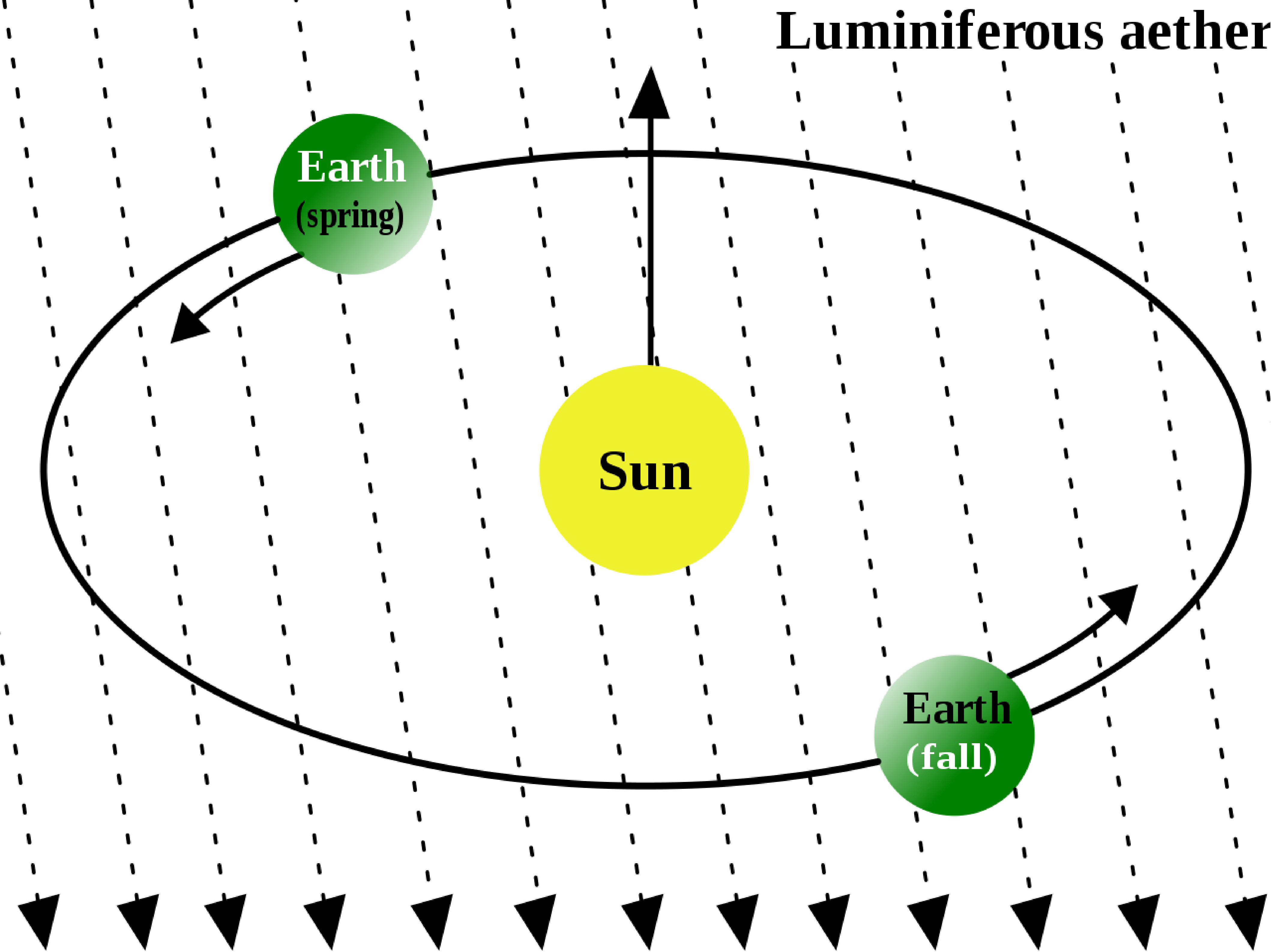
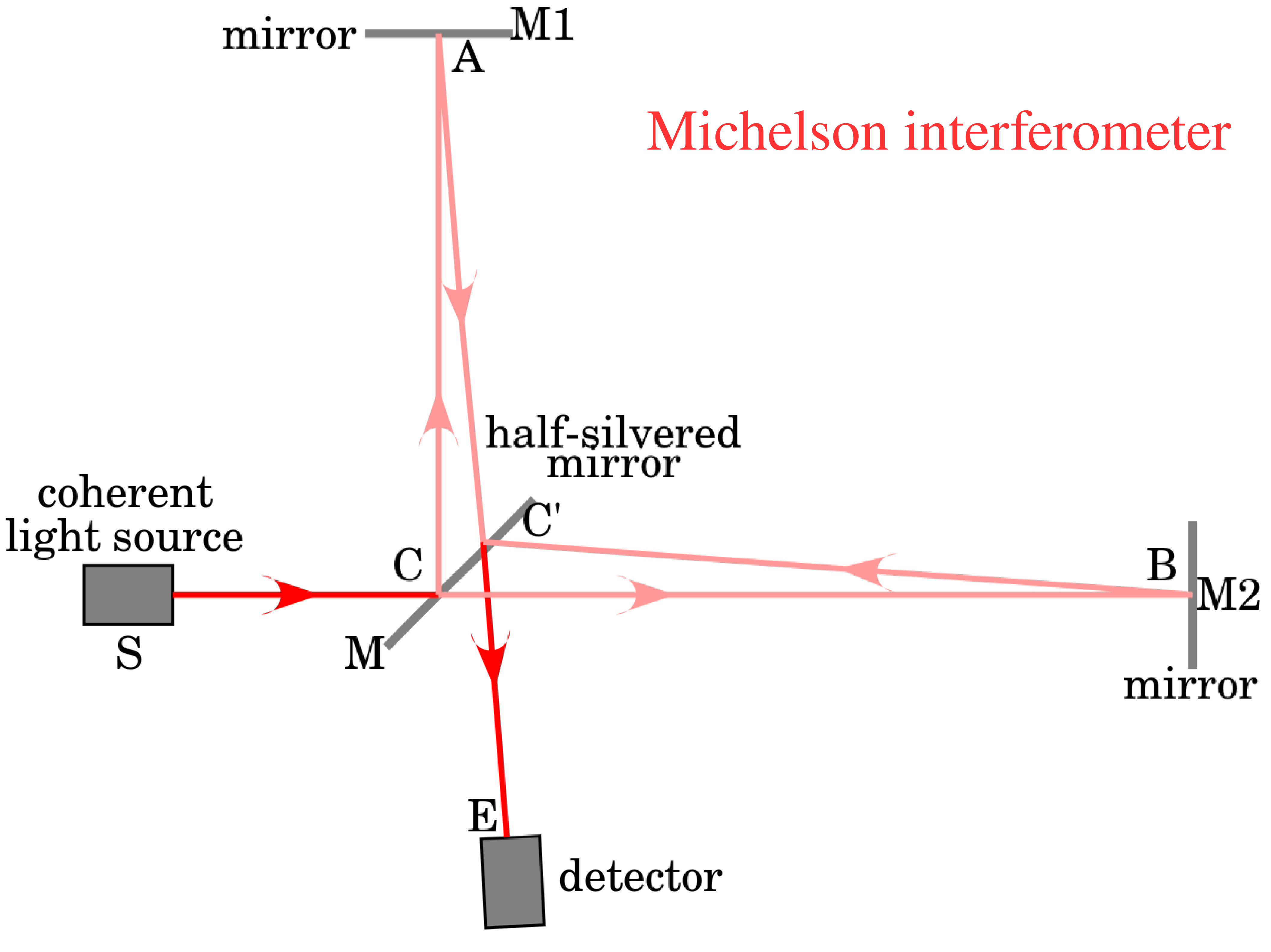

Luminiferous aether
Earth
(spring)
Sun
Earth
(fall)


Michelson interferometer
mirror
M1
A
half-silvered
mirror
coherent
light source
C'
C
B
M2
S
M
mirror
E
detector

Chapter 37 Relativity

Simultaneity and Time Dilation

- **Relativity**: the field of study that measures events: where and when they happen, and by how much any 2 events are separated in space and in time.

