Chapter 2 Motion along a straight line

Motion

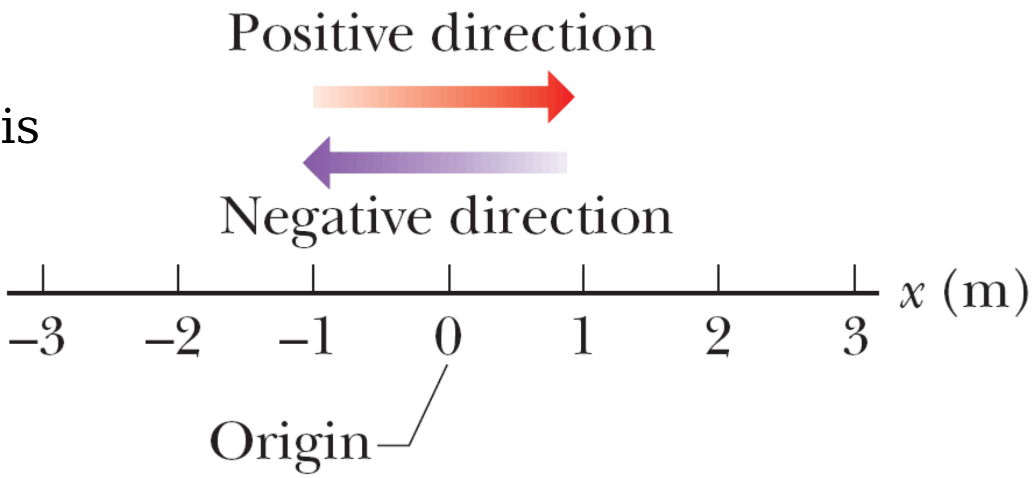
- Moves along a single axis – *one-dimensional motion*.

- In this chapter: 1. the motion is along a straight line only;

  2. we discuss only the motion itself and changes in the motion;

  3. the moving object is either a particle or an object that moves like a particle.

Position and Displacement



- **Origin**: some reference point of a axis

- **Positive direction**: the direction of increasing number

- **Negative direction**: the direction of decreasing number

- **Displacement** Δx: a change from one position x_1 to another position x_2 ,

$$\Delta x = x_2 - x_1$$

Ex:

$$\begin{aligned} \Delta x &= 12\,\text{m} - 5\,\text{m} = 7\,\text{m} \\ \Delta x &= 1\,\text{m} - 5\,\text{m} = -4\,\text{m} \\ \Delta x &= 5\,\text{m} - 5\,\text{m} = 0 \end{aligned}$$

- **Magnitude** of the displacement: $|\Delta x|$

- Displacement has 2 features: (1) its *magnitude*; (2) its *direction*,

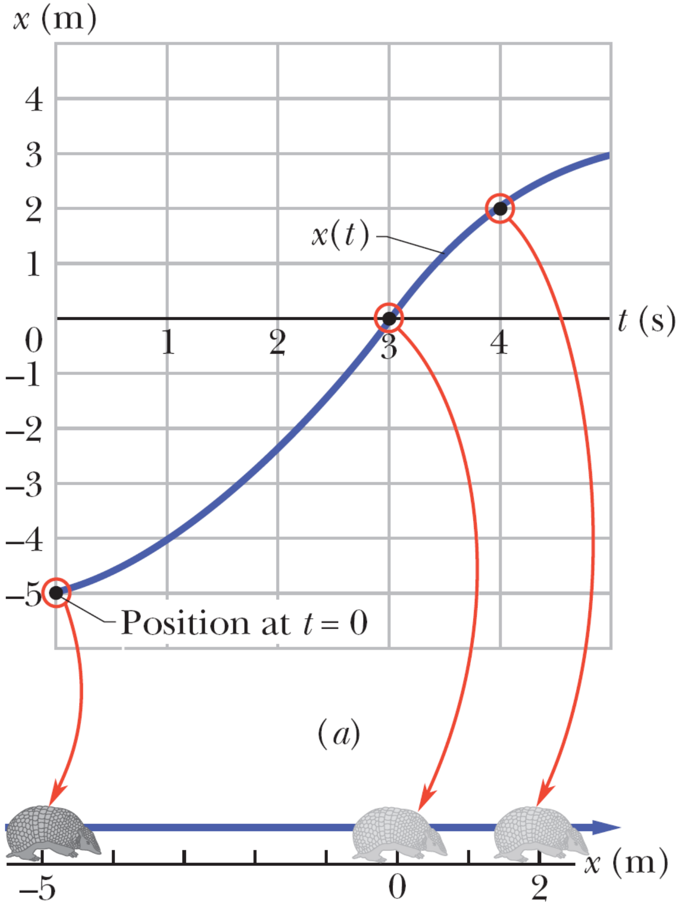
  → an example of a **vector quantity** (see Chapter 3)

Average Velocity and Average Speed

- A compact way to describe position is with a graph of position x plotted as a function of time t — a graph of $x(t)$.

- The graph of $x(t)$ for a stationary particle at x = -2m.

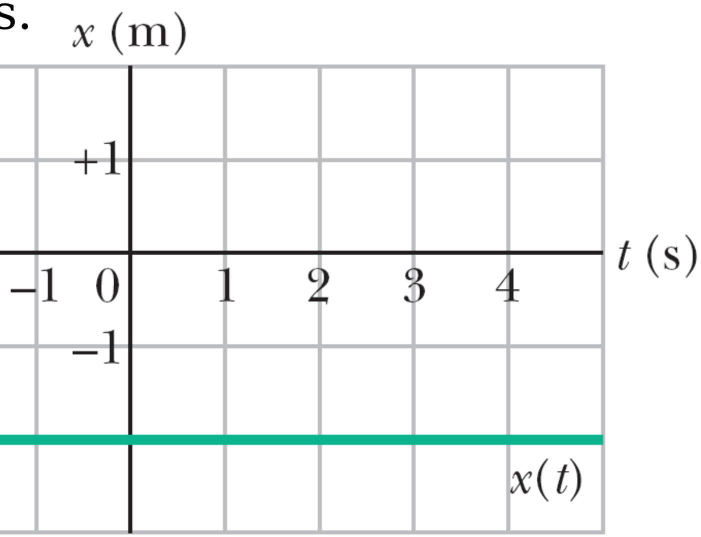
- The graph of $x(t)$ for a moving animal and the path associated with the graph.



- **Average velocity**: the ratio of the displacement Δx that occurs during a particular time interval Δt to that interval,

$$v_{\text{avg}} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

- The unit is always in the form of length/time, eg, m/s.



