Chapter 0 Introduction and Survey

- Cavendish, Coulomb, Faraday, Maxwell, Hertz, ...

- Now classical electrodynamics (EM) rests in a sector of the unified description of particles and interactions known as the *standard model*.

- The unified theoretical framework is generated through principles of continuous gauge invariance of the forces and discrete symmetries of particle properties.

- Classical electrodynamics is a limit of quantum electrodynamics. Quantum electrodynamics is a consequence of a spontaneously broken symmetry in a theory unifying the weak and EM interactions.

- The symmetry breaking leaves the EM force carrier (photon) massless with infinite range, the weak force carriers have masses of the order of 80-90 GeV/c^2 with extremely short range at low energy. The range and strength of the weak interaction are related to the EM coupling ($\alpha \approx \frac{1}{137}$).

Maxwell Equations in Vacuum, Fields, and Sources

- Maxwell equations $\nabla\cdot\mathbf{D}=\rho\,,\quad \nabla\times\mathbf{H}-\frac{\partial\mathbf{D}}{\partial t}=\boldsymbol{J}\,,\quad \nabla\cdot\mathbf{B}=0\,,\quad \nabla\times\mathbf{E}+\frac{\partial\mathbf{B}}{\partial t}=0$

$$\Rightarrow \begin{array}{l}\nabla\cdot\mathbf{E}=\frac{\rho}{\epsilon_0}\\ \nabla\times\mathbf{B}-\frac{1}{c^2}\frac{\partial\mathbf{E}}{\partial t}=\mu_0\boldsymbol{J}\end{array} \quad\Leftarrow\quad \begin{array}{l}\mathbf{D}=\epsilon_0\mathbf{E}\\ \mathbf{B}=\mu_0\mathbf{H}\end{array} \quad\Rightarrow\quad \frac{\partial\rho}{\partial t}+\nabla\cdot\boldsymbol{J}=0 \quad \text{continuity equation}$$

- The Lorentz force equation $\mathbf{F}=q\left(\mathbf{E}+\mathbf{v}\times\mathbf{B}\right)$ considers the motion of charged particle.

- Speed of light in vacuum $c=\frac{1}{\sqrt{\mu_0\,\epsilon_0}}=299{,}792{,}458\ \mathrm{m/s}$, assuming the speed of light is a universal constant.

- Besides $\mathbf{E}$ and $\mathbf{B}$ being convenient replacements for forces produced by charge and current, their important aspects are

(1) they decouples conceptually the sources from the test bodies. This gives $\mathbf{E}$ and $\mathbf{B}$ in meaning in their own right, independent of the sources.

(2) EM fields can exist in regions of space where there are no sources, carrying energy, momentum, angular momentum independent of charges and currents

- $\mathbf{E}$ & $\mathbf{B}$ as ordinary fields is a classical notion. It is thought of as the classical limit of a quantum-mechanical description in terms of real or virtual photons.

- When the number of photons is large but the momentum of an individual photon is small compared to that of material, then the response of material can be decided adequately from a classical description of the EM fields.

- The photoelectric effect is nonclassical for matter, since the quasi-free electrons in the metal change their individual energies by amounts equal to those of the photons, but the photoelectric current can be calculated for the electrons using a classical description of the EM fields.

- The quantum nature of the EM fields must be taken into account in spontaneous emission of radiation by atoms, etc. The average behavior may still be describable in essentially classical terms because of conservation of energy and momentum.

- In classical EM charge density & current density are assumed to be continuous. The magnitudes of point charges are assumed to be arbitrary, but are known to be discrete values in reality.

- The basic unit of charge is the magnitude of the charge on the electron,

$$|q_e| = 4.832068\,(15) \times 10^{-10}\ \mathrm{esu} = 1.60217733\,(49) \times 10^{-19}\ \mathrm{C}$$

- The charges on the proton & on all known particles are integral multiples of q_e.

- The lack of symmetry in the source terms in Maxwell eqns reflects the *absence of magnetic charges* and currents.

Inverse Square Law or the Mass of the Photon

- The distance dependence of the electrostatic law of force was shown to be an inverse square law.

- It is now customary to quote the tests of the inverse square law:

(1) Assume that the force varies as $\frac{1}{r^{2+\epsilon}}$ and quote a value or limit for ϵ.