- Relativity has to do with transforming such measurements between reference frames that move relative to each other.

- In 1905, Albert Einstein published his *special relativity*. "*special*" means that the theory deals only with inertial frames, in which Newton's laws are valid.

- **Entangled**: Einstein demonstrated that space and time are entangled; that is, the time between 2 events depends on how far apart they occur, and vice versa. Also, the entanglement is different for observers who move relative to each other.

- One result is that time does not pass at a fixed rate, instead, that rate is adjustable: Relative motion can change the rate at which time passes.

- Any engineer involved with the Global Positioning System (GPS) must routinely use relativity (both special relativity and *general relativity*) to determine the rate at which time passes on the satellites because that rate differs from the rate on Earth's surface. If the engineers failed to take relativity into account, GPS would become almost useless in less than one day.

The Postulates

- Examine the 2 postulates of relativity, on which Einstein’s theory is based:

1. The Relativity Postulate: The laws of physics are the same for observers in all inertial reference frames. No one frame is preferred over any other.

- Galileo assumed that the laws of *mechanics* were the same in all inertial frames. Einstein extended that idea to include *all* the laws of physics, especially those of electromagnetism and optics.

- This postulate does not say that the measured values of all physical quantities are the same for all inertial observers; It is the laws of physics, which relate these measurements to one another, that are the same.

2. The Speed of Light Postulate: The speed of light in vacuum has the same value c in all directions and in all inertial reference frames.

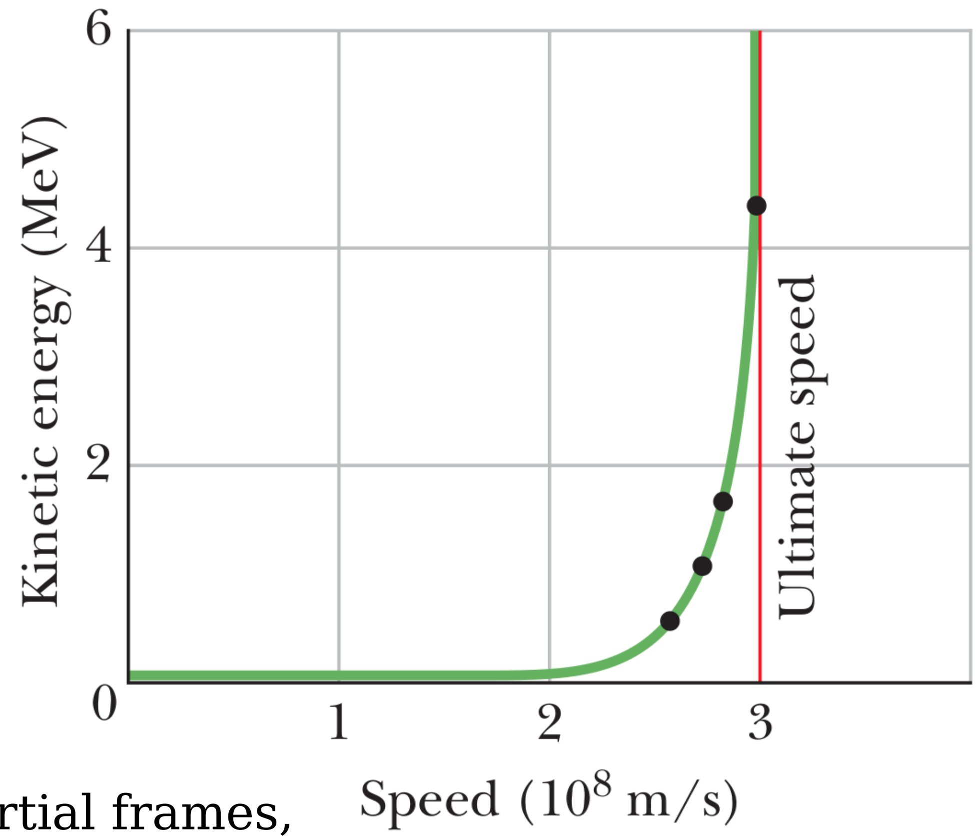
- So there is in nature an *ultimate speed* c, the same in all directions and in all inertial reference frames. Light happens to travel at this ultimate speed.

- No entity that carries energy or information can exceed this limit. Moreover, no particle that has mass can actually reach speed c, no matter how much or for how long that particle is accelerated.

The Ultimate Speed

- The existence of a limit to the speed of accelerated electrons was shown in a 1964 experiment. As the force on a fast electron is increased, the electron's kinetic energy increases toward very large values but its speed does not increase appreciably.

- This ultimate speed has been defined to be exactly $c = 299792458 \text{ m/s}$



Testing the Speed of Light Postulate

- If the speed of light is the same in all inertial frames, then the speed of light emitted by a source moving relative to a lab should be the same as the speed of light emitted by a source at rest in the lab.

- The light source was the neutral pion π^0, an unstable, short-lived particle that can be produce. It decays into 2 gamma rays by the process $\pi^0 \rightarrow \gamma + \gamma$

- In 1964, physicists at CERN generated a beam of π^0's moving at a speed of $0.99975c$ to the lab. The experimenters measured the speed of the gamma rays emitted from these very rapidly moving sources. They found that the speed of the light emitted by the π^0's was the same as it would be if the π^0's were at rest in the lab, namely c.

Coordinate	Value
x	3.58 m
y	1.29 m
z	0 m
t	34.5 s

Measuring an Event

- An **event** is something that happens, and every event can be assigned 3 space coordinates and 1 time coordinate: (1) turn on/off a light (2) an explosion (3) the collision of 2 particles (4) light passes a specific point (5) etc

- Because space & time are entangled with each other in relativity, we can describe these coordinates collectively as *spacetime* coordinates.

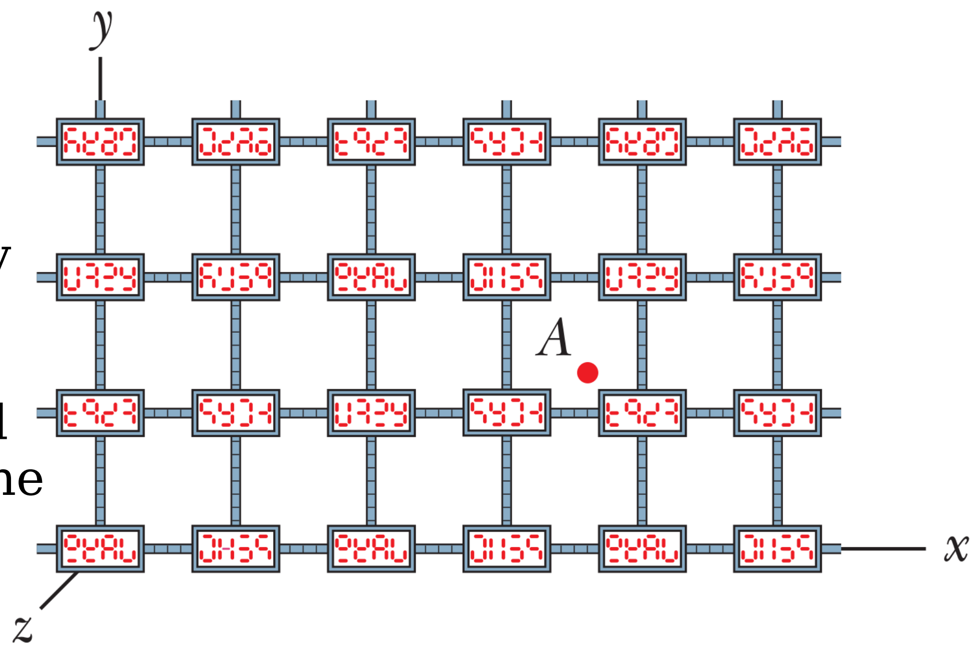
- In general, different observers will assign different spacetime coordinates to the same event.