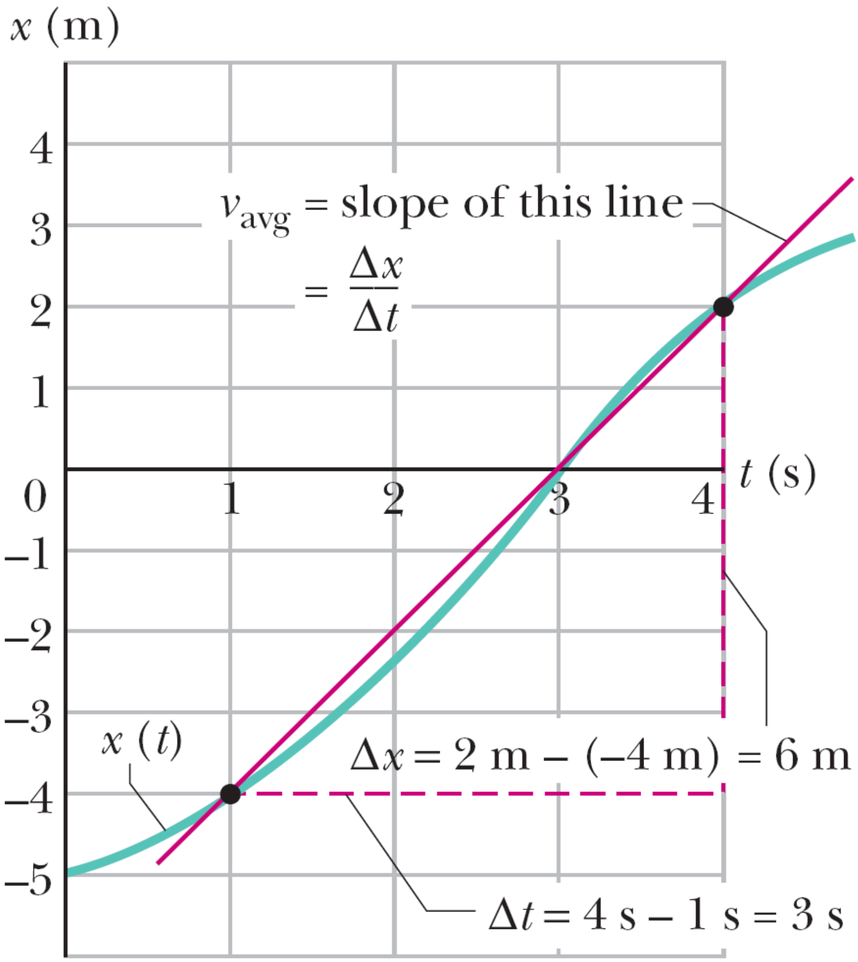
- v_{avg} is the **slope** of the straight line that connects 2 particular points on the $x(t)$ curve.

- v_{avg} is a vector quantity: magnitude & direction,

  positive : slants upward
  negative: slants downward

- The average velocity v_{avg} always has the same sign as the displacement Δx because Δt is always positive.

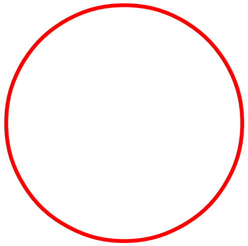
Ex: in the plot $v_{avg} = \dfrac{6\ \mathrm{m}}{3\ \mathrm{s}} = 2\ \mathrm{m/s}$



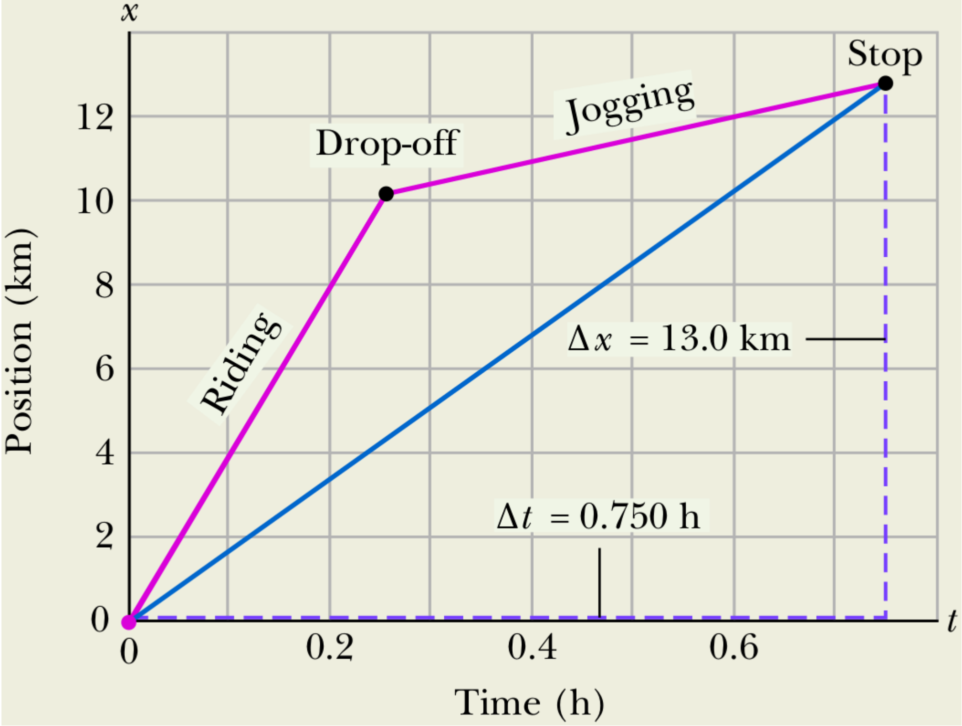
- **Average speed**: $s_{avg} = \dfrac{\text{total distance}}{\Delta t}$

- The difference between average velocity and average speed:

Ex: circular motion



<u>Sample 2.1.1</u>: You get a lift from a car service to take you to a state park along a straight road due east (directly toward the east) for 10 km at an average velocity of 40 km/h. From the drop-off point, you jog along a straight path due east for 3 km, which takes 0.5 h.



(a) What is your overall displacement from your starting point to the point where your jog ends?

$$\Delta x = x_2 - x_1 = 13 - 0 = 13\text{ km}$$

(b) What is the time interval Δt from the beginning of your movement to the end of the jog?

$$\Delta t_{car} = \frac{\Delta x_{car}}{v_{avg,car}} \quad \Leftarrow \quad v_{avg,car} = \frac{\Delta x_{car}}{\Delta t_{car}}$$

$$= \frac{10\text{ km}}{40\text{ km/h}} = 0.25\ h \quad \Rightarrow \quad \Delta t = \Delta t_{car} + \Delta t_{car} = 0.25\text{ h} + 0.5\text{ h} = 0.75\text{ h}$$

(c) What is your average velocity v_{avg} from the starting point to the end of the jog? Find it both numerically and graphically.

