Chapter 2 Motion along a straight line

Motion

- Moves along a single axis – *one-dimensional motion*.

- In this chapter: 1. the motion is along a straight line only;

  2. we discuss only the motion itself and changes in the motion;

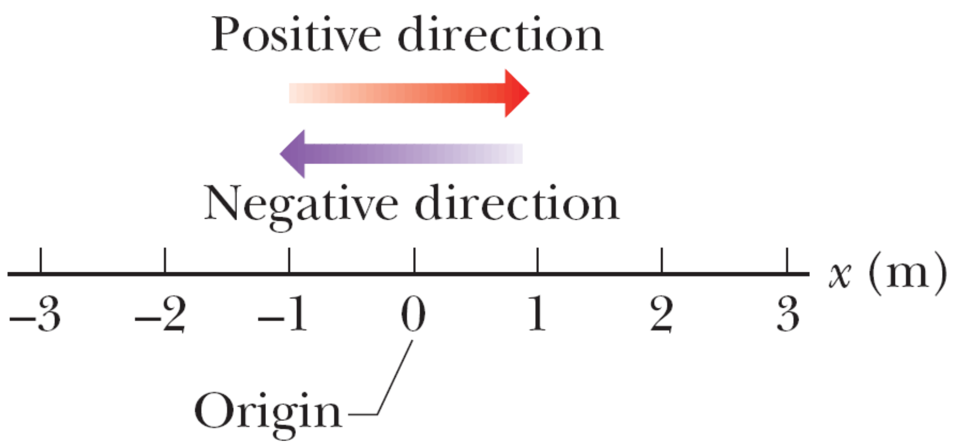
  3. the moving object is either a particle or an object that moves like a particle.

Position and Displacement

- **Origin**: some reference point of a axis

- **Positive direction**: the direction of increasing number

- **Negative direction**: the direction of decreasing number



- **Displacement** Δx: a change from one position x_1 to another position x_2,

$$\Delta x = x_2 - x_1$$

Ex:

$$\Delta x = (12\text{ m}) - (5\text{ m}) = 7\text{ m}$$
$$\Delta x = (1\text{ m}) - (5\text{ m}) = -4\text{ m}$$
$$\Delta x = (5\text{ m}) - (5\text{ m}) = 0$$

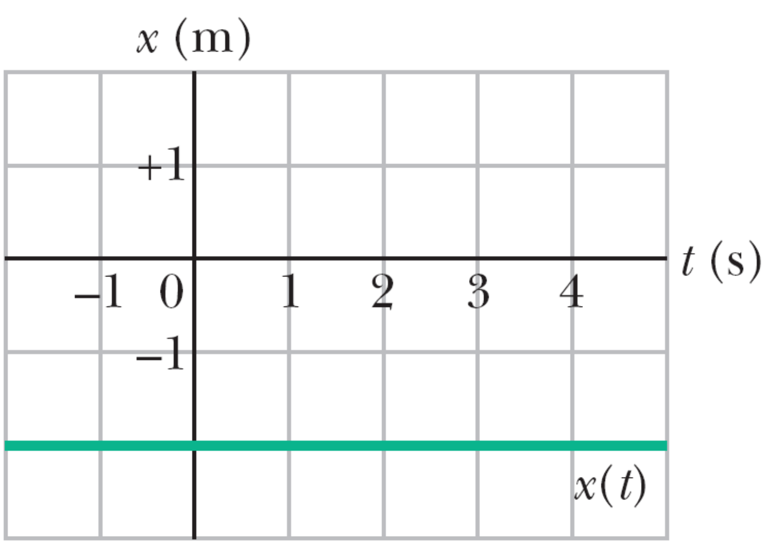
- **Magnitude** of the displacement: $|\Delta x|$

- Displacement has 2 features: (1) its *magnitude*; (2) its *direction*,
  → an example of a **vector quantity** (see Chapter 3)

