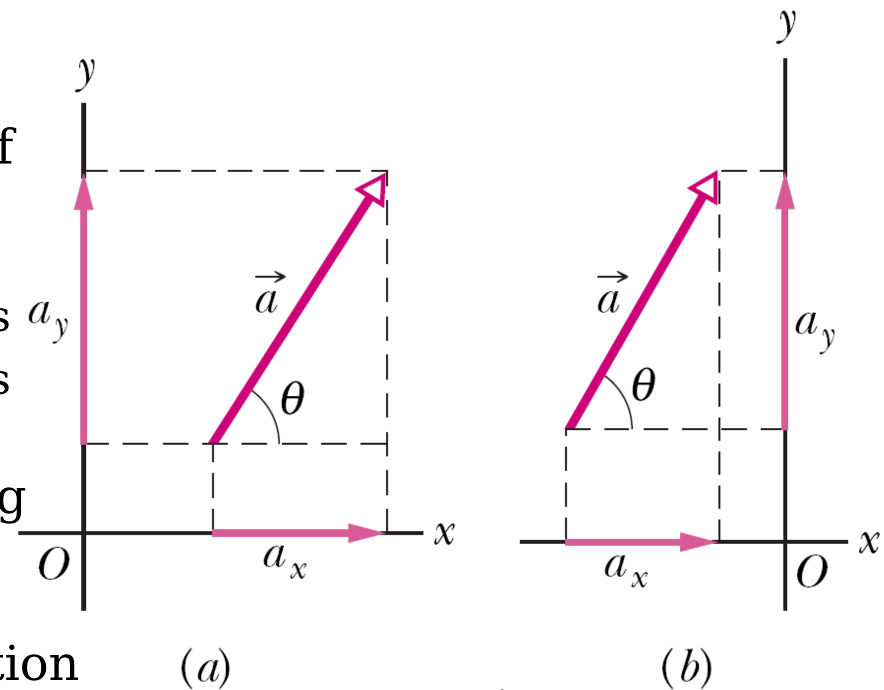


(a) (b)

Components of Vectors

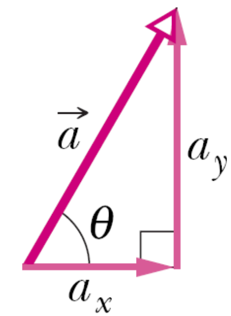
- A **component** of a vector is the projection of the vector on an axis.

x component: the projection of a vector on an x axis
y component: the projection of a vector on an y axis



- **Resolving the vector**: the process of finding the components of a vector

- A component of a vector has the same direction (along an axis) as the vector.

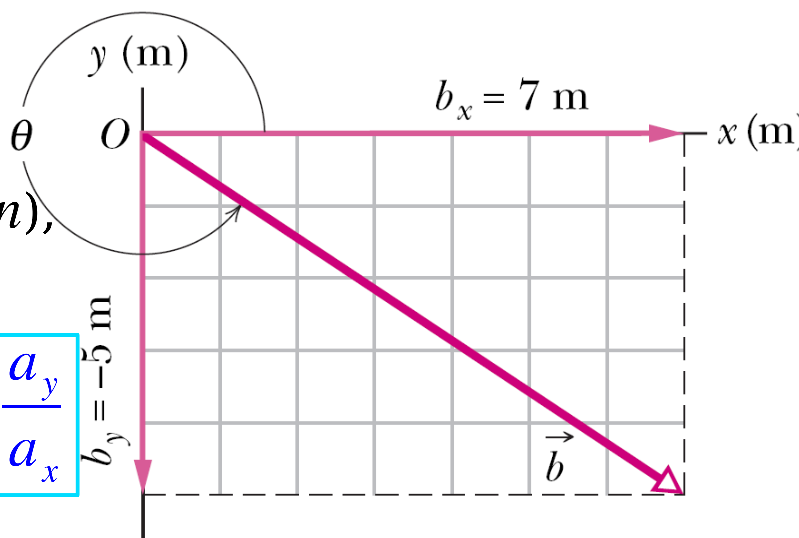


$$a_x = a\cos\theta\ ,\ a_y = a\sin\theta$$

θ is the angle that the vector $\vec{a}$ makes with the positive direction of the *x* axis, $a = |\vec{a}|$.