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{13\text{ km}}{0.75\text{ h}} = 17.3\text{ km/h}$$

Instantaneous Velocity & Speed

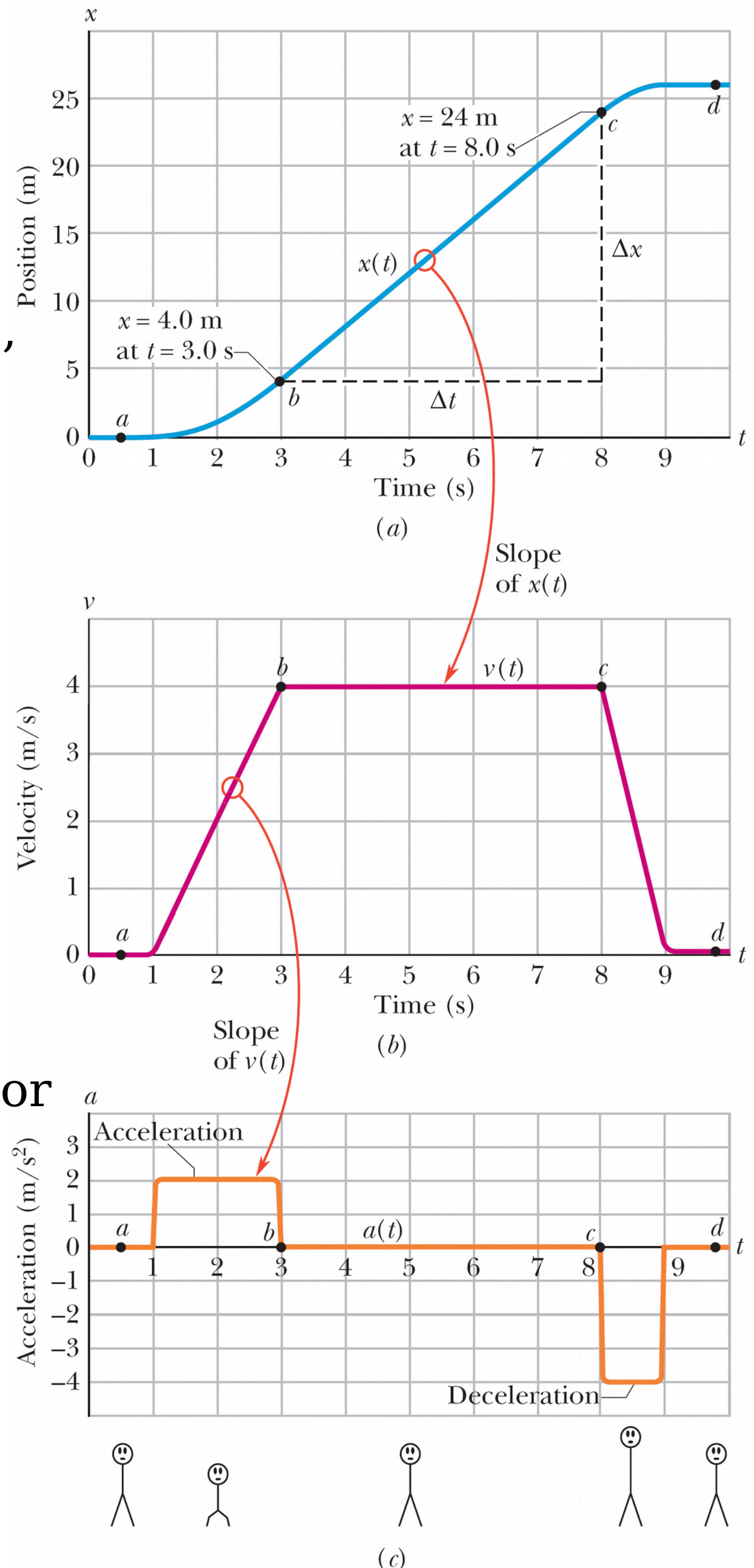
- The **instantaneous velocity** (or simply **velocity**) at any instant comes from the average velocity by shrinking the time interval Δt closer and closer to 0,

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{d x}{d t}$$

- 2 features:
  (1) v is the derivative of x with respect to t;
  (2) v is the slope of the particle's position-time curve at the instant.

- **Speed**: the magnitude of velocity

Sample 2.2.1: The figure is an $x(t)$ plot for an elevator cab that is initially stationary, then moves upward (which we take to be the positive direction of x), and then stops. Plot $v(t)$.



Acceleration (a vector quantity)

- **Average acceleration**: $a_{\text{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$
- **Instantaneous acceleration** (or simply **acceleration**): $a = \frac{\mathrm{d}\,v}{\mathrm{d}\,t}$
- The acceleration of a particle at any instant is the 2nd derivative of its position $x(t)$ with respect to time, $a = \frac{\mathrm{d}\,v}{\mathrm{d}\,t} = \frac{\mathrm{d}}{\mathrm{d}\,t}\left(\frac{\mathrm{d}\,x}{\mathrm{d}\,t}\right) = \frac{\mathrm{d}^2\,x}{\mathrm{d}\,t^2}$
- The unit of acceleration is in the form of length/time^2, eg, m/s^2.
- see figure (c).
- Human's body reacts to accelerations but not to velocities. why?

Newton's 2nd law

- Accelerations are sometimes expressed in terms of g unit, with $1\ g = 9.8\ \mathrm{m/s^2}$.

Ex: fighter jet's pilot

calculus part one

Definition of a derivative: $\frac{dx}{dt} = \frac{d}{dt} x(t) = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{(t+\Delta t) - t}$

If $x = t^n$ then $\boxed{\frac{dx}{dt} = \frac{dt^n}{dt} = n\, t^{n-1}}$

$$x = t^n \quad \Rightarrow \quad \frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{(t+\Delta t)^n - t^n}{(t+\Delta t) - t} = \lim_{\Delta t \to 0} \frac{(t+\Delta t)^n - t^n}{\Delta t}$$

$$(t+\Delta t)^n = \sum_{m=0}^{n} \frac{n!}{m!\,(n-m)!} (\Delta t)^m\, t^{n-m}$$

$$= t^n + n\, t^{n-1}\, \Delta t + \frac{n(n-1)}{2}\, t^{n-2} (\Delta t)^2 + \cdots + (\Delta t)^n$$

$$\Rightarrow \quad \frac{(t+\Delta t)^n - t^n}{\Delta t} = n\, t^{n-1} + \Delta t \left(\frac{n(n-1)}{2}\, t^{n-2} + \cdots \right)$$

$$\Rightarrow \quad \frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{(t+\Delta t)^n - t^n}{\Delta t} = n\, t^{n-1}$$

Sample 2.3.1: A particle's position on the x axis: $x(t) = 4 - 27\,t + t^3$

(a) velocity and acceleration? $v = \frac{d\,x}{d\,t} = 3\,t^2 - 27 \quad \Rightarrow \quad a = \frac{d\,v}{d\,t} = 6\,t$