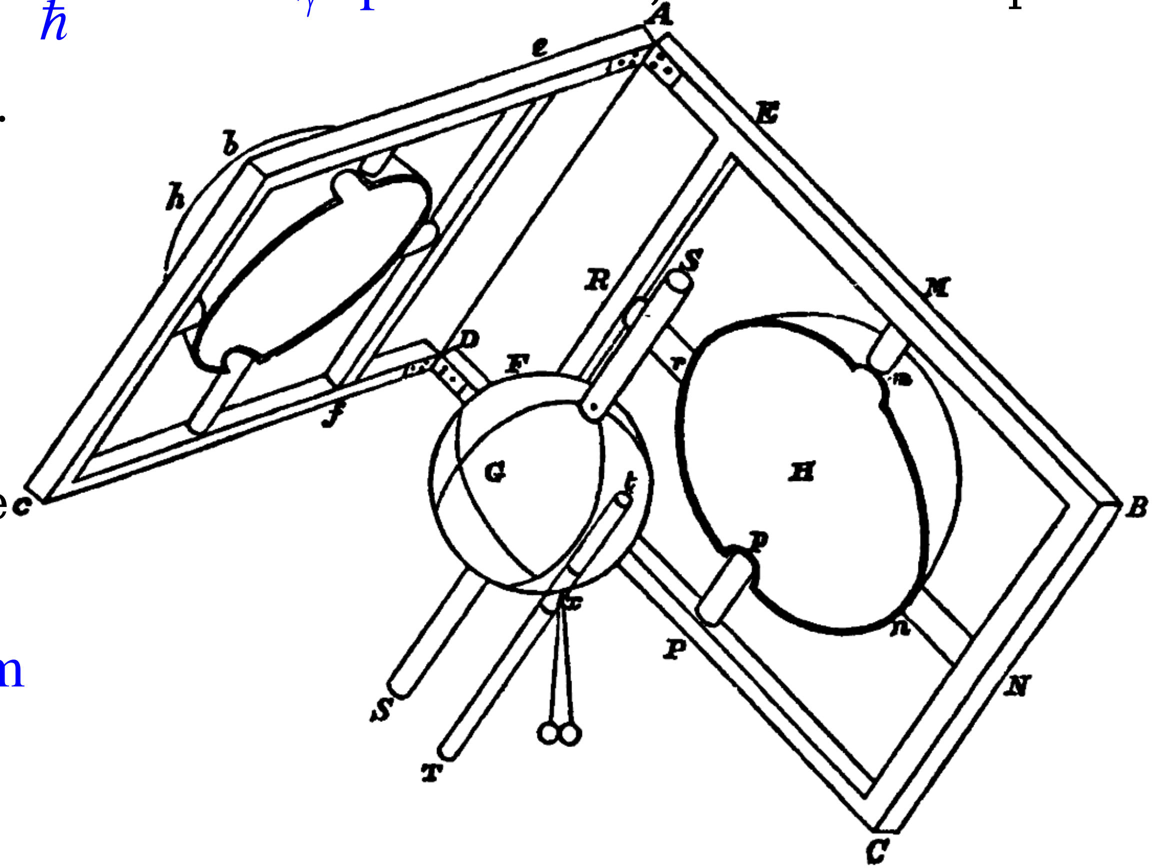
(2) Assume that the electrostatic potential has the "Yukawa" form $\frac{e^{-\mu r}}{r}$ and quote a value of limit for μ or $\frac{1}{\mu}$: $\mu = \frac{m_\gamma c}{\hbar}$ $\Leftarrow$ m_γ : photon mass, the inverse square law test is sometimes phrased of an upper limit on m_γ.