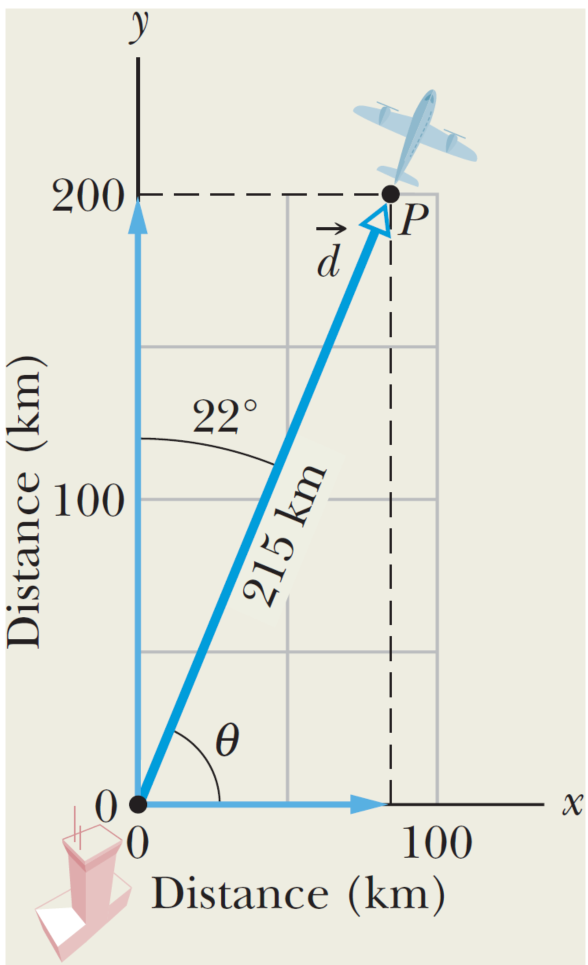
- Reconstruct a vector from its component.

- $\vec{a}$ is determined by a_x and a_y (*component notation*), or by a and θ (*magnitude- angle notation*).

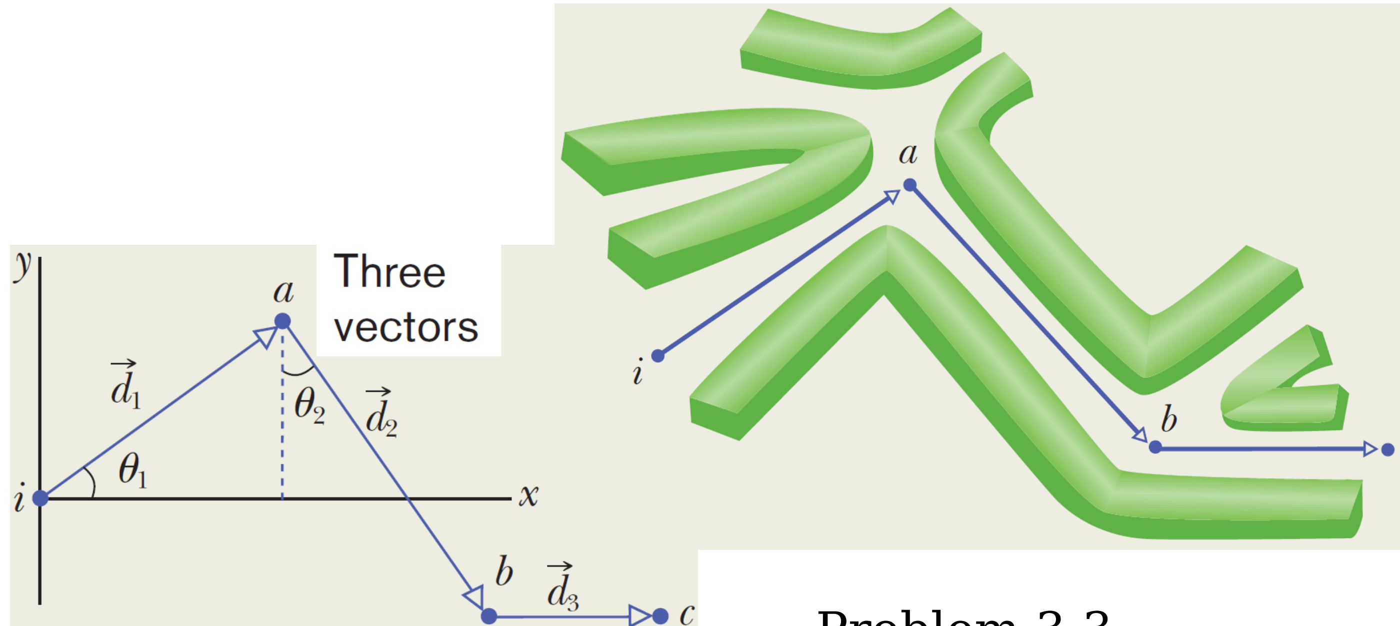


- For transformation $$a = \sqrt{a_x^2 + a_y^2}\ ,\quad \text{and}\quad \tan\theta = \frac{a_y}{a_x}$$