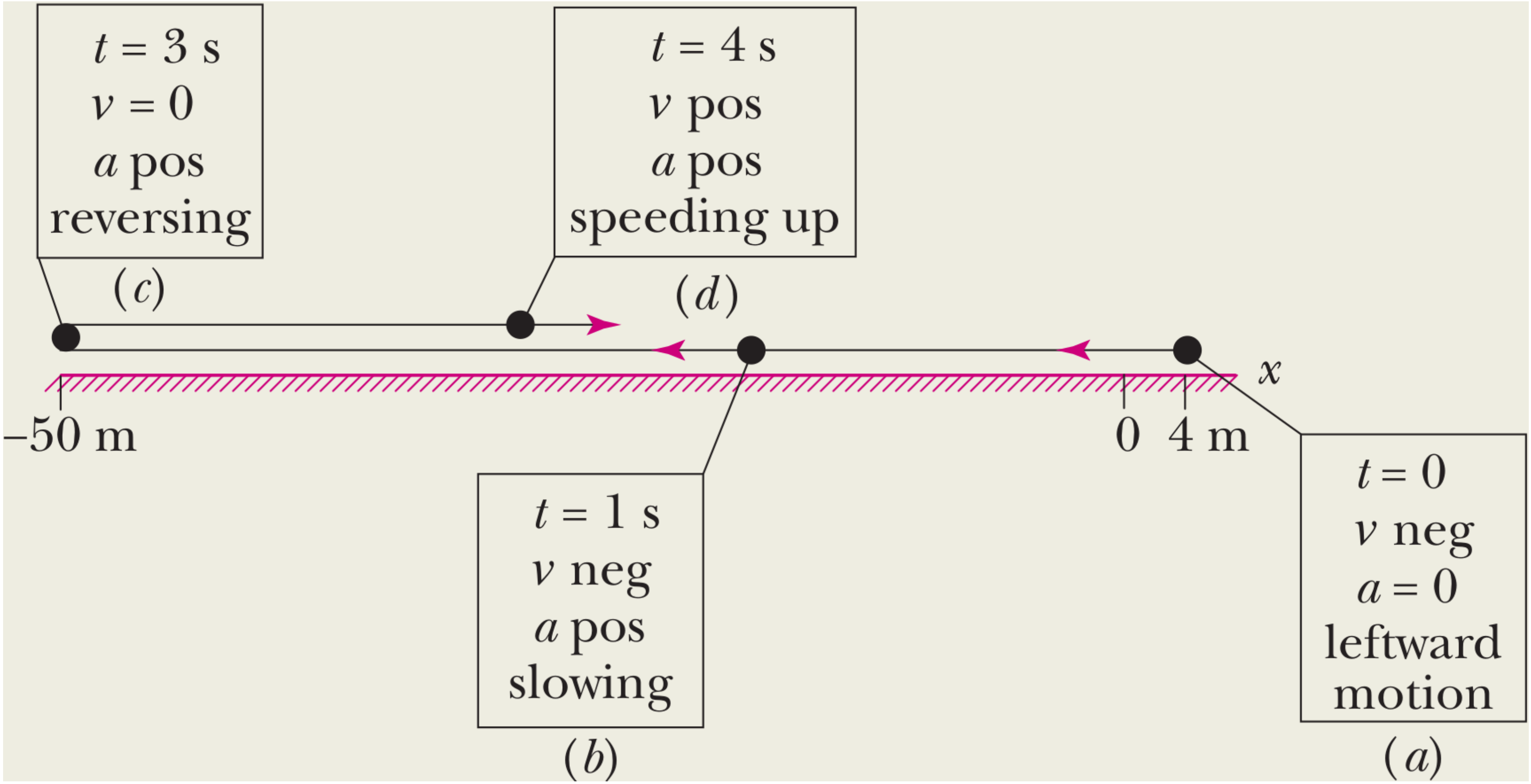
(b) When v=0? $0 = v = 3\,t^2 - 27 \quad \Rightarrow \quad t = \pm 3\text{ s}$

(c) Describe the motion for $t \geq 0$.

$t = 0 \quad \Rightarrow \quad x = 4\text{ m}, \quad v = -27\text{ m/s}, \quad a = 0$

$0 < t < 3\text{ s} \quad \Rightarrow \quad v < 0, \quad a > 0, \quad x(3\text{ s}) = -50\text{ m} \quad \Rightarrow$ slowing

$t > 3\text{ s} \quad \Rightarrow \quad v > 0, \quad a > 0$



Constant Acceleration: A Special Case

- In this section the equation are valid only for constant acceleration.

$$a = a_{avg} = \frac{v - v_0}{t - 0} \quad \Rightarrow \quad v = v_0 + a\,t \qquad (1)$$

v_0 is the velocity at time $t = 0$ and v is the velocity at any later time t, a is the acceleration.

- The function is linear and the plot is a straight line.

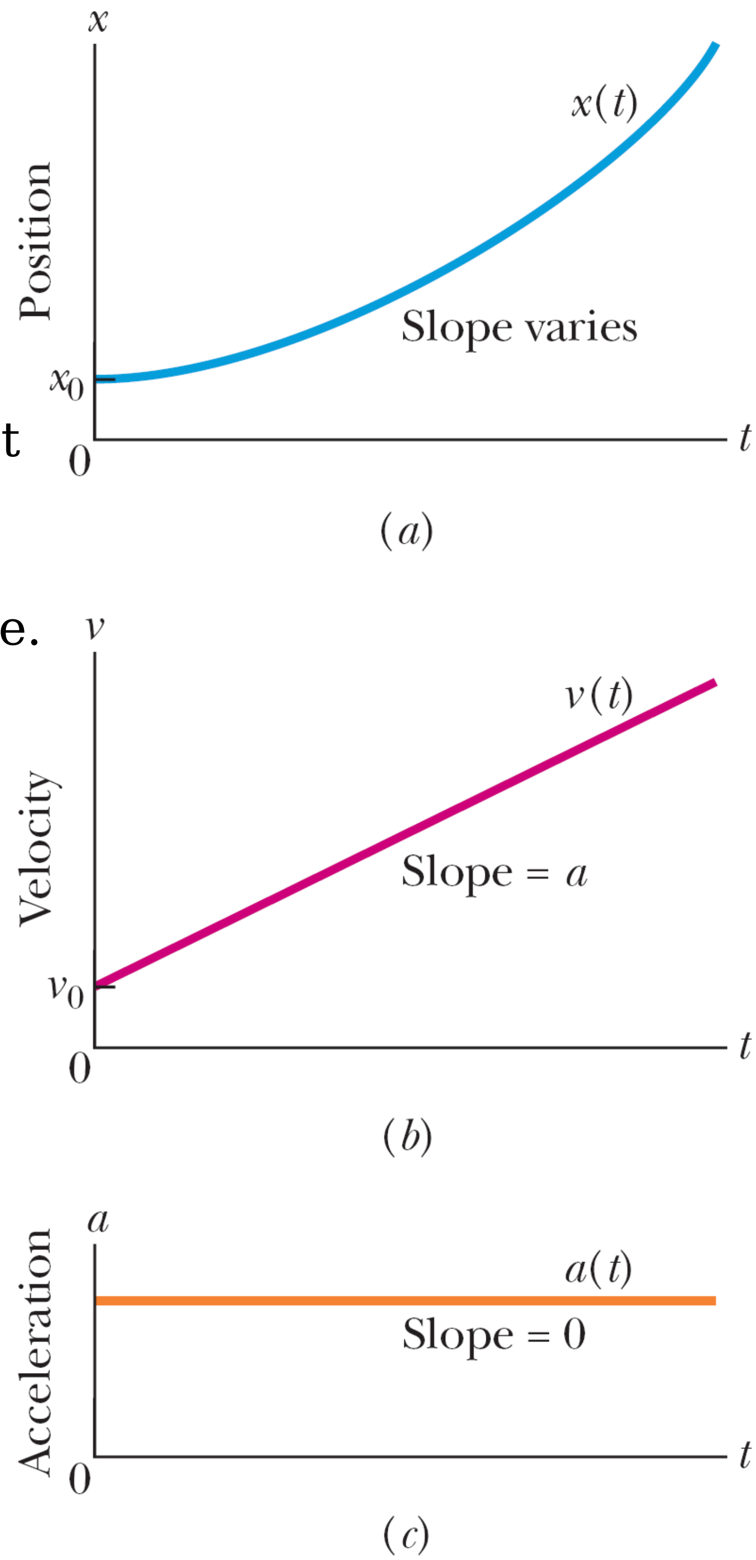
$$v_{avg} = \frac{x - x_0}{t - 0} \quad \Rightarrow \quad x = x_0 + v_{avg}\,t$$