- The current limit is

$$\epsilon = (2.7 \pm 3.1) \times 10^{-16}$$
$$\Rightarrow \quad m_\gamma < 1.6 \times 10^{-51}\ \text{kg}$$

- The surface measurements of the earth magnetic field give the value

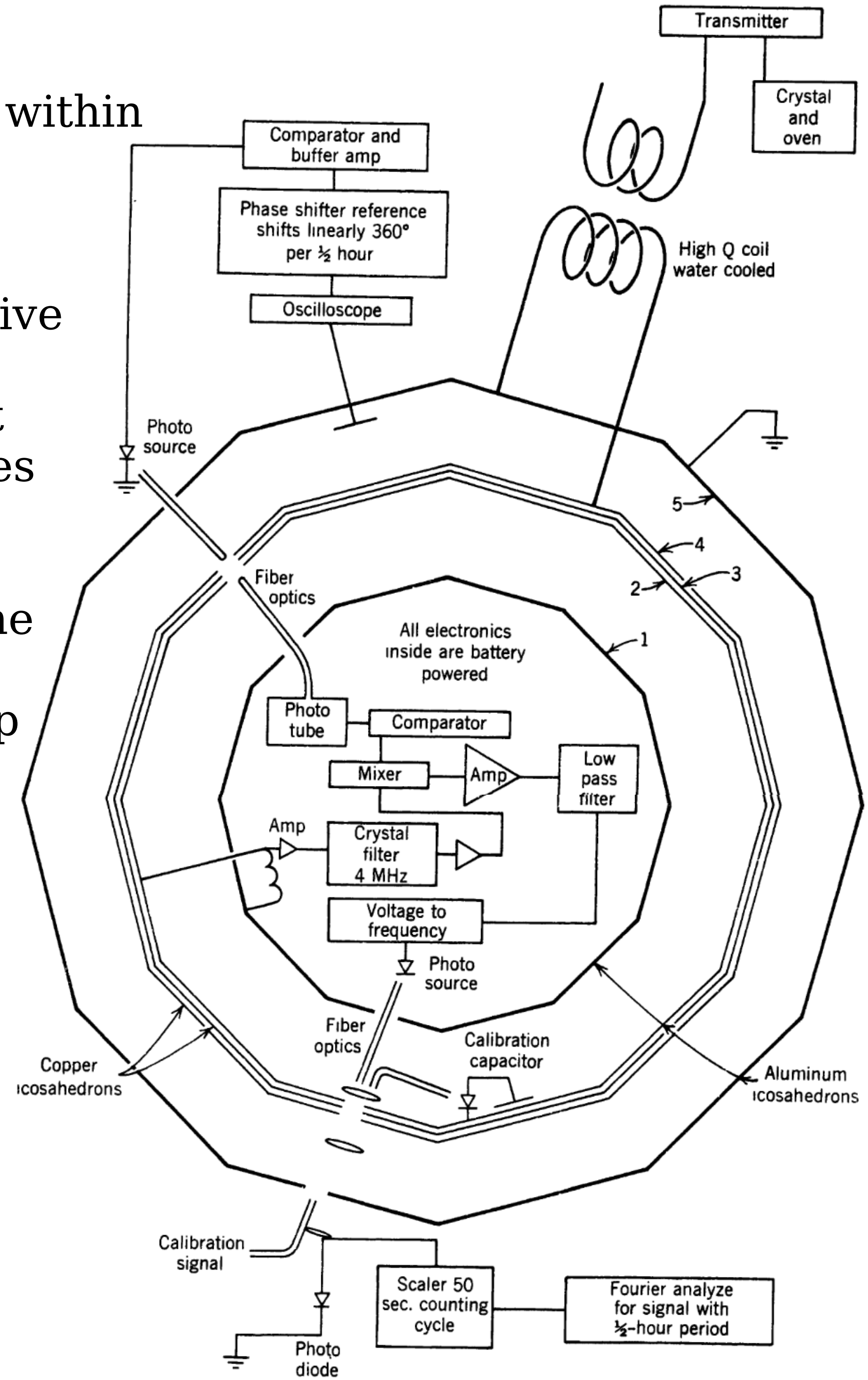
$$m_\gamma < 4 \times 10^{-51}\ \text{kg} \quad \Leftrightarrow \quad \frac{1}{\mu} > 10^{8}\ \text{m}$$



- All these assumptions can be tested only within the framework of the validity of quantum mechanics, linear superposition, etc.

- Elastic scattering experiments with positive and negative electrons at center of mass energies of up to 100 GeV have shown that quantum electrodynamics holds to distances of the order of 10^{-18} m.

- The photon mass can be taken to be 0 (the inverse square force law holds) over the whole classical range of distances and deep into the quantum domain. The inverse square law holds over at least 25 orders of magnitude in the length scale!