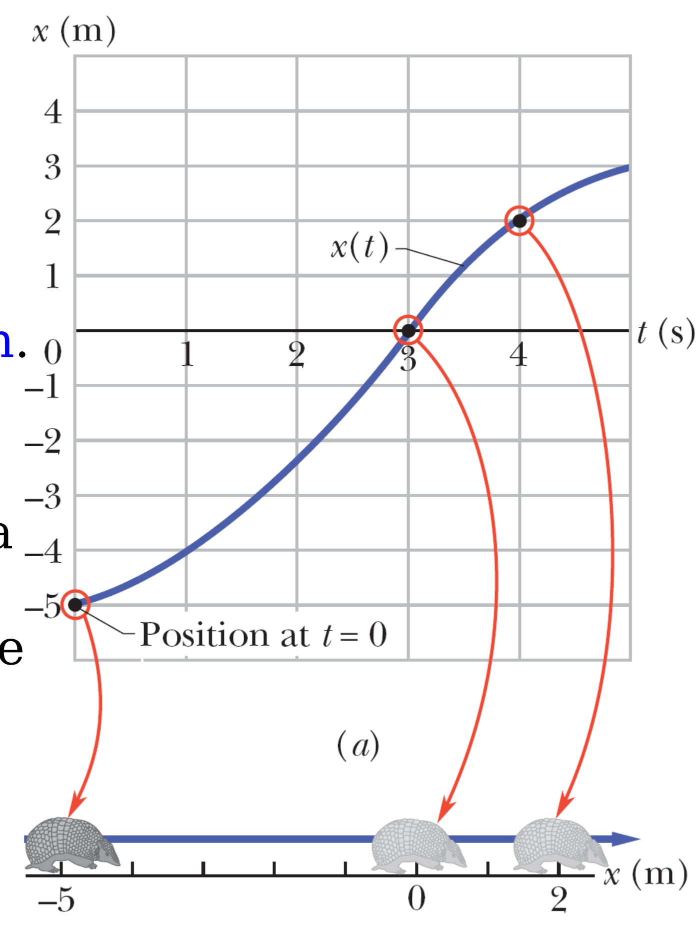
Average Velocity and Average Speed

- A compact way to describe position is with a graph of position x plotted as a function of time t — a graph of $x(t)$.

- The graph of $x(t)$ for a stationary particle at x = -2m.



- The graph of $x(t)$ for a moving animal and the path associated with the graph.



- **Average velocity**: the ratio of the displacement Δx that occurs during a particular time interval Δt to that interval,

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

- The unit is always in the form of length/time, eg, m/s.

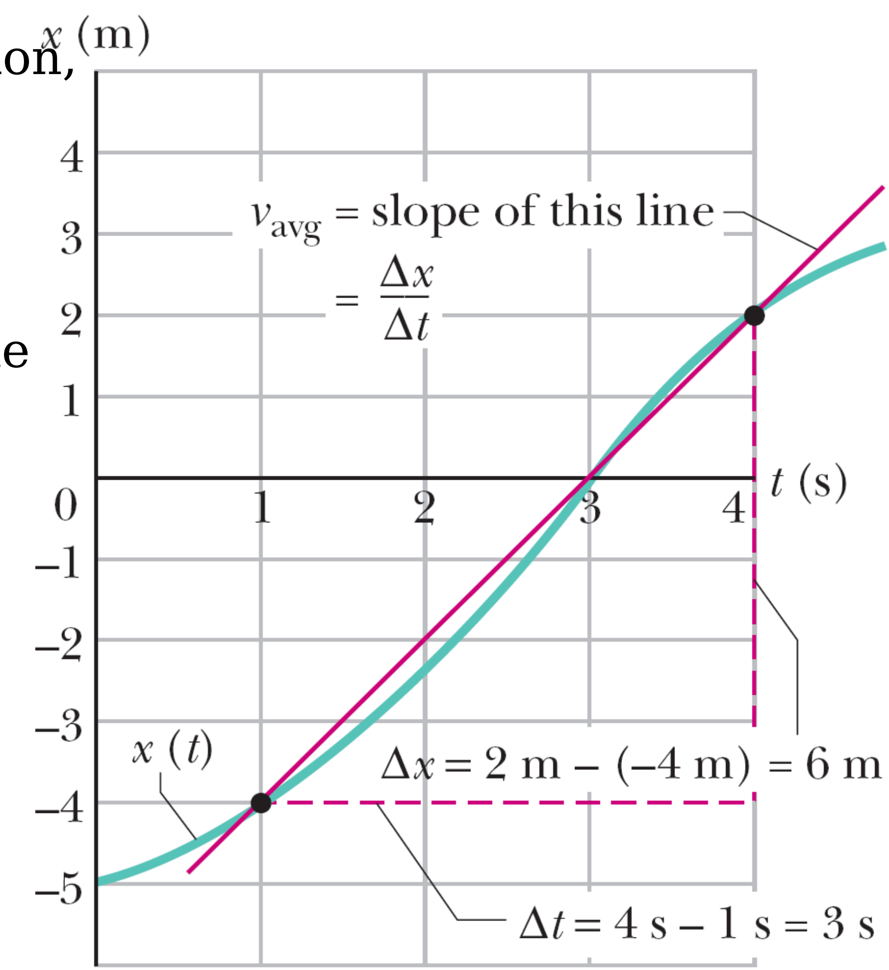
- v_{avg} is the **slope** of the straight line that connects two particular points on the $x(t)$ curve.