since $$v_{avg} = \frac{1}{2}\left(v_0 + v\right) = v_0 + \frac{1}{2}\,a\,t$$

then $$x - x_0 = v_0\,t + \frac{1}{2}\,a\,t^2 \qquad (2)$$

- Equation (1) & (2) are the *basic equations for constant acceleration*.

- 5 quantities are involved in any problem about constant acceleration — $x - x_0$, v, t, a, v_0.



(a)

(b)

(c)

- In each equation 3 quantities are given and one is to be found.

$$v^2 = v_0^2 + 2a(x - x_0)$$

- 3 additional equations: $x - x_0 = \frac{1}{2}(v_0 + v)t$

$$x - x_0 = vt - \frac{1}{2}at^2$$

Equations for motion with constant acceleration

Equation	Missing Quantity
$v = v_0 + at$	$x - x_0$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
$v^2 = v_0^2 + 2a(x - x_0)$	t
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
$x - x_0 = vt - \frac{1}{2}at^2$	v_0

Sample 2.4.1: **Autonomous car passing slower car**: Both cars have length L=4.50 m, speed v_0=22 m/s, distance=3L. Let a=3.5 m/s^2 until v=27 m/s. Let t_1 & d_1 be the time and distance. Let t_2 be the time from the end of the acceleration to when B is ahead of A by 3L. The total time $t_{tot}=t_1+t_2$. Find (a) t_1 (b) d_1 (c) x_{B2} (d) x_{A2} (e) x_{B2} in terms of x_{A2} and L (f) t_2 (g) t_{tot}

$$t_1 = \frac{v - v_0}{a} = \frac{27 - 22}{3.5} = 1.4285 \text{ s}$$

$$v^2 = v_0^2 + 2\,a\,d_1$$

$$\Rightarrow \quad d_1 = \frac{v^2 - v_0^2}{2\,a} = \frac{27^2 - 22^2}{2 \times 3.5} = 35 \text{ m}$$

$$x_{B2} = d_1 + v\,t_2\,, \quad x_{A2} = 4\,L + v_0\left(t_1 + t_2\right)$$

$$\Rightarrow \quad x_{B2} = x_{A2} + 4\,L$$

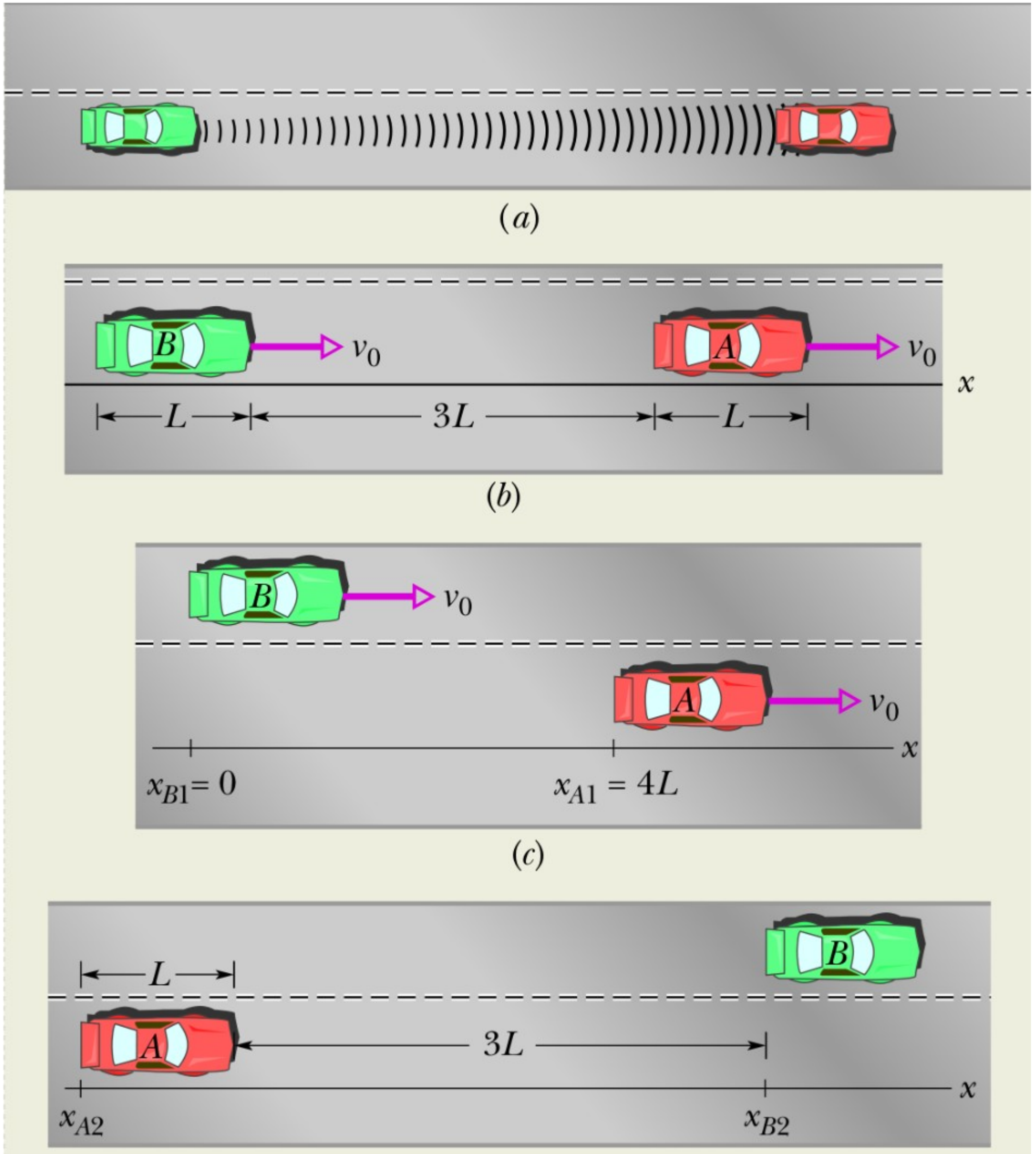
$$\Rightarrow \quad d_1 + v\,t_2 = 8\,L + v_0\left(t_1 + t_2\right)$$

$$\Rightarrow \quad t_2\left(v - v_0\right) = 8\,L + v_0\,t_1 - d_1$$

$$\Rightarrow \quad t_2 = \frac{8\,L + v_0\,t_1 - d_1}{v - v_0}$$

$$= \frac{8 \cdot 4.5 + 22 \cdot 1.4285 - 35}{27 - 22} = 6.4854 \text{ s}$$

$$t_{tot} = t_1 + t_2 = 1.4285 + 6.4854 \simeq 7.91 \text{ s}$$



(a)