- **Travel Times**: suppose a balloon bursts 1 km to your right while a firecracker pops 2 km to your left, both at 9 am. Because light from the firecracker pop has farther to go, it arrives at your eyes later than does light from the balloon burst, and thus the pop will seem to have occurred later than the burst.

- To sort out the actual times, we construct an imaginary array of measuring rods & clocks throughout the observer's inertial frame. This construction allows us to find the coordinates, as follows:

1. The Space Coordinates: Imagine the observer's coordinate system fitted with a close-packed, 3D array of measuring rods, one set of rods parallel to each of the 3 coordinate axes. These rods provide a way to determine coordinates along the axes. The observer needs only read the 3 space coordinates to locate the position of an event.

2. The Time Coordinate: Imagine every point of intersection in the array of rods includes a clock, illuminated by the light by the event.



- The array of clocks must be synchronized properly. We don't know whether moving the clocks will change their rates. (Actually, it will.) We must put the clocks in place and then synchronize them.

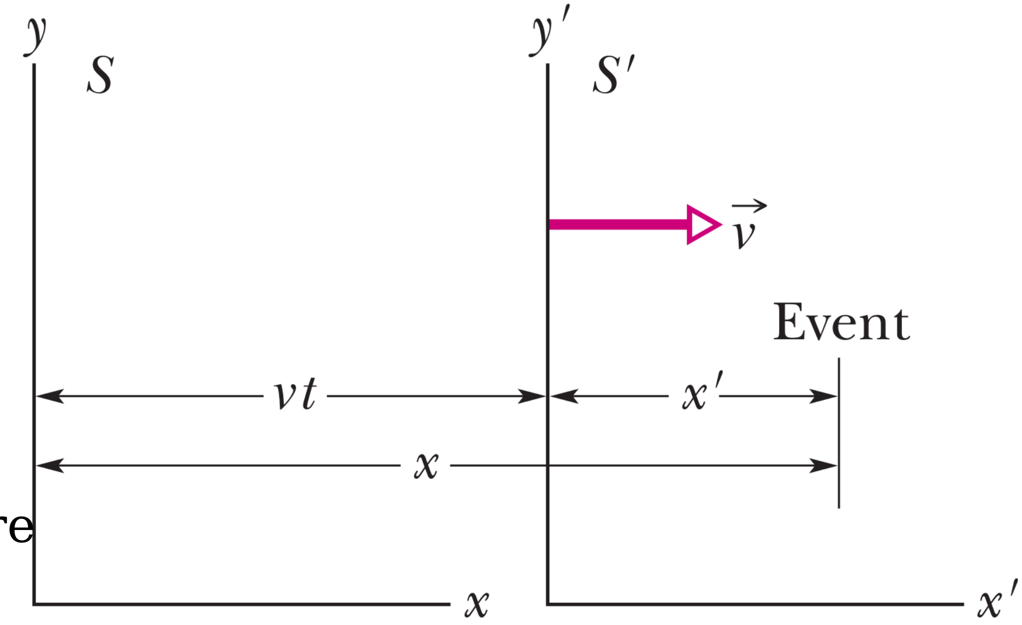
- We choose light to send out our synchronizing signals because light travels at the greatest possible speed, the limiting speed c.

- The observer sends out a pulse of light when the origin clock reads $t=0$. When the light pulse reaches the location of a clock, it is set to $t=r/c$.

3. The Spacetime Coordinates: The observer can now assign spacetime coordinates to an event by simply recording the time on the clock nearest the event and the position as measured on the nearest measuring rods.

- If there are 2 events, the observer computes their time separation as the time difference on clocks near each and their space separation from the coordinate differences on rods near each. We thus avoid the practical problem of calculating the travel times of the signals to the observer from the events.

The Lorentz Transformation



- We claim that the y and z coordinates, which are perpendicular to the motion, are not affected by the motion

$$y' = y, \quad z' = z$$

Galilean Transformation Equations

- Before Einstein, the 4 coordinates were related by the *Galilean transformation equations*:

$$x' = x - vt$$
$$t' = t$$

Galilean transformation equations, approximately valid at low speeds.

- The 2nd equation effectively claims that time passes at the same rate for observers in both reference frames.

The Lorentz Transformation Equations

- The equations above work well when v is small compared to c, but they are incorrect when v is greater than about $0.1\,c$.

- The equations that are correct for any physically possible speed are called the **Lorentz transformation equations**.

- The equations can be derived from the postulates of relativity, they will be showed in an auxiliary page.

$$c = c' \quad \Rightarrow \quad 0 = c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 - (y_2 - y_1)^2 - (z_2 - z_1)^2$$
$$= c^2(t'_2 - t'_1)^2 - (x'_2 - x'_1)^2 - (y'_2 - y'_1)^2 - (z'_2 - z'_1)^2 \quad \text{for light}$$

$\Rightarrow$ define an (infinitesimal) interval $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$ for 2 events

$$\Rightarrow \quad ds^2 = ds'^2 \quad \Leftarrow \quad ds'^2 = a(v)\, ds^2, \quad ds^2 = a(v)\, ds'^2 \quad \Rightarrow \quad a = \pm 1 \quad \Rightarrow \quad a = 1$$

Moves along x-axis $\Rightarrow$ $dy = dy'$, $dz = dz'$ $\Rightarrow$ $c^2 dt^2 - dx^2 = c^2 dt'^2 - dx'^2$

$$\begin{cases} dx' = A(dx - v\, dt) \\ dt' = B\, dt + D\, dx \end{cases} \text{generalized Galilean transformation} \quad \Rightarrow \quad \begin{matrix} A = B = \pm\gamma \\ D = -\dfrac{\beta}{c} B \end{matrix} \quad \Leftarrow \quad ds^2 = ds'^2$$

Choose **+** for v approaching 0 continuously.