- v_{avg} is a vector quantity: magnitude & direction,

  positive : slants upward
  negative: slants downward

- The average velocity v_{avg} always has the same sign as the displacement Δx because Δt is always positive.

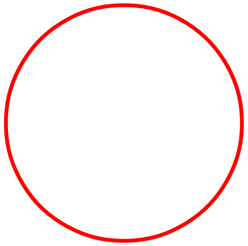
Ex: in the plot $v_{avg} = \frac{6\text{ m}}{3\text{ s}} = 2\text{ m/s}$



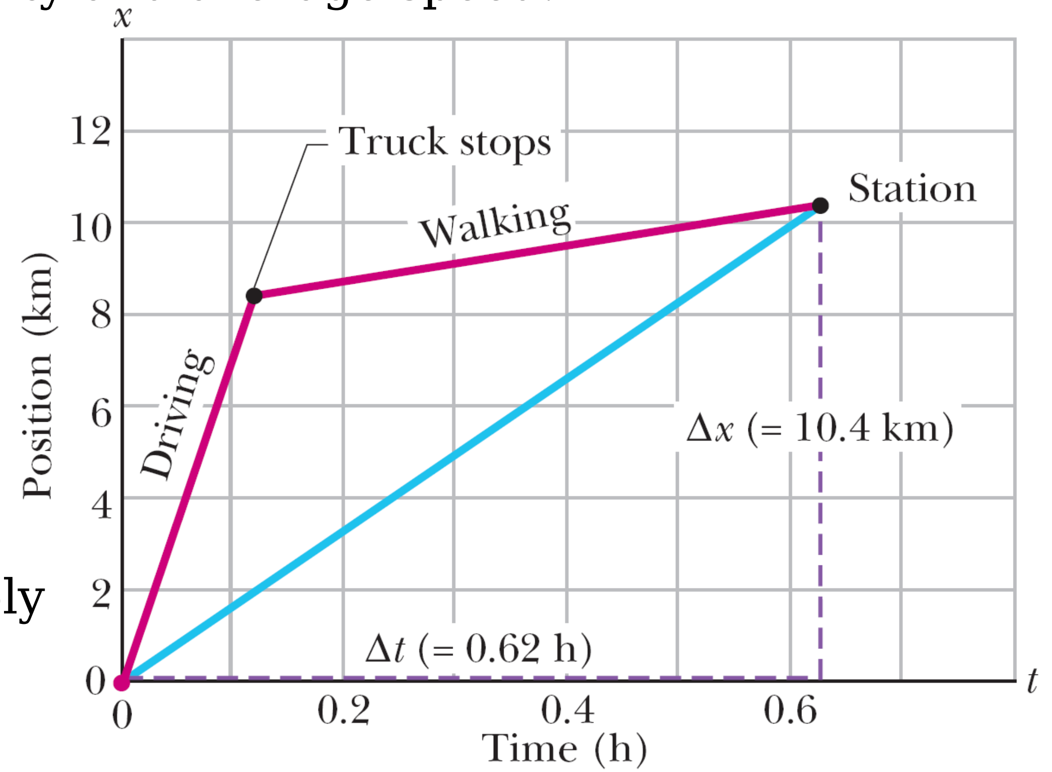
- **Average speed**: $s_{avg} = \frac{\text{total distance}}{\Delta t}$

- The difference between average velocity and average speed:

Ex: circular motion



problem 2-1



Instantaneous Velocity & Speed

- The **instantaneous velocity** (or simply **velocity**) at any instant comes from the average velocity by shrinking the time interval Δt closer and closer to 0,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- 2 features: (1) v is the derivative of x with respect to t;
  (2) v is the slope of the particle's position-time curve at the instant.