(b)

(c)

Sample 2.4.2: The head of woodpecker is moving forward at a speed of 7.49 m/s when the beak makes the first contact with a tree limb. The beak stops after penetrating the limb by 1.87 mm. Assume the acceleration to be constant, find the acceleration magnitude in term of g.

$$v = 0\,, \quad v_0 = 7.49\ \mathrm{m/s}\,, \quad x - x_0 = 1.87 \times 10^{-3}\ \mathrm{m}$$

$$\Rightarrow \quad \text{using} \quad v^2 = v_0^2 + 2\,a\,(x - x_0)$$

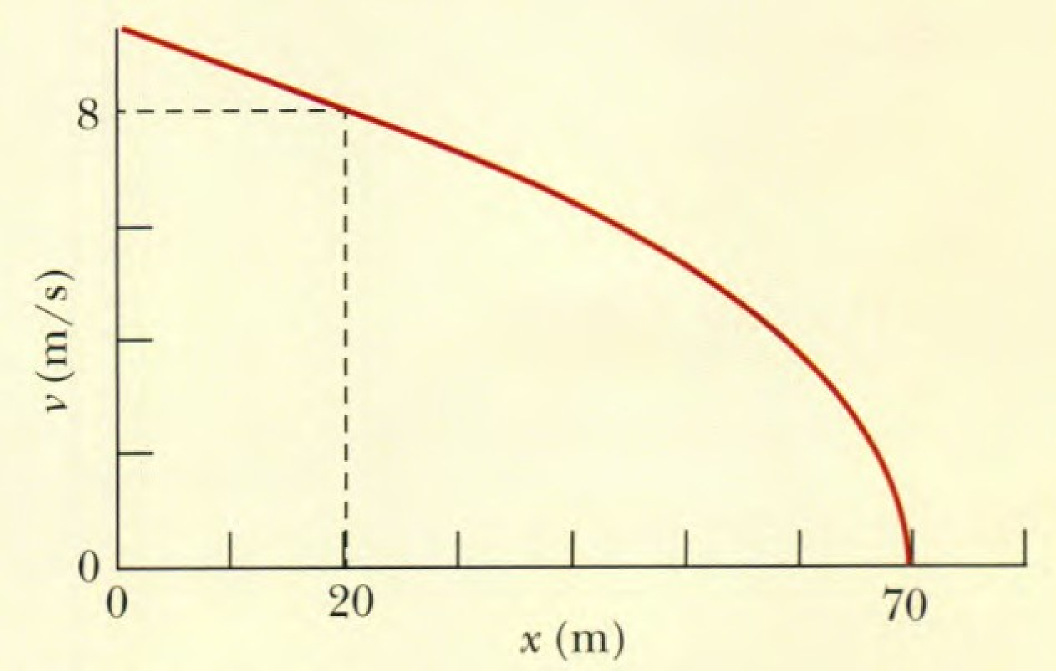
$$\Rightarrow \quad 0^2 = (7.49)^2 + 2\,a \times 1.87 \times 10^{-3}$$

$$\Rightarrow \quad a = -1.5 \times 10^{-4}\ \mathrm{m/s^2}$$

$$\Rightarrow \quad |a| = 1.53 \times 10^{3}\,g \quad \Leftarrow \quad g = 9.8\ \mathrm{m/s^2}$$

How can a woodpecker withstand such huge acceleration without concussion?

Sample 2.4.3: The figure gives a particle's velocity v versus its position as it moves along the x axis with constant acceleration. What is its velocity at position x=0?



Coordinate pairs

$$(x_1, v_1) = (20\ \mathrm{m}, 8\ \mathrm{m/s})$$

$$(x_2, v_2) = (70\ \mathrm{m}, \quad 0 \quad)$$

$$\Rightarrow \quad \text{using} \quad v^2 = v_0^2 + 2a(x - x_0)$$

$$\Rightarrow \quad v_2^2 = v_1^2 + 2a(x_2 - x_1)$$

$$\Rightarrow \quad 0^2 = 8^2 + 2a(70 - 20)$$

$$\Rightarrow \quad a = -0.64\ \mathrm{m/s^2}$$

$$\text{If } x_3 = 0, \quad \text{what is } v_3? \quad \Rightarrow \quad v_3^2 = v_1^2 + 2a(x_3 - x_1)$$

$$\Rightarrow \quad v_3^2 = 8^2 + 2(-0.64)(0 - 20) = 89.6 \quad \Rightarrow \quad v_3 = 9.46\ \mathrm{m/s}$$

Another Look at Constant Acceleration

- Rewrite the definition of acceleration

$$\mathrm{d}v = a\,\mathrm{d}t \quad \Rightarrow \quad \int \mathrm{d}v = \int a\,\mathrm{d}t = a\int \mathrm{d}t \quad \Leftarrow \quad a = \mathrm{const} \quad \Rightarrow \quad v = a\,t + C$$

- To evaluate the constant C, $\quad v_0 \Leftarrow v\left(t = 0\right) = \left(a\right)\left(0\right) + C = C \quad \Rightarrow \quad v = v_0 + a\,t$

- Rewrite the definition of velocity $\quad \mathrm{d}x = v\,\mathrm{d}t$

$$\Rightarrow \quad x \Leftarrow \int \mathrm{d}x = \int v\,\mathrm{d}t = \int \left(v_0 + a\,t\right)\mathrm{d}t = v_0\int \mathrm{d}t + a\int t\,\mathrm{d}t = v_0\,t + \frac{1}{2}a\,t^2 + C'$$

- To evaluate the constant C', $\quad x_0 \Leftarrow x\left(t = 0\right) = C' \quad \Rightarrow \quad x = x_0 + v_0\,t + \frac{1}{2}a\,t^2$

Free-Fall Acceleration

- Objects accelerate downward at a certain constant rate, the magnitude is presented by g.

- Independent of the object's characteristics, such as mass, density, or shape (neglecting the effects of the air); it is the same for all objects.

- At sea level in Earth's mid-latitudes the value is $9.8\ \mathrm{m/s^2}$ (or $32\ \mathrm{ft/s^2}$).

The free-fall acceleration near Earth's surface is $a = -g = -9.8\ \mathrm{m/s^2}$, and the *magnitude* is $g = 9.8\ \mathrm{m/s^2}$.
Do not substitute $-9.8\ \mathrm{m/s^2}$ for g.