Linear Superposition

- The Maxwell equations in vacuum are *linear* in the fields $\mathbf{E}$ and $\mathbf{B}$.

- Macroscopically all sorts of experiments test linear superposition at the level of 0.1% accuracy. At the macroscopic and at the atomic level, linear superposition is remarkably valid.

- The Born-Infeld theory is proposed to test the nonlinearity of EM

$$\frac{\epsilon}{\epsilon_0} = \frac{\mu_0}{\mu} = \frac{b}{\sqrt{b^2 + c^2 B^2 - E^2}} \quad \Leftarrow \quad b : \text{maximum field strength}$$

Fields are obviously modified at short distances; all EM energies are finite.

- There is no evidence of this kind of classical nonlinearity.

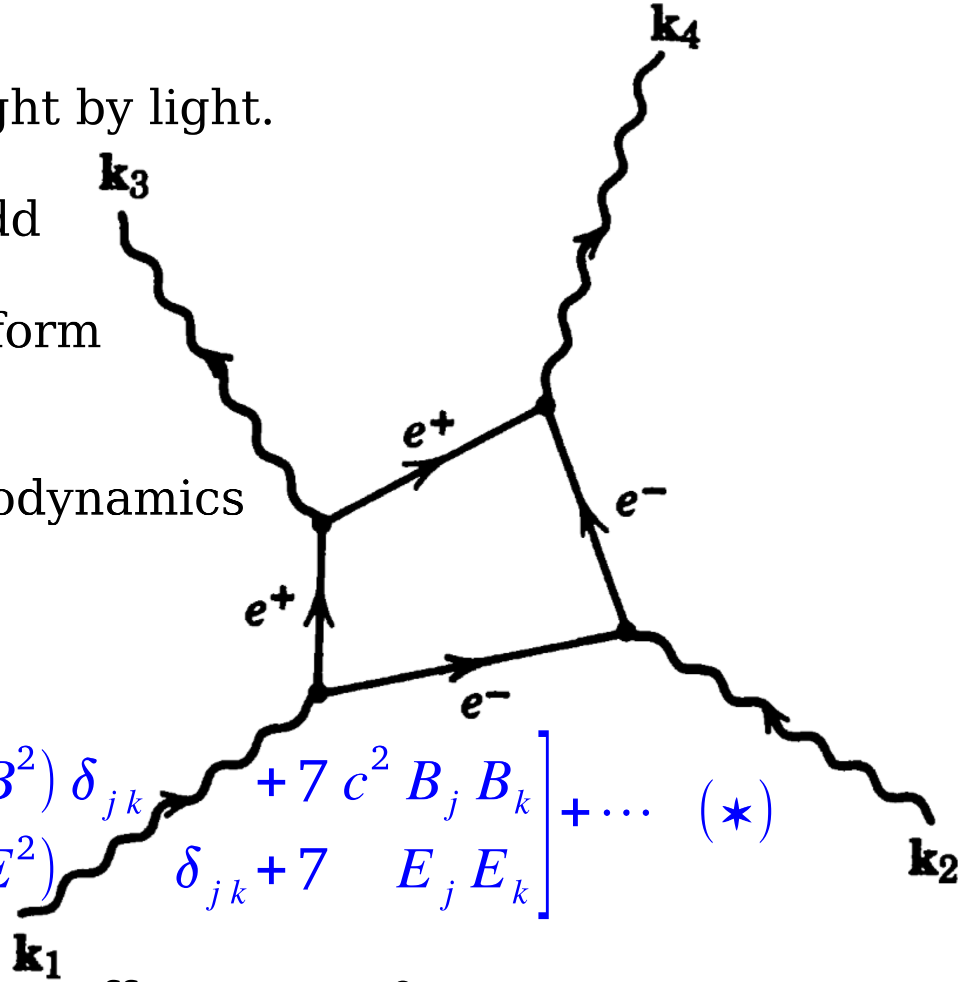
- The quantum mechanics of many-electron atoms is described to high precision with the interactions given by a *linear superposition* of pairwise potentials.

- At the present time there is no evidence for any classical nonlinear behavior of vacuum fields at short distances.

- A *quantum-mechanical nonlinearity* of EM fields comes from the uncertainty principle permitting the pair creation by 2 photons and then the pair annihilation with the emission of 2 different photons.