$$\Rightarrow \quad \begin{cases} c\, dt' = \gamma(c\, dt - \beta\, dx) \\ dx' = \gamma(dx - \beta c\, dt) \\ dy' = dy \\ dz' = dz \end{cases} \quad \Leftarrow \quad \beta = \frac{v}{c}, \quad \gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\mathrm{d}s^2 = \mathrm{d}s'^2 \quad \Rightarrow \quad c^2\,\mathrm{d}t^2 - \mathrm{d}x^2 = c^2\,\mathrm{d}t'^2 - \mathrm{d}x'^2 \quad \Leftarrow \quad \mathrm{d}y = \mathrm{d}y', \quad \mathrm{d}z = \mathrm{d}z'$$

$$\Rightarrow \quad c^2\,\mathrm{d}t^2 - \mathrm{d}x^2 = c^2\left(B\,\mathrm{d}t + D\,\mathrm{d}x\right)^2 - A^2\left(\mathrm{d}x - v\,\mathrm{d}t\right)^2$$
$$= \left(c^2 B^2 - A^2 v^2\right)\mathrm{d}t^2 + 2\left(c^2 B D + A^2 v\right)\mathrm{d}t\,\mathrm{d}x - \left(A^2 - c^2 D^2\right)\mathrm{d}x^2$$

$$\Rightarrow \quad \begin{array}{c} B^2 - \beta^2 A^2 = 1 \\ c\,B\,D + A^2\beta = 0 \\ A^2 - c^2 D^2 = 1 \end{array} \quad \Rightarrow \quad A^2 = -\frac{c}{\beta} B\,D \quad \Rightarrow \quad \begin{array}{c} B\left(B + c\,\beta\,D\right) = 1 \\ -\frac{c}{\beta} D\left(B + c\,\beta\,D\right) = 1 \end{array}$$

$$\Rightarrow \quad D = -\frac{\beta}{c} B \quad \Rightarrow \quad B^2\left(1 - \beta^2\right) = 1 \quad \Rightarrow \quad B = \pm\gamma \quad \Rightarrow \quad A^2 = B^2 \quad \Rightarrow \quad A = \pm\gamma$$

- $$\begin{aligned} x' &= \gamma(x - \beta ct) \\ y' &= y \\ z' &= z \\ ct' &= \gamma(ct - \beta x) \end{aligned}$$ Lorentz transformation equations, valid at all physically possible speeds.

- This entanglement of space & time are showed in the 1st and the last equations.

- It is a formal requirement of relativistic equations that they should reduce to classical equations if we let $c \to \infty$. So, if c were infinitely great, all finite speeds would be "low" and classical equations would never fail.

- From the equations above, we have $$\begin{aligned} x &= \gamma(x' + \beta ct') \\ ct &= \gamma(ct' + \beta x') \end{aligned}$$

- If the S' frame has a positive velocity relative to an observer in the S frame, then the S frame has a negative velocity relative to an observer in the S' frame.

- To be more general than $t=t'=0$, let's rewrite the Lorentz transformations in terms of any pair of events 1 & 2,

$$\Delta x = x_2 - x_1 \quad \& \quad \Delta t = t_2 - t_1 \quad \text{in } S$$
$$\Delta x' = x'_2 - x'_1 \quad \& \quad \Delta t' = t'_2 - t'_1 \quad \text{in } S'$$

$$\Rightarrow \quad \begin{array}{l} 1.\quad \Delta x = \gamma(\Delta x' + \beta c \Delta t') \\ 2.\quad c\Delta t = \gamma(c\Delta t' + \beta \Delta x') \end{array} \quad \& \quad \begin{array}{l} 1'.\quad \Delta x' = \gamma(\Delta x - \beta c \Delta t) \\ 2'.\quad c\Delta t' = \gamma(c\Delta t - \beta \Delta x) \end{array}$$

Some Consequences of the Lorentz Equations

Simultaneity

- Since $\Delta t = \gamma(\Delta t' + v\Delta x'/c^2)$, if 2 events occur at different places in S', $\Delta x' \neq 0$. It follows that even if the events are simultaneous in S' ($\Delta t' = 0$), they will not be simultaneous in S. In S

$$\Delta t = \gamma \frac{v}{c^2} \Delta x' \quad \text{simultaneous events in } S'$$

- The spatial separation $\Delta x'$ guarantees a temporal separation Δt.

Time Dilation

- If 2 events occur at the same place in S' ($\Delta x' = 0$) but at different times ($\Delta t' \neq 0$)

$$\Delta t = \gamma \Delta t' \quad \text{events in same places in } S'$$

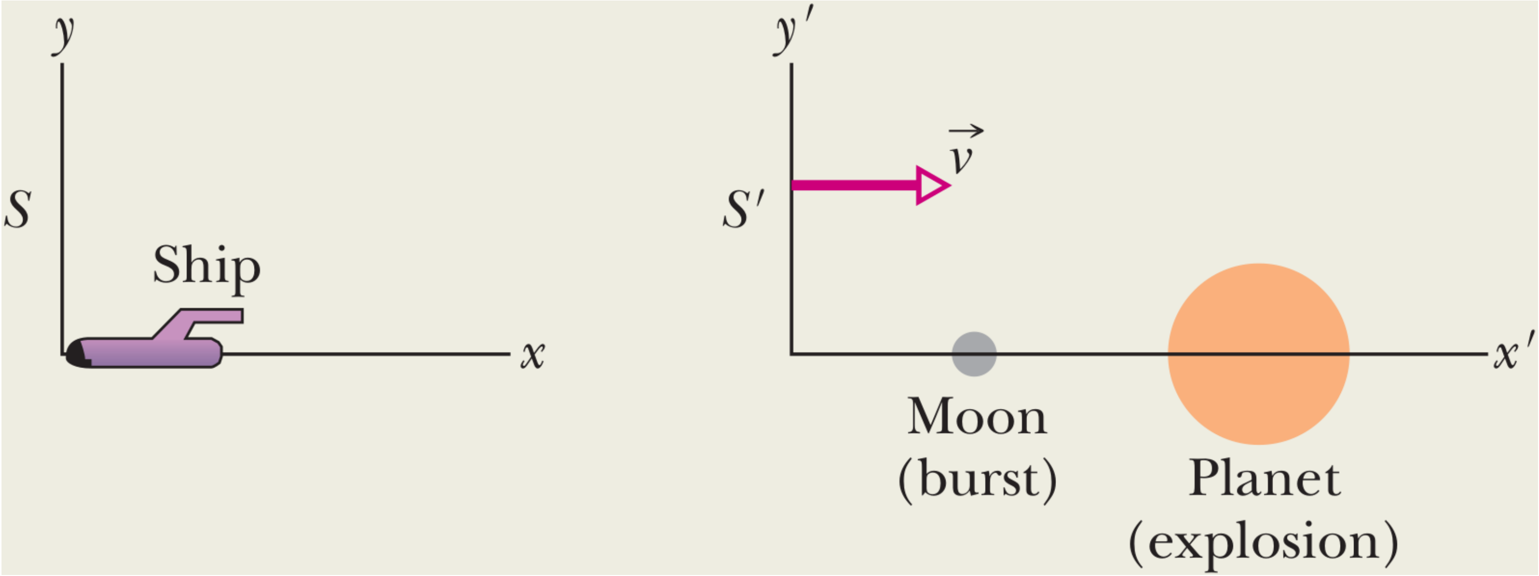
$$\Rightarrow \quad \Delta t = \gamma \Delta t_0 \quad \text{time dilation}$$

Length Contraction

- Since $\Delta x' = \gamma(\Delta x - v\Delta t)$, if a rod lies parallel to the x and x' axes and is at rest in S', and $\Delta x'$ is the proper length L_0 of the rod . For the rod moving in S, Δx can be identified as the length L of the rod in S only if the coordinates of the rod's end points are measured *simultaneously* ($\Delta t = 0$)

$$\Rightarrow \quad L = \frac{L_0}{\gamma} \quad \text{length contraction}$$

Problem 37.5



The Relativity of Simultaneity

- Suppose that Sam notes that 2 events occur at the same time. Suppose also that Sally, who is moving at a constant velocity $\vec{v}$ with respect to Sam, also records these same 2 events. Sally will find that they occur at different time.