- **Speed**: the magnitude of velocity

problem 2-2

Acceleration (a vector quantity)

- **Average acceleration**:

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

- **Instantaneous acceleration** (or simply **acceleration**):

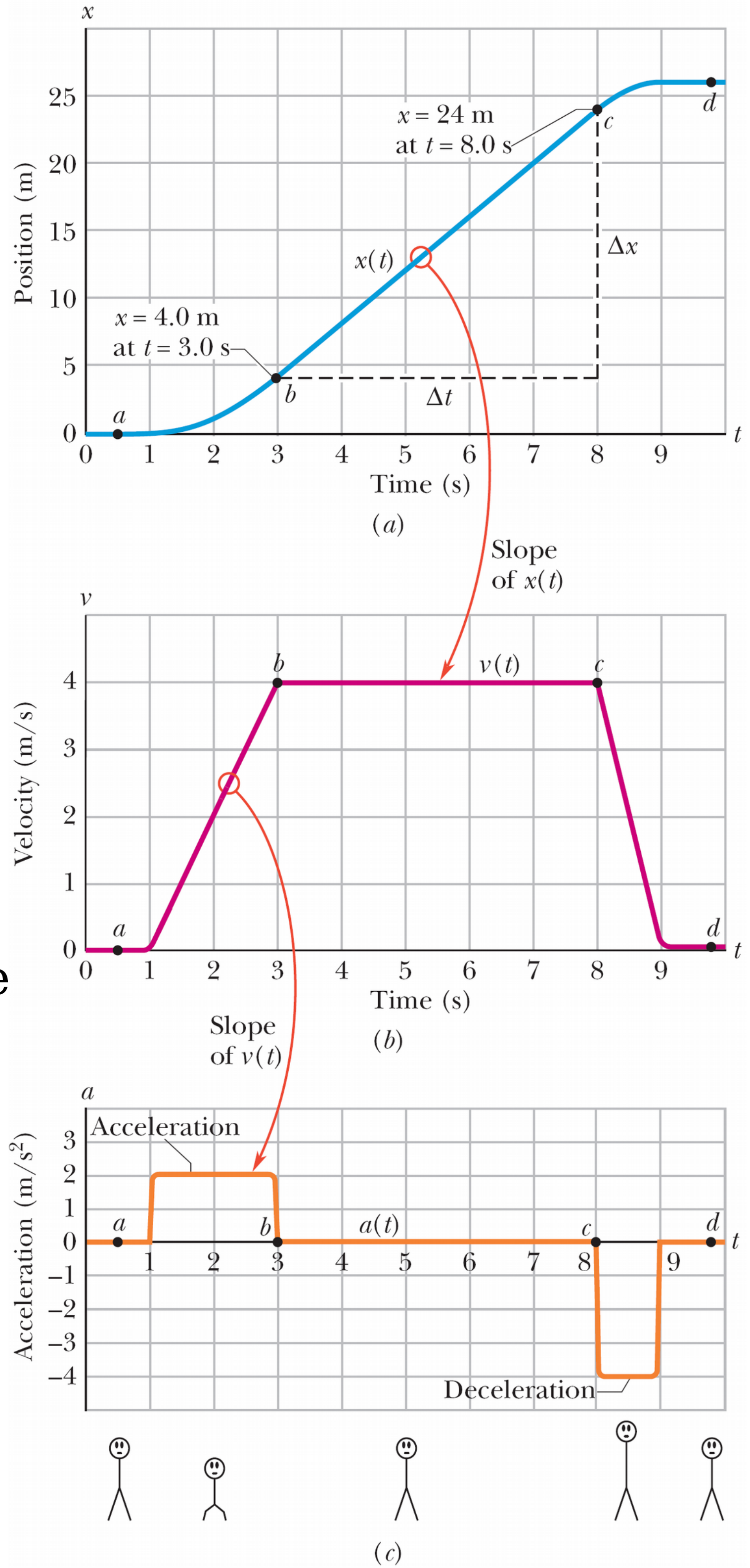
$$a = \frac{dv}{dt}$$

- The acceleration of a particle at any instant is the 2nd derivative of its position $x(t)$ with respect to time

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

- The unit of acceleration is in the form of length/time^2, eg, m/s^2.

- see figure (c).



- Human's body reacts to accelerations but not to velocities. why?

Newton's 2nd law

- Accelerations are sometimes expressed in terms of g unit, with $1\,g = 9.8\ \mathrm{m/s^2}$.