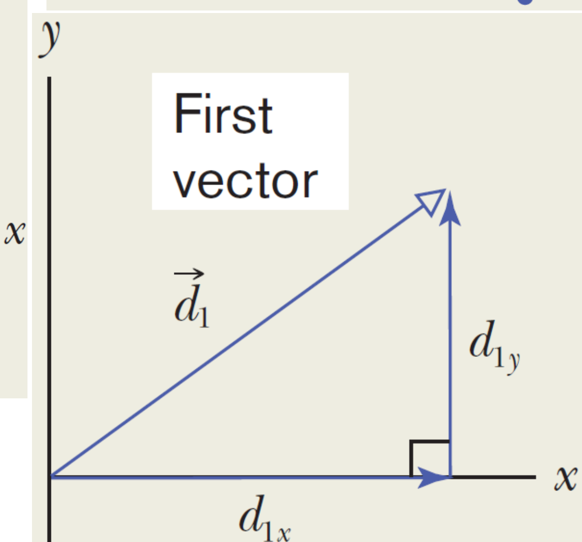
- For 3D, we need a magnitude and 2 angles (a, θ, ϕ) or 3 components (a_x, a_y, a_z) to specify a vector.

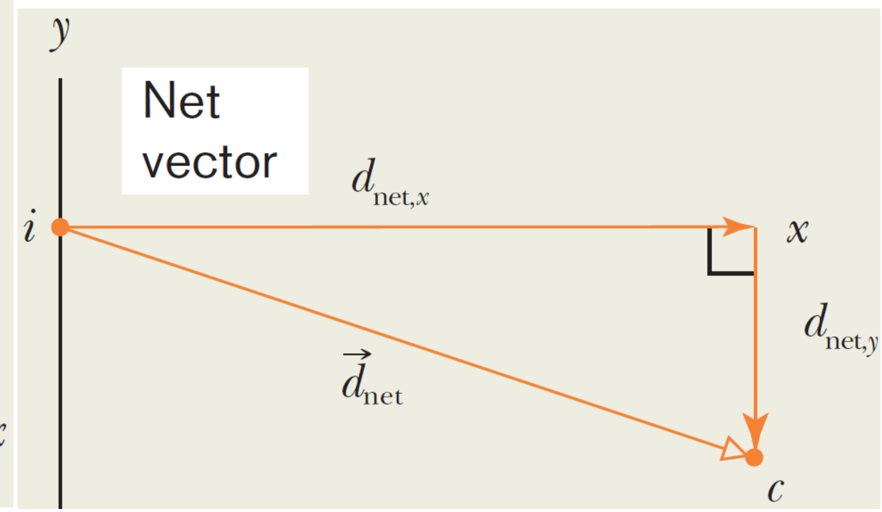


Problem 3-2



Problem 3-3





Angles — Degrees and Radians

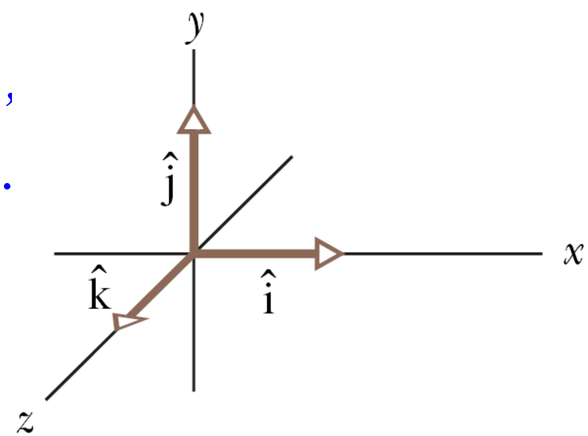
$$360^\circ\,(\text{degree}) = 2\pi\,(\text{radian})$$

Ex: $$40^\circ = 40^\circ \times \frac{2\pi}{360^\circ} = 0.70\,(\text{rad})$$

Unit Vectors

- A **unit vector** is a vector that has a magnitude of exactly 1 and points in a particular direction.