- This process is called the scattering of light by light.

- The 2 incident plane waves not merely add coherently with linear superposition, but interact and (with small probability) transform into 2 different plane waves.



- This nonlinear feature of quantum electrodynamics can be expressed as

$$D_j = \epsilon_0 \sum_k \epsilon_{jk} E_k \,, \quad B_j = \mu_0 \sum_k \mu_{jk} H_k$$

$$\text{where } \begin{bmatrix} \epsilon_{jk} \\ \mu_{jk} \end{bmatrix} = \delta_{jk} + \frac{e_G^4 \hbar}{45 \pi m^4 c^7} \begin{bmatrix} 2\left(E^2 - c^2 B^2\right) \delta_{jk} + 7 c^2 B_j B_k \\ 2\left(c^2 B^2 - E^2\right) \delta_{jk} + 7 E_j E_k \end{bmatrix} + \cdots \quad (\ast)$$

- In the classical limit ($\hbar \to 0$), these nonlinear effects go to 0.

- Comparison with the classical Born-Infeld expression

$$b_q = \frac{\sqrt{45\pi}}{2} \sqrt{\frac{e_G^2}{\hbar c}} \frac{e_G}{r_0^2} \simeq 0.51 \frac{e_G}{r_0^2} \quad \Leftarrow \quad r_0 = \frac{e_G^2}{m c^2} \simeq 2.8 \times 10^{-15} \text{ m : classical electron radius}$$

$$\frac{e_G}{r_0^2} = 1.8 \times 10^{20} \text{ V/m : electric field at the surface of a classical electron}$$

- The field-dependent terms in (∗) is called as *vacuum polarization* effects.

- In addition to the scattering of light by light, vacuum polarization causes very small shifts in atomic energy levels.

- Vacuum polarization is manifest by a modification of the electrostatic interaction between 2 charges at short distances, described as a screening of the "bare" charges with distance, or as a "running" coupling constant.

- Since the charge of a particle is defined as the strength of its EM coupling observed at large distances, the presence of a screening action by electron-positron pairs closer to the charge implies that the "bare" charge observed at short distances is larger than the charge defined at large distances.

$$V(r) = \hbar c \frac{Z_1 Z_2 \alpha}{r} \left[1 + \frac{2\alpha}{3\pi} \int_{2m}^{\infty} \frac{\sqrt{\kappa^2 - 4m^2}}{\kappa^2} \left(1 + \frac{2m^2}{\kappa^2} \right) e^{-\kappa r} \, \mathrm{d}\kappa \right]$$

- Due of its short range, the added vacuum polarization energy is unimportant in light atoms. It is, important in high Z atoms and in muonic atoms.

- The idea of a "running" coupling constant is an effective strength of interaction that changes with momentum transfer

$$\tilde{V}(Q^2) = \frac{4\pi Z_1 Z_2 \alpha(Q^2)}{Q^2} \quad \Leftarrow \quad \frac{1}{\alpha(Q^2)} \approx \frac{1}{\alpha(0)} - \frac{1}{3\pi} \ln \frac{Q^2}{m^2 e^{5/3}}$$

Maxwell Equations in Macroscopic Media

- For a small number of definite sources, determination of the fields is possible; but for macroscopic aggregates of matter, the solution of the eqns is impossible.

- 2 aspects:
  (1) the number of individual sources is prohibitively large.
  (2) for macroscopic observations the detailed behavior of the fields is irrelevant.

- What is relevant is the average of a field or a source over a large volume. We call such averaged quantities the macroscopic fields and macroscopic sources.

- The macroscopic field quantities $$\mathbf{D} = \epsilon_0 \mathbf{E} + \left(\mathbf{P} - \sum_\beta \frac{Q'_{\alpha\beta}}{x_\beta} \mathbf{e}_\alpha + \cdots \right), \quad \mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \left(\mathbf{M} + \cdots \right)$$

- The charge & current densities are macroscopic averages of the *free* charge & current densities in medium. The *bound* charges & currents are in $\mathbf{P}$, $\mathbf{M}$, & $\mathbf{Q}'$.