Ex: fighter jet's pilot

problem 2-3

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calculus part one

definition of a derivative: $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t} x(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{(t+\Delta t) - t}$

If $x = t^n$ then $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\mathrm{d}t^n}{\mathrm{d}t} = n\,t^{n-1}$

Constant Acceleration: A Special Case

- In this section the equation are valid only for constant acceleration.

$$a = a_{\mathrm{avg}} = \frac{v - v_0}{t - 0} \quad \Rightarrow \quad v = v_0 + a\,t \qquad (1)$$

v_0 is the velocity at time $t = 0$ and v is the velocity at any later time t, a is the acceleration.

$$\frac{\mathrm{d}\,x}{\mathrm{d}\,t} = \frac{\mathrm{d}}{\mathrm{d}\,t}\,x(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{(t+\Delta t) - t}$$

$$x = t^{n} \quad \Rightarrow \quad \frac{\mathrm{d}\,x}{\mathrm{d}\,t} = \lim_{\Delta t \to 0} \frac{(t+\Delta t)^{n} - t^{n}}{(t+\Delta t) - t} = \lim_{\Delta t \to 0} \frac{(t+\Delta t)^{n} - t^{n}}{\Delta t}$$

$$\begin{aligned}(t+\Delta t)^{n} &= \sum_{m=0}^{n} \frac{n!}{m!\,(n-m)!}(\Delta t)^{m}\,t^{n-m} \\ &= t^{n} + n\,t^{n-1}\,\Delta t + \frac{n(n-1)}{2}\,t^{n-2}(\Delta t)^{2} + \cdots + (\Delta t)^{n}\end{aligned}$$

$$\Rightarrow \quad \frac{(t+\Delta t)^{n} - t^{n}}{\Delta t} = n\,t^{n-1} + \Delta t\left[\frac{n(n-1)}{2}\,t^{n-2} + \cdots\right]$$

$$\Rightarrow \quad \frac{\mathrm{d}\,x}{\mathrm{d}\,t} = \lim_{\Delta t \to 0} \frac{(t+\Delta t)^{n} - t^{n}}{\Delta t} = n\,t^{n-1}$$

- The function is linear and the plot is a straight line.

$$v_{avg} = \frac{x - x_0}{t - 0} \quad \Rightarrow \quad x = x_0 + v_{avg}\, t$$

since $\quad v_{avg} = \frac{1}{2}(v_0 + v) = v_0 + \frac{1}{2} a\, t$

then $\quad x - x_0 = v_0 t + \frac{1}{2} a\, t^2 \qquad (2)$

- Equation (1) & (2) are the *basic equations for constant acceleration*.

- 5 quantities are involved in any problem about constant acceleration — $x{-}x_0$, v, t, a, v_0.

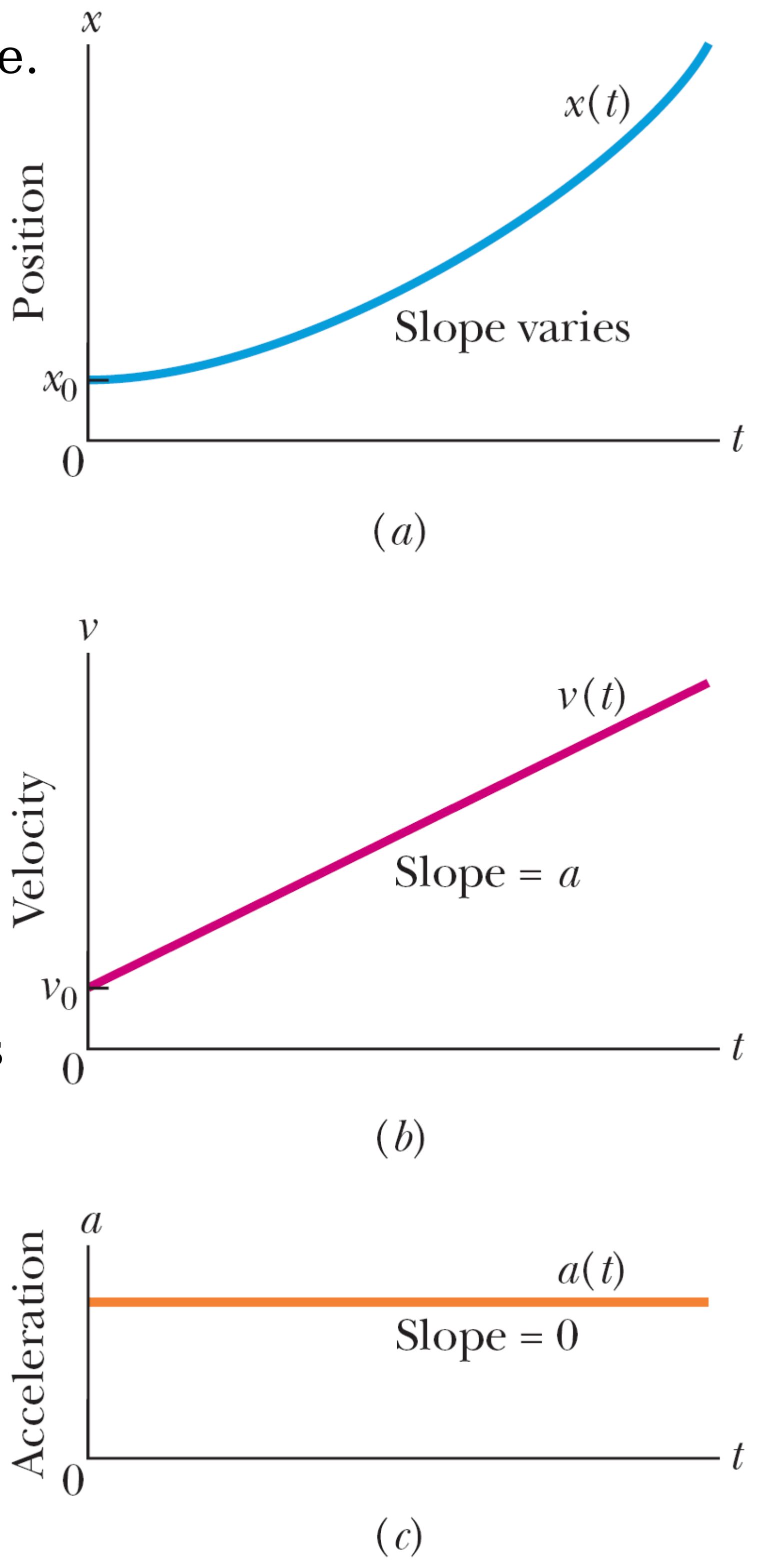
- In each equation 3 quantities are given and one is to be found.