- use **right-handed coordinate system**

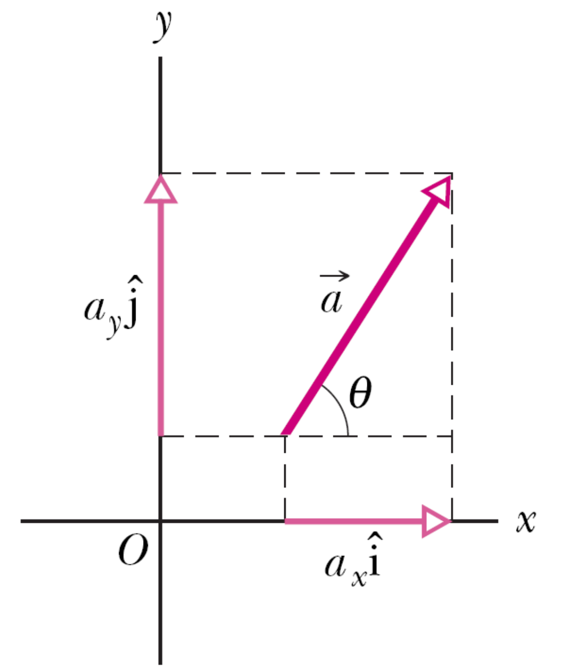


Example of usage: $\vec{a} = a_x\,\hat{\mathrm{i}} + a_y\,\hat{\mathrm{j}}$, $\vec{b} = b_x\,\hat{\mathrm{i}} + b_y\,\hat{\mathrm{j}}$.

vector components: $a_x\,\hat{\mathrm{i}}$, $a_y\,\hat{\mathrm{j}}$

scalar components: a_x, a_y.

Example: $\vec{d} = -(2.6\text{ km})\,\hat{\mathrm{i}} + (0.025\text{ km})\,\hat{\mathrm{j}} + (3.9\text{ km})\,\hat{\mathrm{k}}$



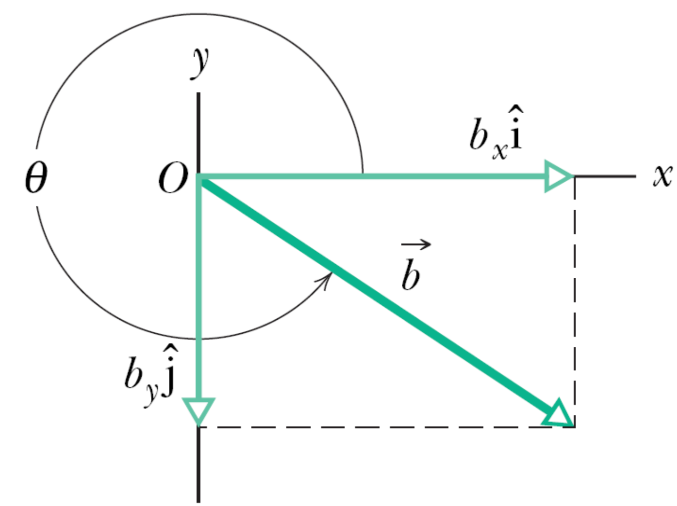
(a)

Adding Vectors by Components

- Decompose $\vec{r} = \vec{a} + \vec{b}$

  into

$$r_x = a_x + b_x$$
$$r_y = a_y + b_y$$
$$r_z = a_z + b_z$$



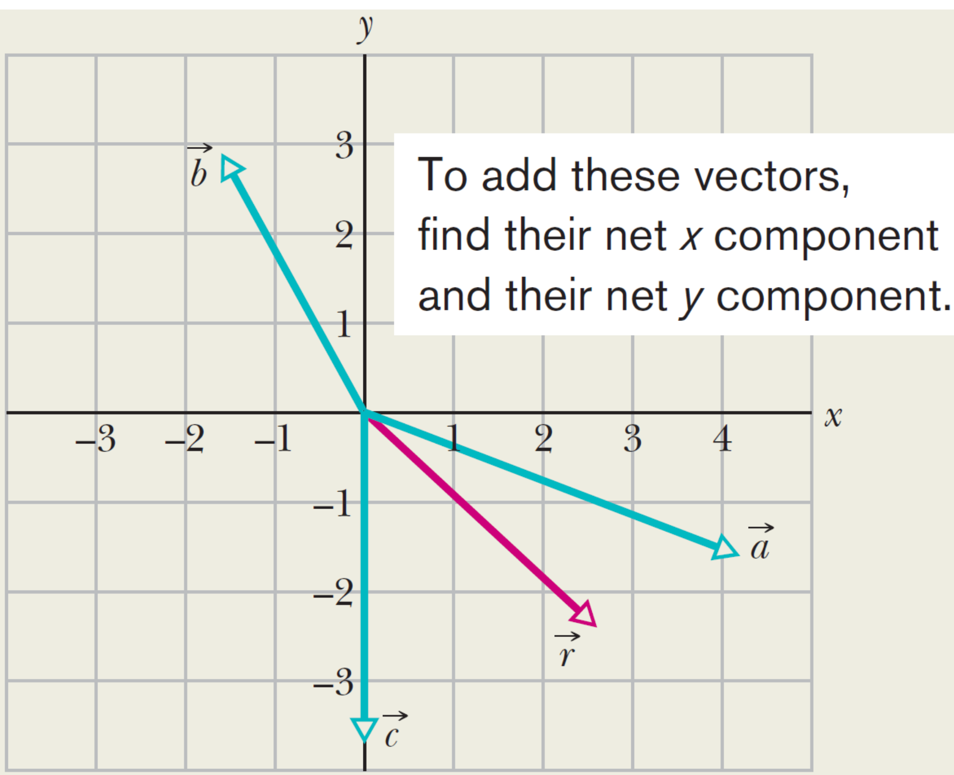
(b)