If 2 observers are in relative motion, they will not, in general, agree as to whether 2 events are simultaneous. If one observer finds them to be simultaneous, the other generally will not.

- Their observations are equally valid, no reason to favor one over the other.

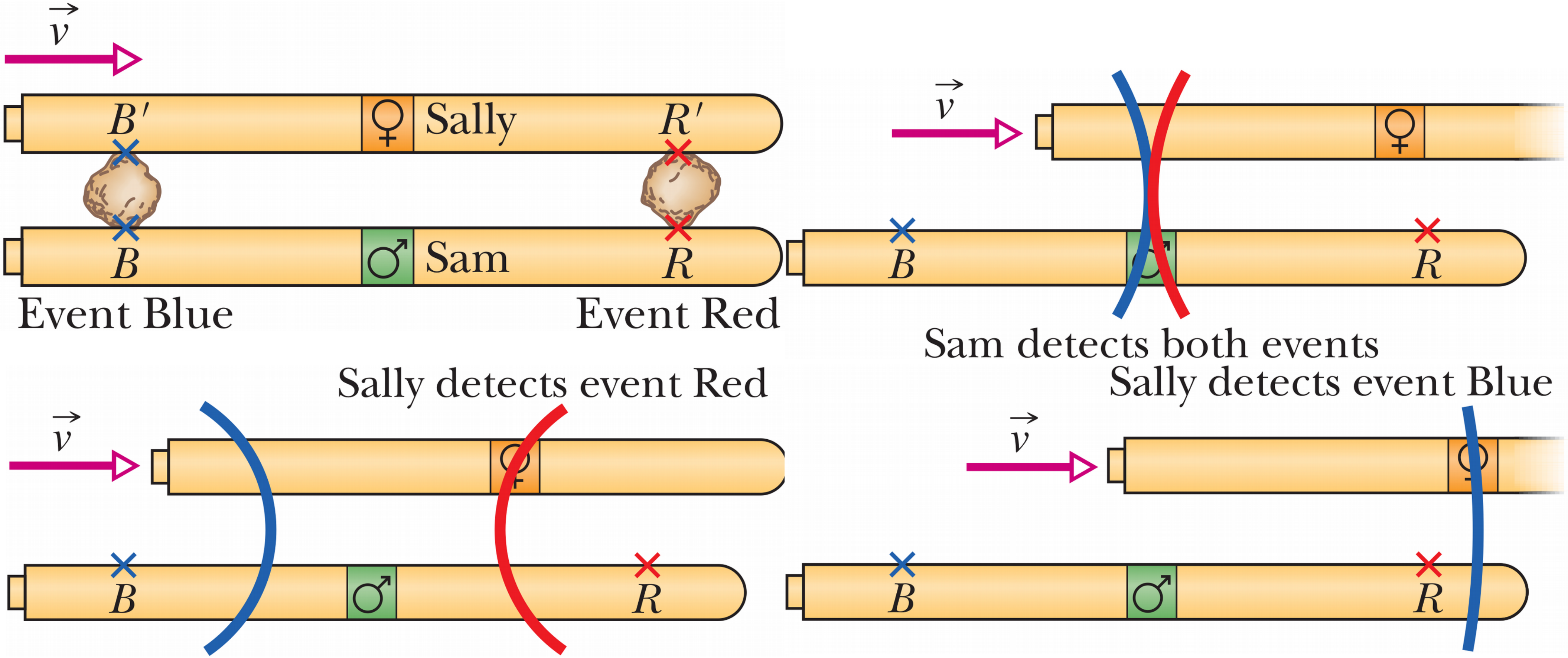
Simultaneity is not an absolute concept but rather a relative one, depending on the motion of the observer.

- If the relative speed of the observers is much less than the speed of light, then measured departures from simultaneity are small that they are not noticeable.

A Closer Look at Simultaneity

- Suppose that the wavefronts from the 2 events happen to reach Sam at the same time. And Sam finds, by measurement, that he was stationed exactly spatially halfway between the 2 events.

- Sam thinks that Red’s light & Blue’s reached him at the same time, and he was halfway between the 2 sources. So these 2 events were simultaneous events.



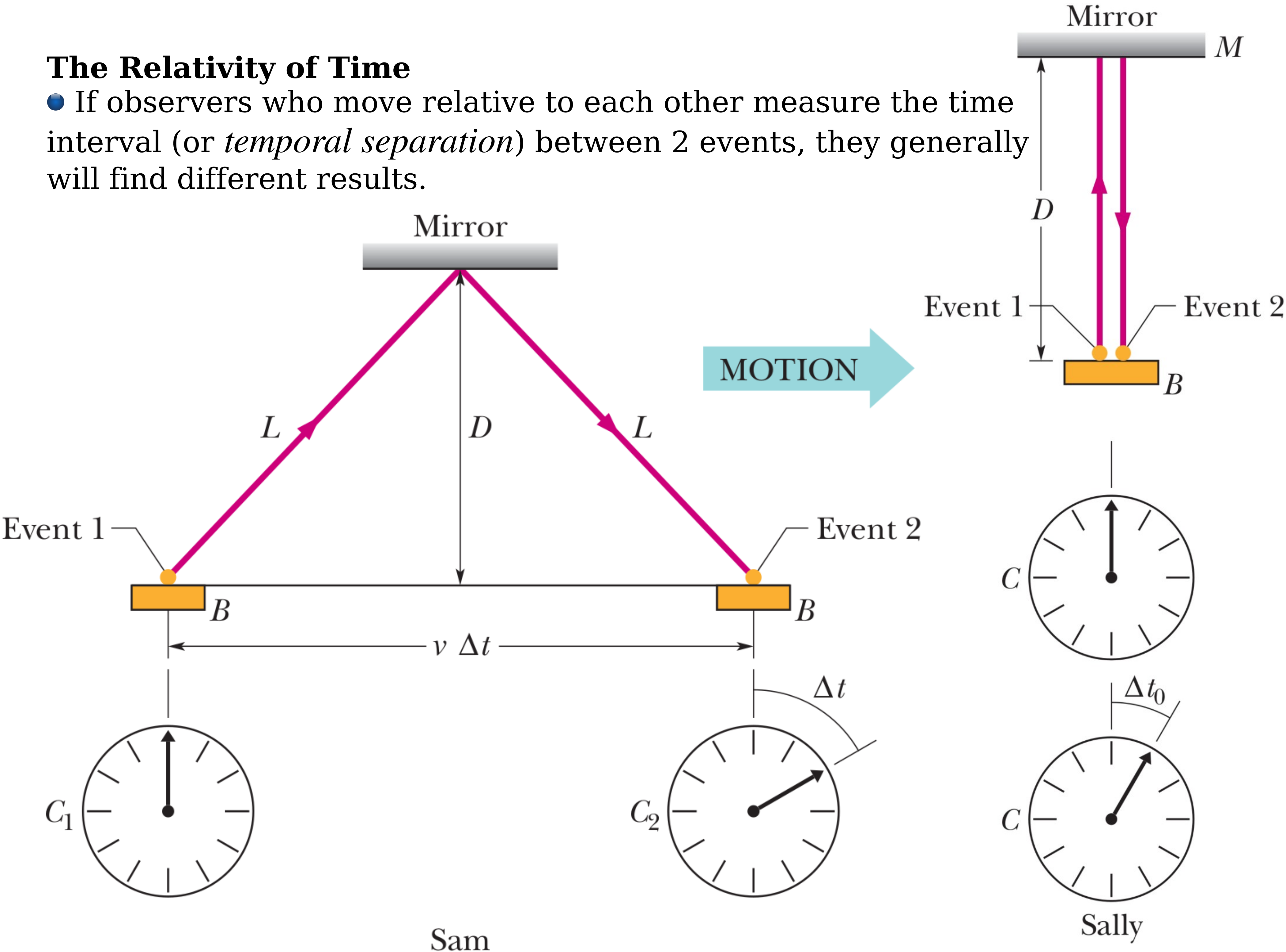
- Sally and Red's wavefront are moving *toward* each other, while she and Blue's wavefront are moving in the *same direction*. Thus, Red's wavefront reaches Sally *before* Blue's wavefront does.