- 3 additional equations:

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$x - x_0 = \frac{1}{2}(v_0 + v)\, t$$

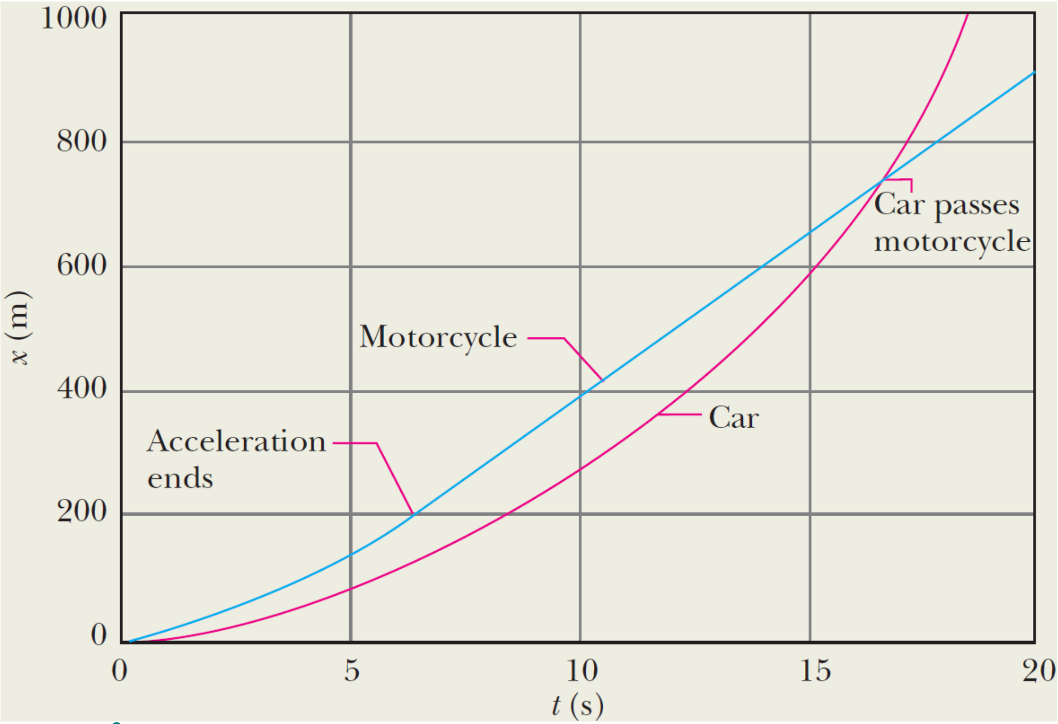
$$x - x_0 = v\, t - \frac{1}{2} a\, t^2$$