- 2 vectors must be equal if their corresponding components are equal.

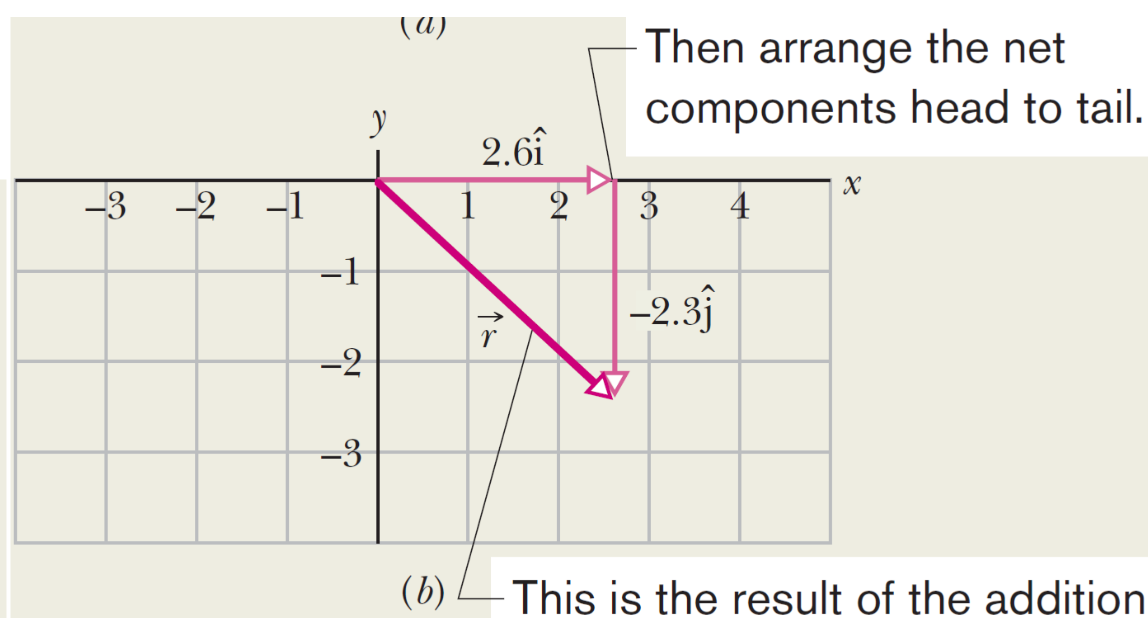
- add 2 vectors, we must
  (1) resolve the vectors into their scalar components;
  (2) combine these scalar components, axis by axis, to get the components of the sum;
  (3) combine the components to get the sum vector.

- This procedure also applies to vector subtractions:

$$\vec{d} = \vec{a} - \vec{b} = \vec{a} + (-\vec{b}) = d_x\,\hat{\mathrm{i}} + d_y\,\hat{\mathrm{j}} + d_z\,\hat{\mathrm{k}} \quad \Rightarrow \quad \begin{aligned} d_x &= a_x - b_x \\ d_y &= a_y - b_y \\ d_z &= a_z - b_z \end{aligned}$$



problem 3-4



Vectors and the Laws of Physics

- an infinite number of different pairs of components of a vector

- Which is the right pair of components?