Sample 2.5.1: **Timing a fall over Niagara Falls** One man fell 48m to the water. Assume his initial velocity was zero, and neglect the effect of the air on the ball during the fall.

(a) How long did he fall to reach the water surface?

$$y-y_0=v_0\,t-\frac{g}{2}t^2 \quad\Rightarrow\quad -48-0=0\cdot t-\frac{9.8}{2}t^2$$

$$\Rightarrow\quad t^2=48/4.9 \quad\Rightarrow\quad t=3.1\text{ s}$$

(b) Determine his position at each full second.

Use $y-y_0=v_0\,t-\frac{g}{2}t^2$. See table.

(c) What was his velocity as he reached the water surface?

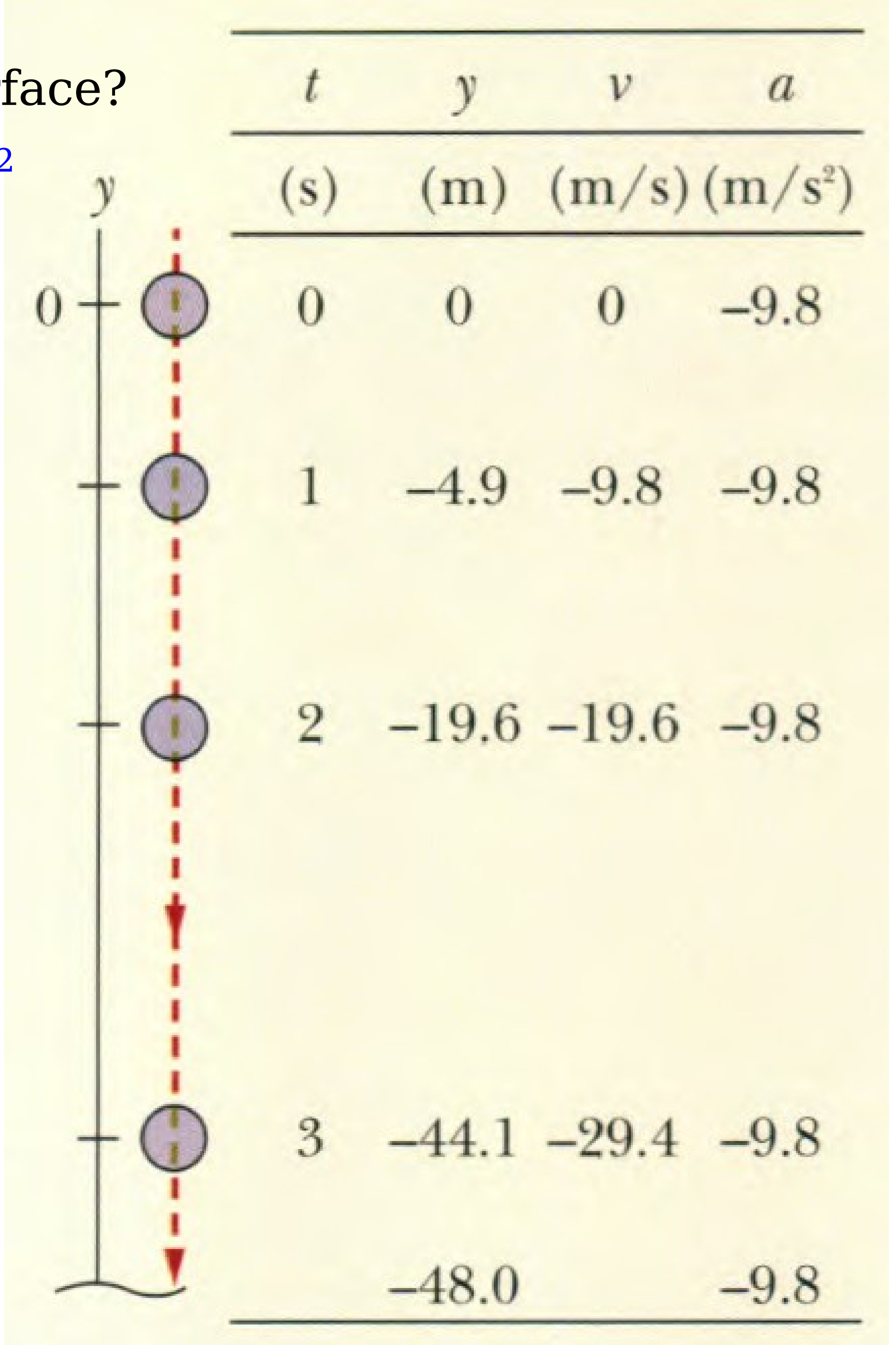
$$v^2=v_0^2-2\,g\,(y-y_0)=0-2\cdot 9.8\,(-48-0)$$

$$\Rightarrow\quad v\approx -31\text{ m/s}$$

(d) What was his velocity at each count of one full second? Was he aware of his increasing speed?

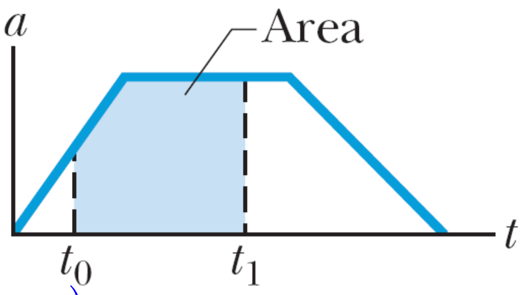
Use $v=v_0-g\,t$. See table.

Without comparing the environment, he won’t aware of his increasing speed until hitting the surface of water.



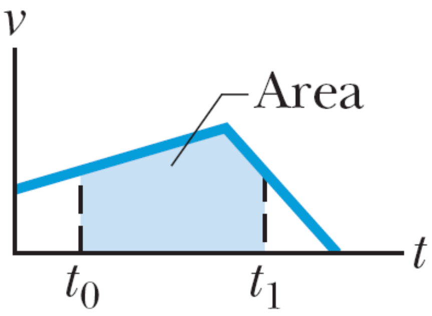
t (s)	y (m)	v (m/s)	a (m/s^2)
0	0	0	–9.8
1	–4.9	–9.8	–9.8
2	–19.6	–19.6	–9.8
3	–44.1	–29.4	–9.8
	–48.0		–9.8

Graphical Integration in Motion Analysis



- $a = \frac{dv}{dt} \Rightarrow v_1 - v_0 = \int_{t_0}^{t_1} a\,dt =$ area between acc. curve & time axis, from t_0 to t_1

the corresponding unit of area $(1\ \mathrm{m/s^2})(1\ \mathrm{s}) = 1\ \mathrm{m/s}$ (velocity)



- When the acceleration curve is above the time axis, the area is positive; when the curve is below the time axis, the area is negative.