Equations for motion with constant acceleration

Equation	Missing Quantity
$v = v_0 + a\,t$	$x - x_0$
$x - x_0 = v_0\,t + \frac{1}{2}\,a\,t^2$	v
$v^2 = v_0^2 + 2\,a\,(x - x_0)$	t
$x - x_0 = \frac{1}{2}\,(v_0 + v)\,t$	a
$x - x_0 = v\,t - \frac{1}{2}\,a\,t^2$	v_0

problem 2-4



Another Look at Constant Acceleration

- Rewrite the definition of acceleration

$$\mathrm{d}\,v = a\,\mathrm{d}\,t \;\Rightarrow\; \int \mathrm{d}\,v = \int a\,\mathrm{d}\,t = a \int \mathrm{d}\,t \;\Leftarrow\; a = \mathrm{const} \;\Rightarrow\; v = a\,t + C$$

- To evaluate the constant C, $v_0 \Leftarrow v\,(t=0) = (a)\,(0) + C = C \;\Rightarrow\; v = v_0 + a\,t$

- Rewrite the definition of velocity $\mathrm{d}\,x = v\,\mathrm{d}\,t$

$$\Rightarrow\; x \Leftarrow \int \mathrm{d}\,x = \int v\,\mathrm{d}\,t = \int (v_0 + a\,t)\,\mathrm{d}\,t = v_0 \int \mathrm{d}\,t + a \int t\,\mathrm{d}\,t = v_0\,t + \frac{1}{2}\,a\,t^2 + C'$$

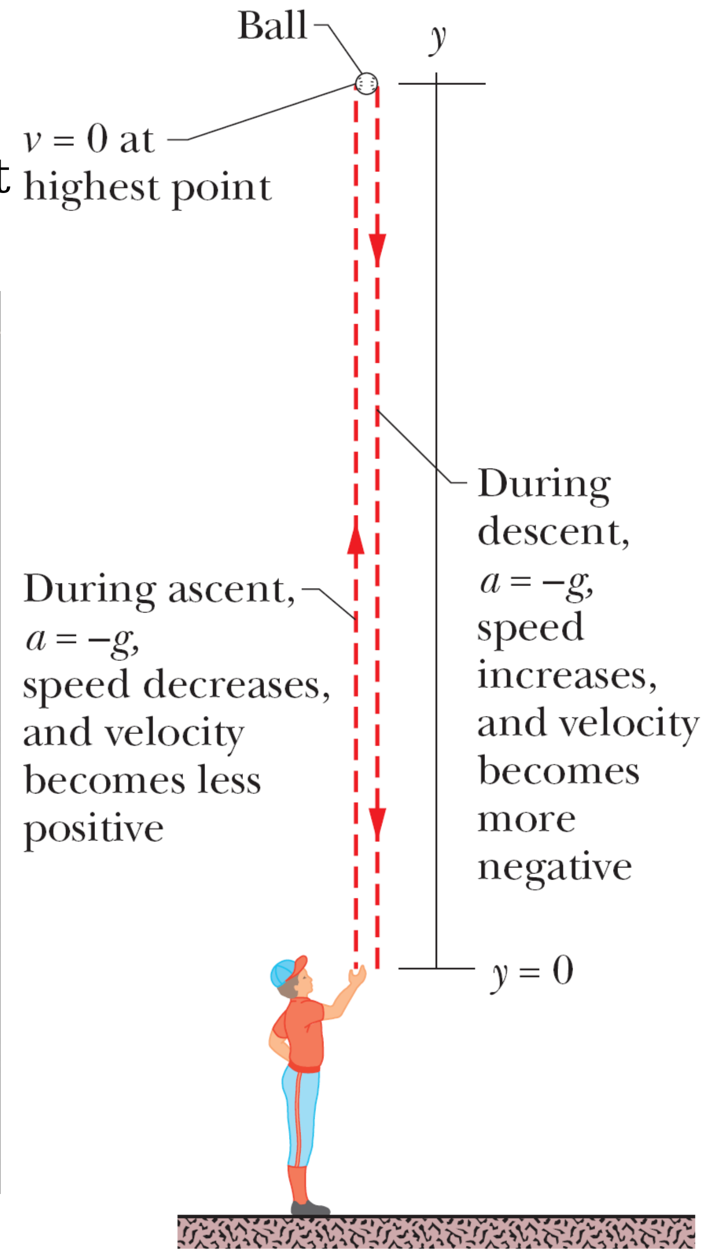
- To evaluate the constant C', $x_0 \Leftarrow x\,(t=0) = C' \;\Rightarrow\; x = x_0 + v_0\,t + \frac{1}{2}\,a\,t^2$

Free-Fall Acceleration

- Objects accelerate downward at a certain constant rate, the magnitude is presented by g.

- Independent of the object's characteristics, such as mass, density, or shape (neglecting the effects of the air); it is the same for all objects.