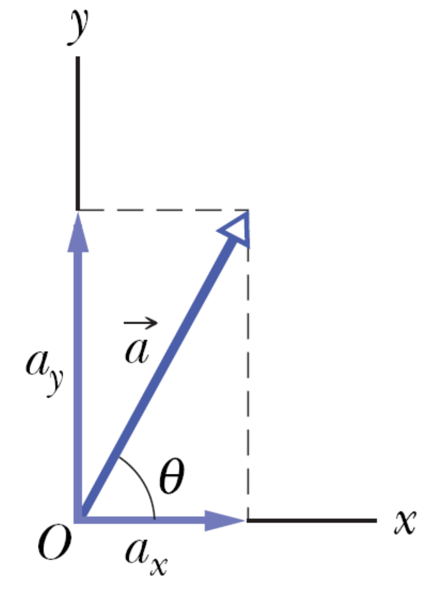
  Ans: they are all equally valid because each pair gives a different way of describing the same vector.

- the relation: $a = \sqrt{a_x^2 + a_y^2} = \sqrt{a_x'^2 + a_y'^2}$ and $\theta = \theta' + \phi$

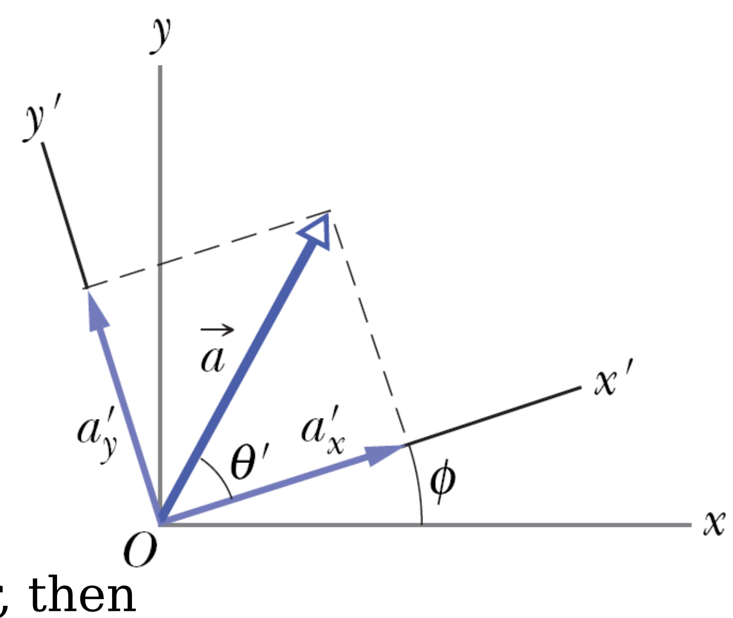
- freedom in choosing a coordinate system

- also true of the relation of physics: independent of the choice of coordinate system

- use the language of vector to present the laws of physics for its simplicity and richness.



(a)



(b)

Multiplying Vectors

- 3 ways in which vectors can be multiplied:

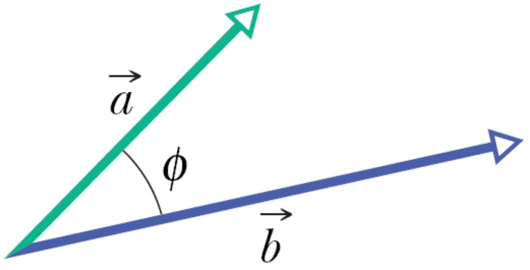
  (1) **Multiplying a Vector by a Scalar**: *s* is a scalar, then

$$\vec{b} = (s)(\vec{a}) = s\,\vec{a}$$
$$\vec{c} = (1/s)(\vec{a}) = \vec{a}/s$$

(2) **Multiplying a Vector by a Vector**:

(2a) **Scalar Product**: its result is a scalar;

(2b) **Vector Product**: its result is a vector.



(a)

The Scalar Product (also **Dot Product**)

- The **scalar product** of 2 vectors is defined as

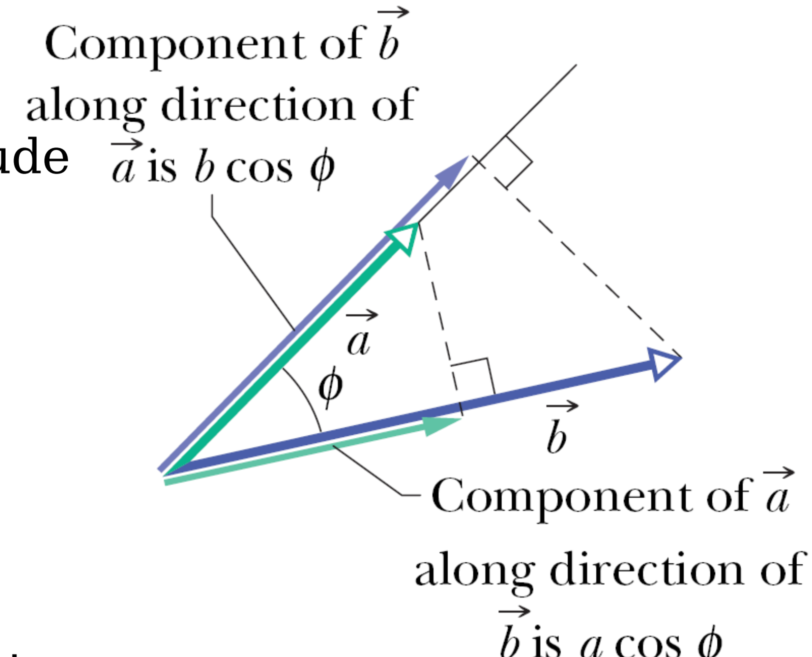
$$\vec{a}\cdot\vec{b} = a\, b \cos\phi$$