- The macroscopic Maxwell eqs are a set of 8 eqns involving $\mathbf{E}$, $\mathbf{B}$, $\mathbf{D}$, and $\mathbf{H}$. The 4 homogeneous eqns can be solved with potentials, but the inhomogeneous eqns cannot be solved until the derived fields $\mathbf{D}$ and $\mathbf{H}$ are known in terms of $\mathbf{E}$ and $\mathbf{B}$.

$$\begin{array}{l} \mathbf{D} = \mathbf{D}[\mathbf{E}, \mathbf{B}] \\ \mathbf{H} = \mathbf{H}[\mathbf{E}, \mathbf{B}] \end{array} \quad \textit{constitutive relations}, \quad \boldsymbol{J} = \boldsymbol{J}[\mathbf{E}, \mathbf{B}] \quad \text{generalized Ohm's law}$$

- In most materials only the *electric and magnetic polarizations* are significant.

- There is big diversity in the electric and magnetic properties of matter with ferroelectric & ferromagnetic materials having nonzero $\mathbf{P}$ or $\mathbf{M}$ in the absence of applied fields, and ordinary dielectric, diamagnetic, & paramagnetic substances.

- For weak fields an applied electric or magnetic field induces an electric or magnetic polarization proportional to the magnitude of the applied field. The response of the medium is called linear

$$D_\alpha = \sum_\beta \epsilon_{\alpha\beta} E_\beta \qquad \Leftarrow \qquad \epsilon_{\alpha\beta} : \text{electric permittivity or dielectric tensor}$$

$$H_\alpha = \sum_\beta \mu'_{\alpha\beta} B_\beta \qquad \qquad \mu'_{\alpha\beta} : \text{inverse magnetic permeability tensor}$$

- For simple materials the linear response is often isotropic. Then $\epsilon_{\alpha\beta}$ & $\mu'_{\alpha\beta}$ are diagonal $\Rightarrow \quad \mathbf{D} = \epsilon\,\mathbf{E}\,, \quad \mathbf{H} = \mu'\,\mathbf{B} = \dfrac{\mathbf{B}}{\mu}$

- At low frequencies ($\nu \leq 10^6$ Hz) solids have dielectric constants typically in the range of $\dfrac{\epsilon_{\alpha\alpha}}{\epsilon_0} \sim 2 - 20$.

- Systems with permanent molecular dipole moments can have much larger and temperature-sensitive dielectric constants, eg, distilled water has a static dielectric constant of $\dfrac{\epsilon}{\epsilon_0} = 88$ at $0°C$, $\dfrac{\epsilon}{\epsilon_0} = 56$ at $100°C$.

- At optical frequencies only the electrons can respond significantly. The dielectric constants are in the range, $\dfrac{\epsilon_{\alpha\alpha}}{\epsilon_0} \sim 1.7 - 10$. Water has $\dfrac{\epsilon_{\alpha\alpha}}{\epsilon_0} = 1.77 - 1.80$ over the visible range, essentially independent of temperature from 0 to 100°C.

- *Diamagnetic* material consists of atoms/molecules with no angular momentum. The response to an applied magnetic field is creating circulating atomic currents to produce a small bulk magnetization opposing the applied field $\Rightarrow \quad \mu'_{\alpha\alpha}\mu_0 > 1$

- If the basic atomic unit of the material has a net angular momentum from unpaired electrons, the substance is *paramagnetic*. The magnetic moment of the odd electron is aligned parallel to the applied field $\Rightarrow \quad \mu'_{\alpha\alpha}\mu_0 < 1$

- *Ferromagnetic* materials are paramagnetic but, due to interactions between atoms, show drastically different behavior. Below the Curie temperature, ferromagnetic substances show spontaneous magnetization, ie, all the magnetic moments in a domain are aligned.

- The application of an external field causes the moments in different domains to line up, leading to the bulk magnetization. Removing the field leaves quite a fraction of the moments still aligned, giving a permanent magnetization.

- Materials that show a linear response to weak fields eventually show nonlinear behavior at high enough field strengths $D_\alpha = \sum \epsilon^{(1)}_{\alpha\beta} E_\beta + \sum \epsilon^{(2)}_{\alpha\beta\gamma} E_\beta E_\gamma + \cdots$

- A large amplitude wave of 2 frequencies ω_1 & ω_2 can generate waves in the medium with frequencies 0, ω_1, ω_2, $2\omega_1$, $2\omega_2$, $\omega_1+\omega_2$, $\omega_1-\omega_2$, ...