- Sally thinks that Red's light reached her before Blue's light did. She found that she too was halfway between the 2 sources. So the events were not simultaneous.

- These reports do not agree. Nevertheless, *both* observers are correct. Note that there is only one wavefront from the site of each event and that *this wavefront travels with the same speed c in both reference frames*, as Einstein states.

The Relativity of Time

- If observers who move relative to each other measure the time interval (or *temporal separation*) between 2 events, they generally will find different results.



The time interval between 2 events depends on how far apart they occur in both space and time; ie, their spatial and temporal separations are entangled.

- we discuss this entanglement by means of an example in a crucial way: *To one of 2 observers, the 2 events occur at the same location*.
- The time interval by Sally: $\Delta t_0 = \frac{2D}{c}$ Sally
- The time interval by Sam: $\Delta t = \frac{2L}{c}$ Sam $\Leftarrow$ $L = \sqrt{\left(\frac{v \Delta t}{2}\right)^2 + D^2}$

$$\Rightarrow \quad L = \sqrt{\left(\frac{v \Delta t}{2}\right)^2 + \left(\frac{c \Delta t_0}{2}\right)^2} \quad \Rightarrow \quad \Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

- $v \leq c \quad \Rightarrow \quad \sqrt{1 - \frac{v^2}{c^2}} \leq 1 \quad \Rightarrow \quad \Delta t \geq \Delta t_0$
- Sam measures a greater time interval between the 2 events than does Sally.
- Sam and Sally have measured the time interval between the same 2 events, but the relative motion between Sam and Sally made their measurements different.
- Relative motion can change the rate at which time passes between 2 events; the key to this effect is that the speed of light is the same for both observers.

When 2 events occur at the same location in an inertial reference frame, the time interval between them, measured in that frame, is called the **proper time interval** or the **proper time**. Measurements of the same time interval from any other inertial reference frame are always greater.

- Sally measures a proper time interval, Sam measures a greater time interval.

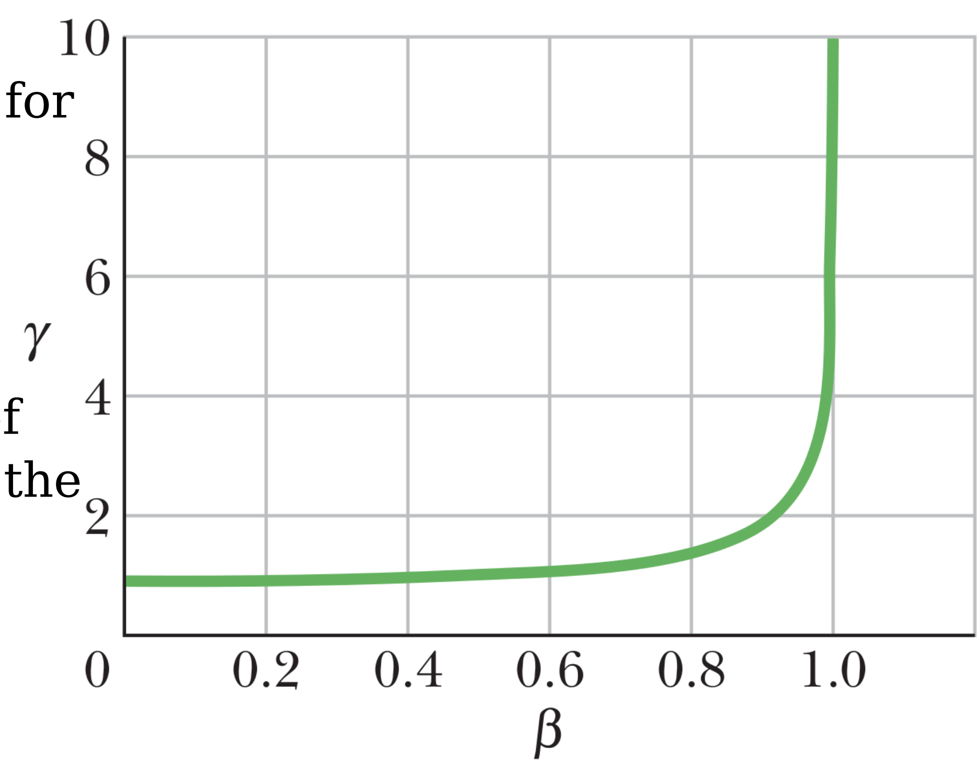
- The amount by which a measured time interval is greater than the proper time interval is called **time dilation**.

- speed parameter $\beta \equiv \frac{v}{c} \le 1$, Lorentz factor $\gamma \equiv \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-v^2/c^2}} \ge 1$

$\Rightarrow$ $\Delta t = \gamma \, \Delta t_0$ time dilation

- Newton's mechanics works well enough for $v<0.1c$, but we must use special relativity for greater values of v.

- From Sally's viewpoint, Sam failed to synchronize his clocks C_1 and C_2 in spite of his insistence that he did. And that is why the time interval he measured between the 2 events was greater than the interval she measured.



2 Tests of Time Dilation

1. Microscopic Clocks: The *lifetime* of a subatomic particle muon in a rest frame is $\Delta t_0 = 2.2\ \mu\text{s}$, a proper time interval.

- If the muons are moving wrt a lab, their lifetimes observed by the lab should yield a greater lifetime (a dilated lifetime).

- For $\beta = 0.9994$, $\gamma \equiv \dfrac{1}{\sqrt{1-\beta^2}} = 28.87 \quad \Rightarrow \quad \Delta t = \gamma\, \Delta t_0 = 63.51\ \mu\text{s}$

2. Macroscopic Clocks: In1971, J. Hafele and R. Keating flew 4 atomic clocks twice around the world, once in each direction, to test special relativity with macroscopic clocks. They verified the predictions of the theory to within 10%. Nowadays the accuracy is within 1%.

- Today, when atomic clocks are transported from one place to another for some purposes, the time dilation caused by their motion is always taken into account.