$v = \frac{dx}{dt} \Rightarrow x_1 - x_0 = \int_{t_0}^{t_1} v\,dt =$ area between velocity curve and time axis, from t_0 to t_1

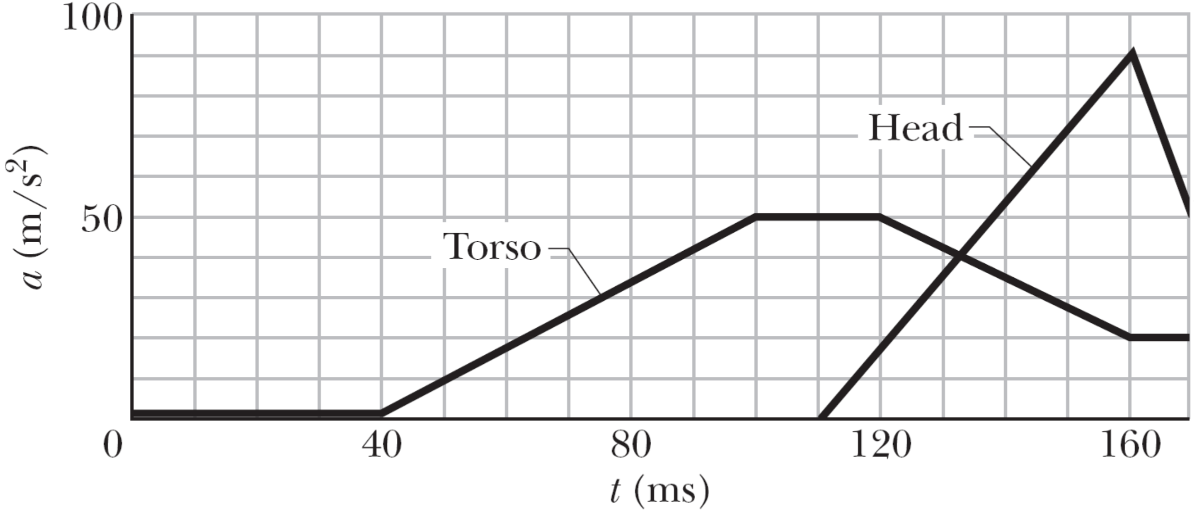
the corresponding unit of area

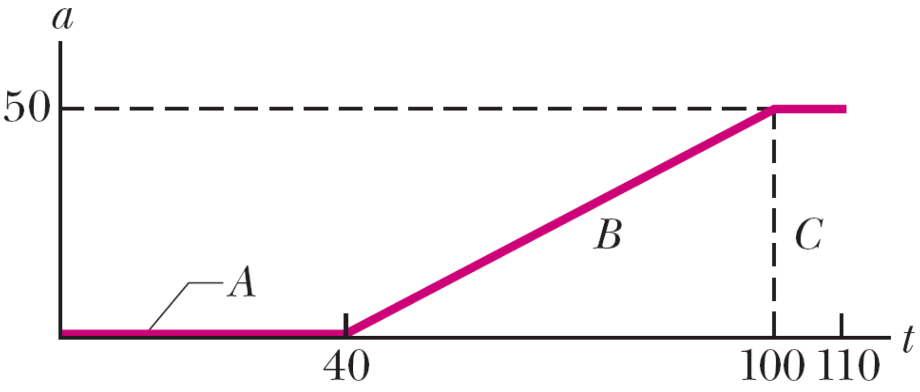
$$(1\ \mathrm{m/s})(1\ \mathrm{s}) = 1\ \mathrm{m} \quad (\text{displacement})$$

- Whether this area is positive or negative is determined as described for the acceleration curve.

The chosen problems: 12, 48, 59

<u>Sample 2.6.1</u>: *Whiplash injury* commonly occurs in a rear-end collision where a front car is hit from behind by a second car. Due to this effect, head restraints were built into cars. If the speed difference of 2 cars is 10.5 km/h. Then the torso acceleration was delayed by 40 ms because during that time interval the seat back had to compress against the driver. The head acceleration was delayed by an additional 70 ms. What was the torso speed when the head began to accelerate?





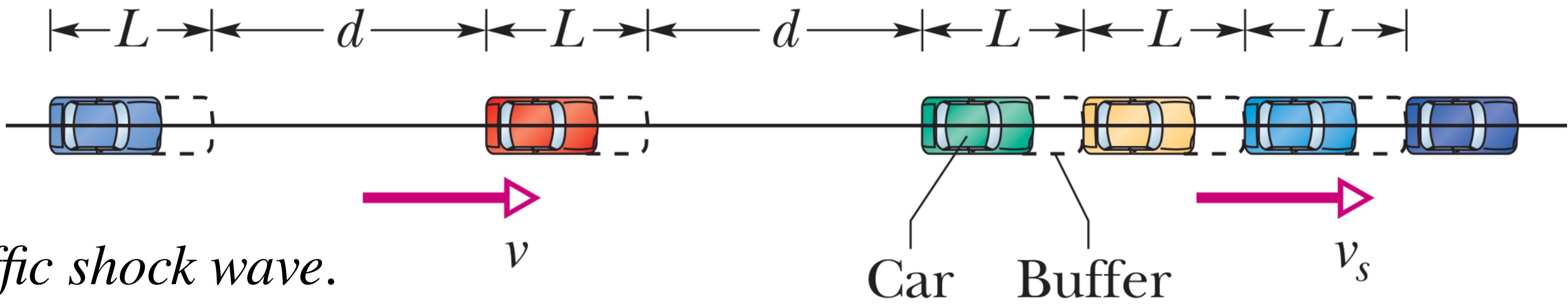
$v_0(t_0=0)=0\,,\quad v_1(t_1=110\text{ ms})=?$

$v_1-v_0=(\text{area between acceleration curve and time axis, from } t_0 \text{ to } t_1)$

$\text{area}_A=0\,,\quad \text{area}_B=\dfrac{1}{2}\times 0.06\times 50=1.5\text{ m/s}\,,\quad \text{area}_C=0.01\times 50=0.5\text{ m/s}$

$\Rightarrow\quad v_1-0=0+1.5+0.5=2\text{ m/s}=7.2\text{ km/hour}$

⇒ It is easy to injure the neck.



Problem 18: *Traffic shock wave.*

It shows a uniformly spaced line of cars moving at speed v=25m/s toward a uniformly spaced line of slow cars moving at speed v_s=4m/s. Assume that each faster car adds length L=12m (car length plus buffer zone) to the line of slow cars when it joins the line, and assume it slows abruptly at the last instant.

(a) For what separation distance d between the faster cars does the shock wave remain stationary? If the separation is twice that amount, what are the (b) speed and (c) direction (upstream or downstream) of the shock wave?

Ans: (a) $t=\dfrac{L}{v_s}=3\text{ s} \;\Rightarrow\; v\,t=d+L \;\Rightarrow\; d=v\,t-L=63\text{ m}$

(b) $d'=2\,d \;\Rightarrow\; \dfrac{L+d'+x}{v}=\dfrac{L+x}{v_s} \;\Rightarrow\; x=+12\text{ m}$

$\Rightarrow\; v_{\text{shock}}=\dfrac{x}{(L+x)/v_s}=+2\text{ m/s} \;\Rightarrow\; \text{downstream} \;\Leftarrow\;$ (c)