- With the development of lasers, nonlinear behavior of this sort has become a research area of its own, called *nonlinear optics*, and also a laboratory tool.

Boundary Conditions at Interfaces Between Different Media

- Apply the divergence theorem to the 1st & 3rd equations of Maxwell equations

$$\Rightarrow \oint_{\mathcal{S}} \mathbf{D}\cdot d\boldsymbol{a} = \int_{\mathcal{V}} \rho\, d^3x\,,\quad \oint_{\mathcal{S}} \mathbf{B}\cdot d\boldsymbol{a} = 0 \quad \Leftarrow \quad d\boldsymbol{a} \equiv da\,\hat{\mathbf{n}}$$

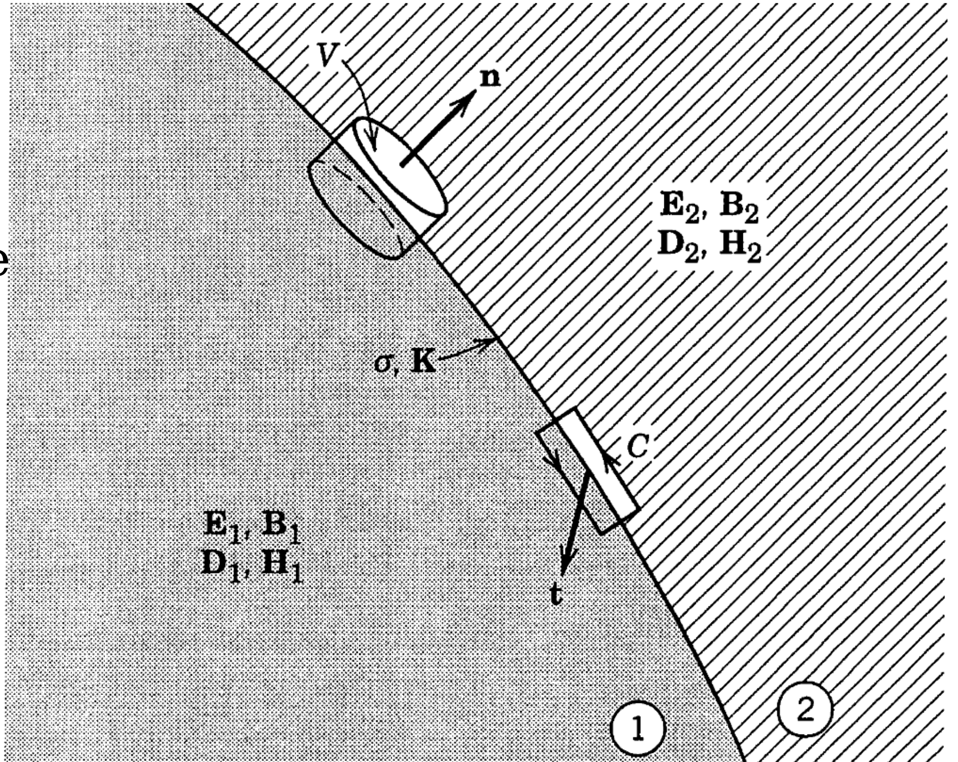
- The 1st relation is Gauss's law that the total flux of $\mathbf{D}$ out through the surface is equal to the charge contained inside. The 2nd is that no net flux of $\mathbf{B}$ flows through a closed surface because of the nonexistence of magnetic charges.

- Apply the Stokes's theorem to the other 2 equations of Maxwell equations

$$\Rightarrow \oint_{\mathcal{C}} \mathbf{H}\cdot d\boldsymbol{\ell} = \int_{\mathcal{S}'} \left(\boldsymbol{J} + \frac{\partial \mathbf{D}}{\partial t} \right)\cdot d\boldsymbol{a}'\,,\quad \oint_{\mathcal{C}} \mathbf{E}\cdot d\boldsymbol{\ell} = -\int_{\mathcal{S}'} \frac{\partial \mathbf{B}}{\partial t}\cdot d\boldsymbol{a}' \quad \Leftarrow \quad d\boldsymbol{a}' \equiv da\,\hat{\mathbf{t}}$$

one is the Ampere-Maxwell law of magnetic fields and the other is Faraday's law of EM induction.