where a is the magnitude of $\vec{a}$, b is the magnitude of $\vec{b}$, and ϕ is the angle between $\vec{a}$ and $\vec{b}$.

- The angle ϕ and 360° - ϕ is the same for the scalar product.

- $\phi = 0$ gives the scalar product its maximum; $\phi = 90°$ makes the scalar product be 0.

- A scalar product can be regarded as the product of 2 quantities:
  (1) the magnitude of one of the vectors, and
  (2) the scalar component of the 2nd vector along the direction of the 1st vector.



(b)

- The commutative law applies to a scalar product:

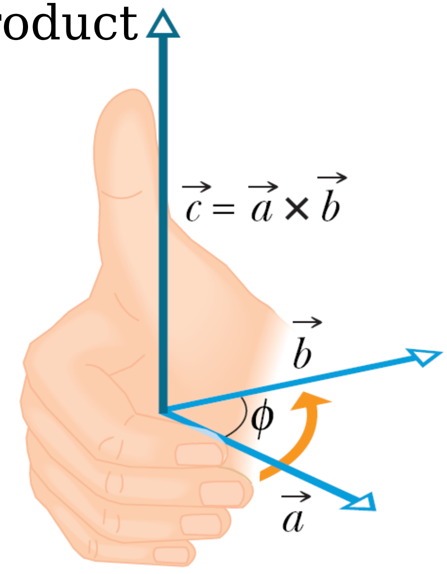
$$\vec{a}\cdot\vec{b}=(a\cos\phi)(b)=(a)(b\cos\phi) \quad\Rightarrow\quad \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$$

- Express the vectors into their components for the scalar product

$$\vec{a}\cdot\vec{b}=(a_x\,\hat{i}+a_y\,\hat{j}+a_z\,\hat{k})\cdot(b_x\,\hat{i}+b_y\,\hat{j}+b_z\,\hat{k})$$

use the distributive law to obtain the formula

$$\vec{a}\cdot\vec{b}=a_x b_x+a_y b_y+a_z b_z$$



(a)

The Vector Product (also Cross Product)

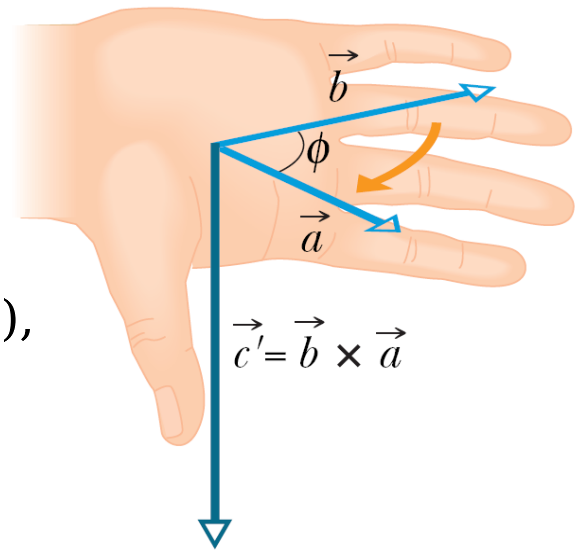
- The **vector product** of 2 vectors is defined as

$$\vec{c}=\vec{a}\times\vec{b} \quad \text{and} \quad c=a\,b\sin\phi$$

where ϕ is the *smaller* angle between $\vec{a}$ and $\vec{b}$.

- The direction of $\vec{c}$ is perpendicular to the plane that contains $\vec{a}$ and $\vec{b}$ with a **right-hand rule**.

- If 2 vectors are parallel (ϕ = 0) or antiparallel (ϕ =180°), their cross product vanishes;
if this 2 vectors are perpendicular to each other, the magnitude of their cross product is maximum.