- At sea level in Earth's mid-latitudes the value is $9.8\ m/s^2$ (or $32\ ft/s^2$).

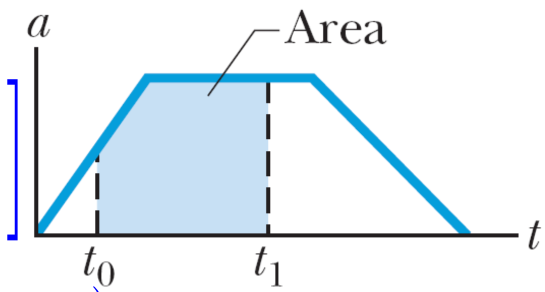


Problem 2-5

The free-fall acceleration near Earth's surface is $a = -g = -9.8\ m/s^2$, and the *magnitude* is $g = 9.8\ m/s^2$. Do not substitute $-9.8\ m/s^2$ for g.

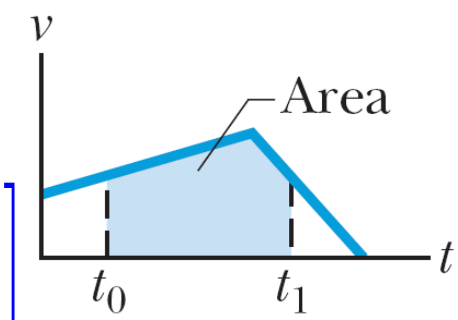
The chosen problem: 34, 43, 56.

Graphical Integration in Motion Analysis



- $a = \dfrac{dv}{dt} \Rightarrow v_1 - v_0 = \displaystyle\int_{t_0}^{t_1} a\,dt = \left[\begin{array}{l}\text{area between acc. curve} \\ \text{\& time axis, from } t_0 \text{ to } t_1\end{array}\right]$

the corresponding unit of area $(1\ \mathrm{m/s^2})(1\ \mathrm{s}) = 1\ \mathrm{m/s} \quad (\text{velocity})$



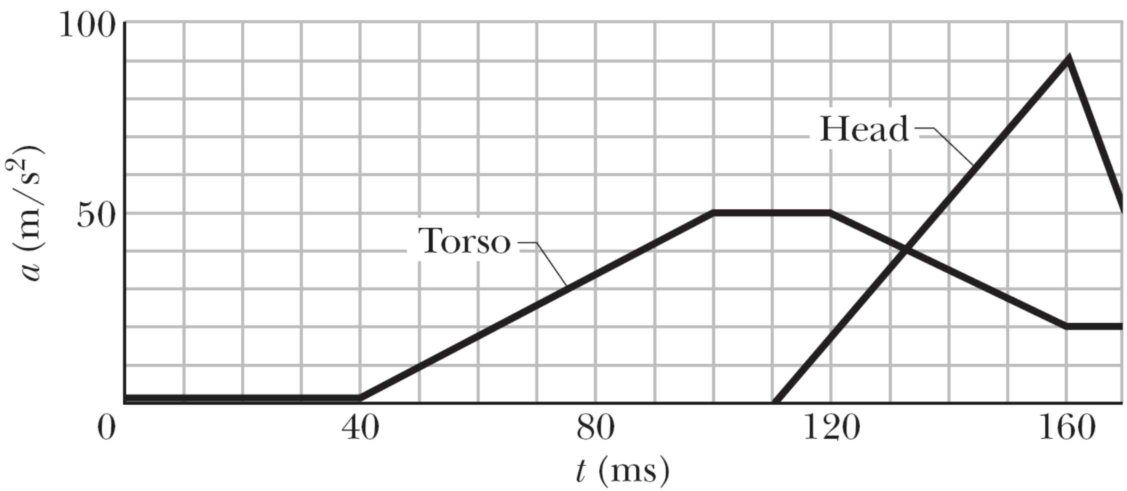
- When the acceleration curve is above the time axis, the area is positive; when the curve is below the time axis, the area is negative.

$v = \dfrac{dx}{dt} \Rightarrow x_1 - x_0 = \displaystyle\int_{t_0}^{t_1} v\,dt = \left[\begin{array}{l}\text{area between velocity curve} \\ \text{and time axis, from } t_0 \text{ to } t_1\end{array}\right]$

the corresponding unit of area

$$(1\ \mathrm{m/s})(1\ \mathrm{s}) = 1\ \mathrm{m} \quad (\text{displacement})$$

- Whether this area is positive or negative is determined as described for the acceleration curve.



Problem 2-6