- These integrals can be used to deduce the relationship of various normal (⊥) and tangential (||) components of the fields on either side of a surface between different media, perhaps with a surface charge or current density at the interface.



- $$\oint_{S} \mathbf{D}\cdot d\boldsymbol{a} = (\mathbf{D}_2-\mathbf{D}_1)\cdot\Delta\boldsymbol{a} = \sigma\,\Delta a \Leftarrow \int_{\mathcal{V}} \rho\, d^3x \Rightarrow \begin{array}{l}(\mathbf{D}_2-\mathbf{D}_1)_\perp \equiv (\mathbf{D}_2-\mathbf{D}_1)\cdot\hat{\mathbf{n}} = \sigma \\ (\mathbf{B}_2-\mathbf{B}_1)_\perp \equiv (\mathbf{B}_2-\mathbf{B}_1)\cdot\hat{\mathbf{n}} = 0\end{array}$$
  $$\oint_{S} \mathbf{B}\cdot d\boldsymbol{a} = (\mathbf{B}_2-\mathbf{B}_1)\cdot\Delta\boldsymbol{a} = 0$$
  $$\Rightarrow\quad D_2^\perp - D_1^\perp = \sigma\,,\quad B_2^\perp = B_1^\perp$$
- The normal component of $\mathbf{B}$ is continuous and the discontinuity of the normal component of $\mathbf{D}$ at any point is equal to the surface charge density at that point.
- $$\oint_{C} \mathbf{E}\cdot d\boldsymbol{\ell} = \hat{\mathbf{t}}\times\hat{\mathbf{n}}\cdot(\mathbf{E}_2-\mathbf{E}_1)\,\Delta\ell = 0 = -\frac{\partial\mathbf{B}}{\partial t}\cdot\hat{\mathbf{t}}\,\Delta a \Leftarrow -\int_{S'} \frac{\partial\mathbf{B}}{\partial t}\cdot\hat{\mathbf{t}}\, da$$
  $$\oint_{C} \mathbf{H}\cdot d\boldsymbol{\ell} = \hat{\mathbf{t}}\times\hat{\mathbf{n}}\cdot(\mathbf{H}_2-\mathbf{H}_1)\,\Delta\ell = \boldsymbol{K}\cdot\hat{\mathbf{t}}\,\Delta\ell = \left(\boldsymbol{J}+\frac{\partial\mathbf{D}}{\partial t}\right)\cdot\hat{\mathbf{t}}\,\Delta a$$
  $$\Leftarrow \int_{S'} \left(\boldsymbol{J}+\frac{\partial\mathbf{D}}{\partial t}\right)\cdot\hat{\mathbf{t}}\, da$$
  $$\Delta a\to 0 \;\Rightarrow\; \begin{array}{l}(\mathbf{E}_2-\mathbf{E}_1)_\parallel = \hat{\mathbf{n}}\times(\mathbf{E}_2-\mathbf{E}_1) = 0 \\ (\mathbf{H}_2-\mathbf{H}_1)_\parallel = \hat{\mathbf{n}}\times(\mathbf{H}_2-\mathbf{H}_1) = \boldsymbol{K}\end{array} \Rightarrow\; \mathbf{E}_2^\parallel = \mathbf{E}_1^\parallel\,,\quad \mathbf{H}_2^\parallel - \mathbf{H}_1^\parallel = \boldsymbol{K}\times\hat{\mathbf{n}}$$
- The tangential component of $\mathbf{E}$ across an interface is continuous, while the tangential component of $\mathbf{H}$ is discontinuous by the magnitude of the surface current density and whose direction is parallel to $\boldsymbol{K}\times\hat{\mathbf{n}}$.

Some Remarks on Idealizations in Electromagnetism

- When a conducting object is *grounded*, it is assumed to be connected by a fine conducting wire to a remote reservoir of charge as the common zero of potential.

- Objects held at fixed potentials are similarly connected to one side of a voltage source, the other side of which is connected to the common "ground."

- An idealization in macroscopic electromagnetism is the idea of a surface charge density or a surface current density.

- One limit is that the "surface" distribution is confined to a region near the surface that is *macroscopically small, but microscopically large*.

- The other limit is set by quantum-mechanical effects in the atomic structure of materials.