(b)

- The order of the vector multiplication is important:

$$\vec{a}\times\vec{b} = -(\vec{b}\times\vec{a})$$

the commutative law does **not** apply to a vector product.

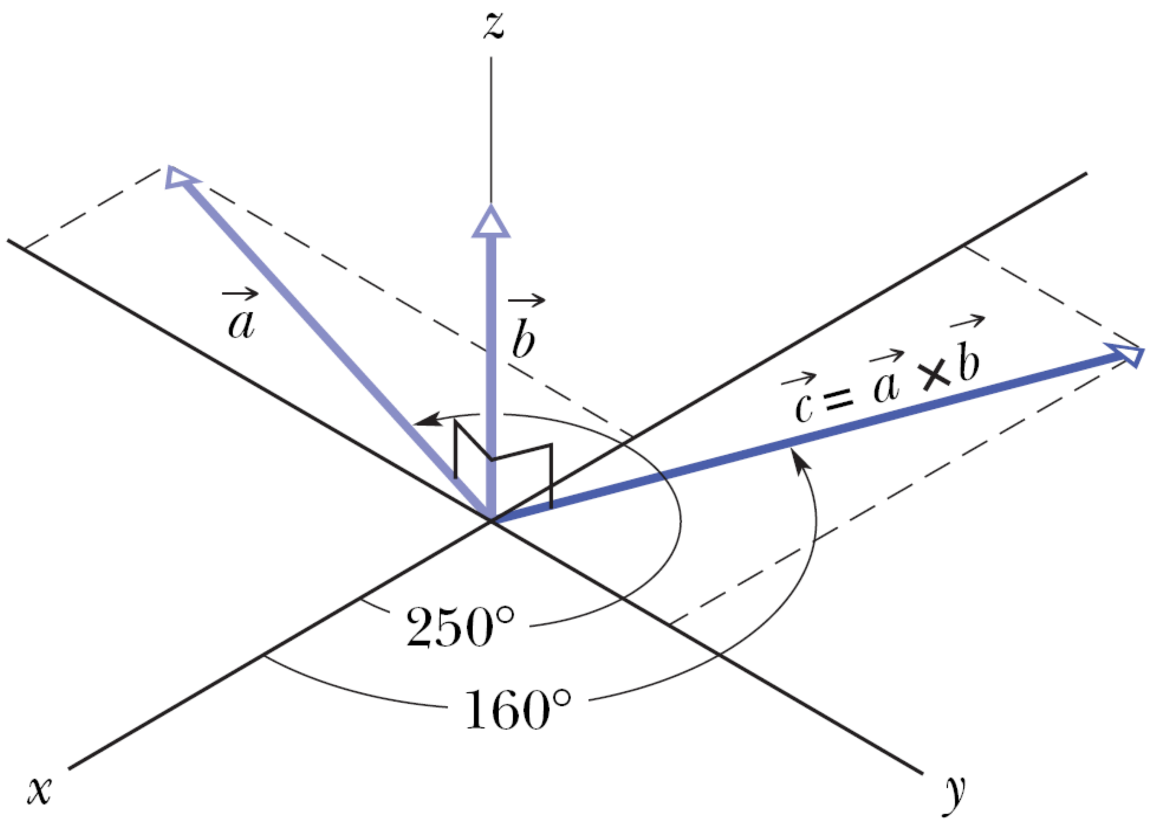
- In unit-vector notation $\vec{a}\times\vec{b} = (a_x\hat{\mathrm{i}} + a_y\hat{\mathrm{j}} + a_z\hat{\mathrm{k}})\times(b_x\hat{\mathrm{i}} + b_y\hat{\mathrm{j}} + b_z\hat{\mathrm{k}})$

Since $a_x\hat{\mathrm{i}}\times b_x\hat{\mathrm{i}} = a_x b_x(\hat{\mathrm{i}}\times\hat{\mathrm{i}}) = 0$ and $a_x\hat{\mathrm{i}}\times b_y\hat{\mathrm{j}} = a_x b_y(\hat{\mathrm{i}}\times\hat{\mathrm{j}}) = a_x b_y\hat{\mathrm{k}}$

it gives

$$\vec{a}\times\vec{b} = (a_y b_z - a_z b_y)\hat{\mathrm{i}} + (a_z b_x - a_x b_z)\hat{\mathrm{j}} + (a_x b_y - a_y b_x)\hat{\mathrm{k}}$$

$$= \begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$



Problem 3.5

Problem 3.6

Problem 3.7

The chosen problems: 26, 32, 43.