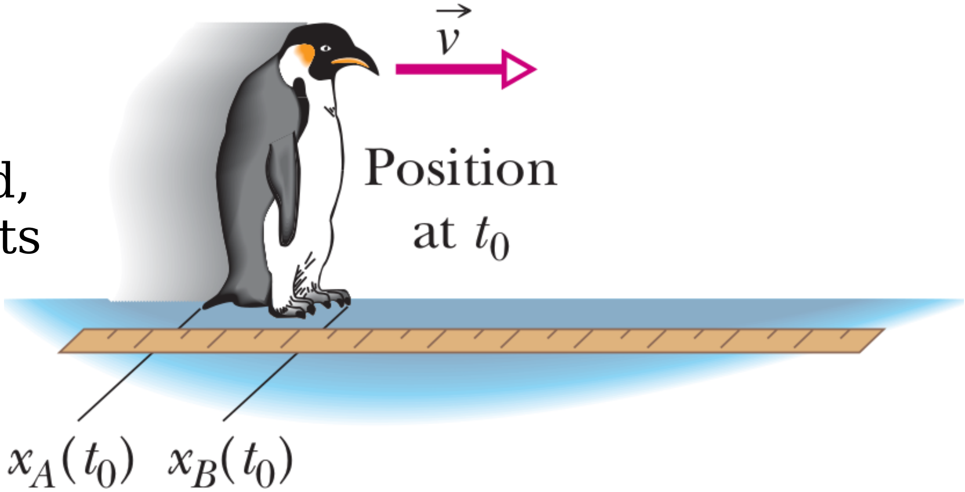
Problem 37.1

Problem 37.2

The Relativity of Length

- If you measure the length of a moving rod, you must note the positions of the end points simultaneously.

- Because simultaneity is relative, length should also be a relative quantity.

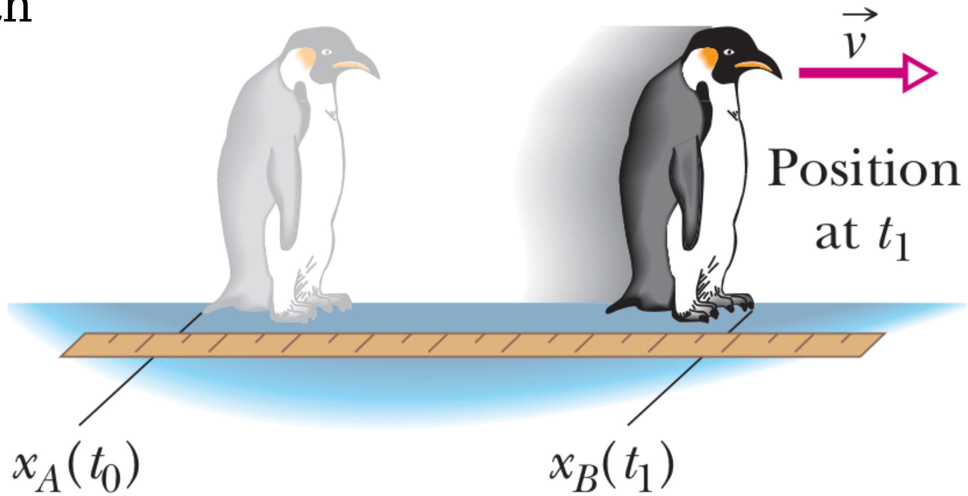


- If there is relative motion at v between you and the rod along the length of the rod, with simultaneous measurements its length L is

$$L = L_0\sqrt{1-\beta^2} = \frac{L_0}{\gamma} \quad \text{length contraction}$$

L_0 is the length of a rod at rest.

- $\gamma \geq 1 \quad \Rightarrow \quad L \leq L_0$



- The relative motion causes a *length contraction*, and L is called a *contracted length*. A greater speed v results in a greater contraction.

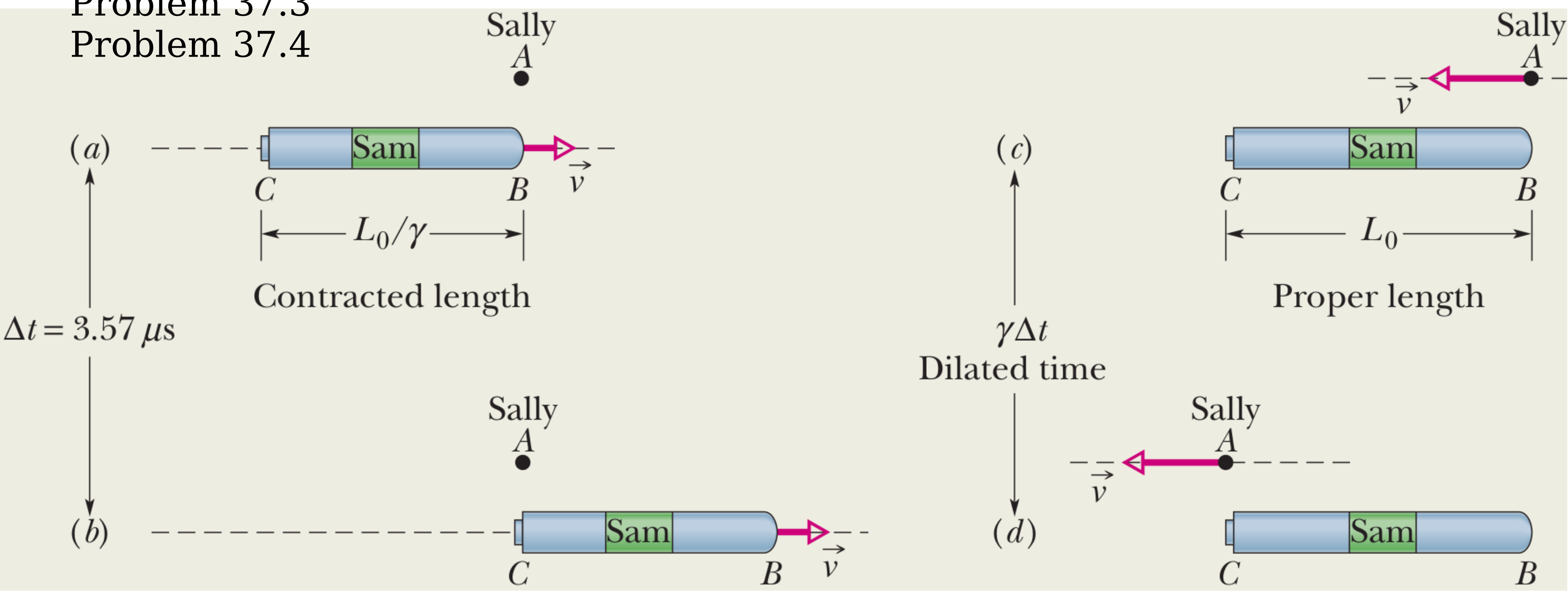
The length L_0 of an object measured in the rest frame of the object is its **proper length/rest length**. Measurements of the length from any reference frame that is in relative motion parallel to the length are always less than the proper length.

- When you measure a contracted length for a moving rod, to the rod you did not locate the 2 ends of the rod simultaneously. To the rod, you first located the rod's front end and then, slightly later, its rear end, and that is why you measured a length less than the proper length.

Proof: Sam, at rest with a rod, get its proper length L_0. Sally moves through the rod in a time $\Delta t \Rightarrow L_0 = v\Delta t$, from Sam. Δt is not a proper time interval because the 2 events occur at 2 different places. For Sally, she finds that the 2 events measured by Sam occur *at the same place*. and she measures the interval Δt_0 as a proper time interval $\Rightarrow L = v\Delta t_0$ from Sally.

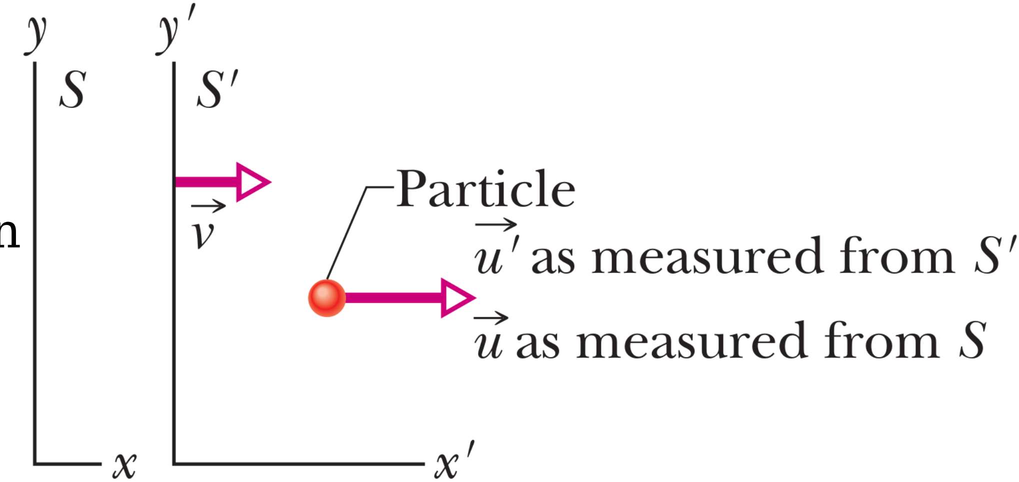
$$\Rightarrow \frac{L}{L_0} = \frac{v\,\Delta t_0}{v\,\Delta t} = \frac{1}{\gamma} \quad \Rightarrow \quad L = \frac{L_0}{\gamma}$$

Problem 37.3
Problem 37.4



The Relativity of Velocities

- Use the Lorentz transformation to compare the velocities that 2 observers in different inertial reference frames would measure for the same moving particle.



- Let the particle move with constant velocity parallel to the x & x' axes, send out 2 signals as it moves. Each observer measures the space interval and the time interval between these 2 events.

$$\Delta x = \gamma \left(\Delta x' + v \Delta t' \right)$$
$$\Delta t = \gamma \left(\Delta t' + \frac{v}{c^2} \Delta x' \right) \Rightarrow \frac{\Delta x}{\Delta t} = \frac{\Delta x' + v \Delta t'}{\Delta t' + v \Delta x' / c^2} = \frac{\Delta x' / \Delta t' + v}{1 + v \left(\Delta x' / \Delta t' \right) / c^2}$$

To the limit

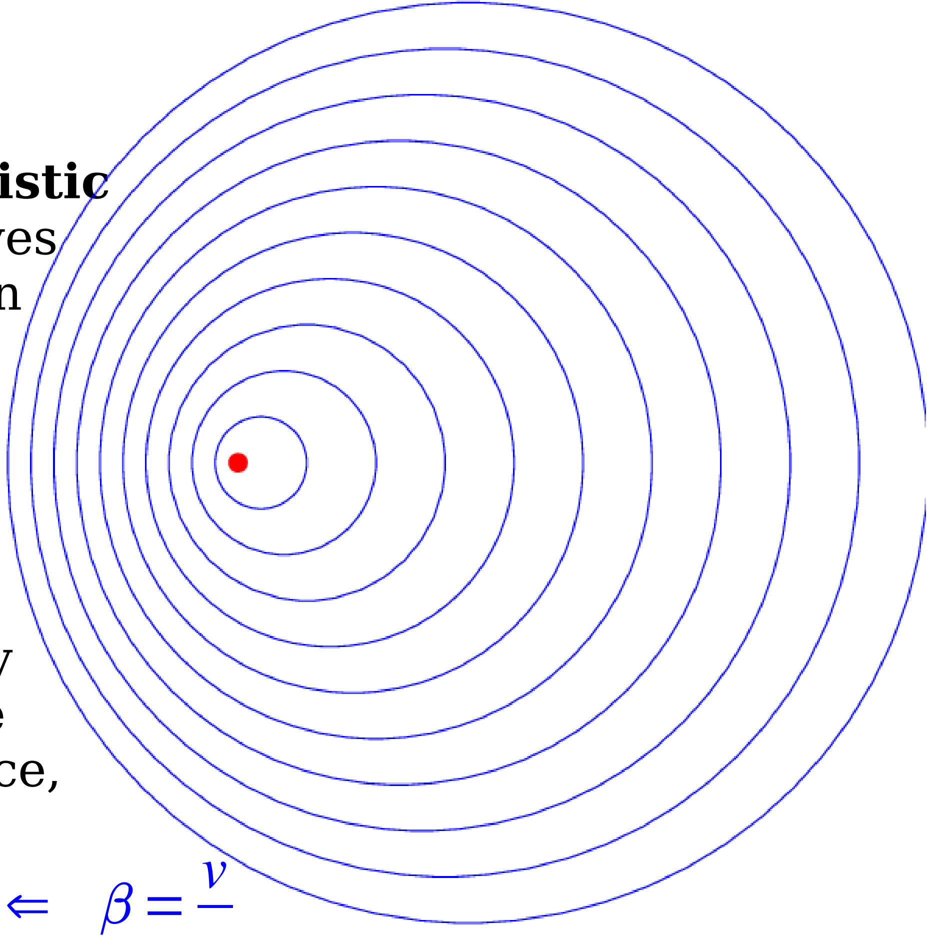
$$u \equiv \frac{dx}{dt} \quad \text{the velocity of the particle in } S$$

$$u' \equiv \frac{dx'}{dt'} \quad \text{the velocity of the particle in } S'$$

$$\Rightarrow \quad u = \frac{u' + v}{1 + u' v / c^2} \quad \text{relativistic velocity transformation}$$

- As $c \to \infty$, it reduces to the classical velocity transformation equation, $u = u' + v$

Doppler Effect for Light



- Different from the sound wave, the **relativistic (longitudinal) Doppler effect** for light waves depends on only the relative velocity between source and detector, as measured from the reference frame of either.

- Let f_0 represent the **proper frequency** of the source, and f represent the frequency detected by an observer moving with velocity $\vec{v}$ relative to that rest frame. Then, when the direction of $\vec{v}$ is directly away from the source,

$$\lambda = (c+v)\,T = c\,(1+\beta)\,\frac{T_0}{\sqrt{1-\beta^2}} = \lambda_0\sqrt{\frac{1+\beta}{1-\beta}} \quad \Leftarrow \quad \beta = \frac{v}{c}$$

$$\Rightarrow \quad f = f_0\sqrt{\frac{1-\beta}{1+\beta}} \quad \text{source and detector separating}$$

$$\Rightarrow \quad \lambda = \lambda_0\sqrt{\frac{1+\beta}{1-\beta}} \quad \text{source and detector separating}$$

- When the direction of $\vec{v}$ is directly toward the source, we must change the signs of β in the equations.

- For an increasing separation, the measured wavelength is greater than the proper wavelength, $\lambda > \lambda_0$. Such a Doppler shift is called as being a *red shift*.

- For a decreasing separation, $\lambda < \lambda_0$, and the Doppler shift is a *blue shift*.

Low-Speed Doppler Effect

- $\beta \ll 1 \quad \Rightarrow \quad f = f_0\left(1 - \beta + \frac{1}{2}\beta^2\right)$

- The low-speed equation for the Doppler effect with sound waves has the same first 2 terms but a different coefficient in the 3rd term. Thus, the relativistic effect for low-speed light sources and detectors shows up only with the β^2 term.

- A police radar uses the Doppler effect with microwaves to measure the speed of a car. The radar emits a microwave at f_0 along the road. A car toward the radar intercepts the beam but at a frequency blue-shifted by the Doppler effect due to the car's motion toward the radar. The car reflects the beam back toward the radar. Because the car is toward the radar, the detector intercepts a reflected beam that is further blue-shifted. The radar compares that detected frequency with f_0 and computes the speed of the car.

Astronomical Doppler Effect

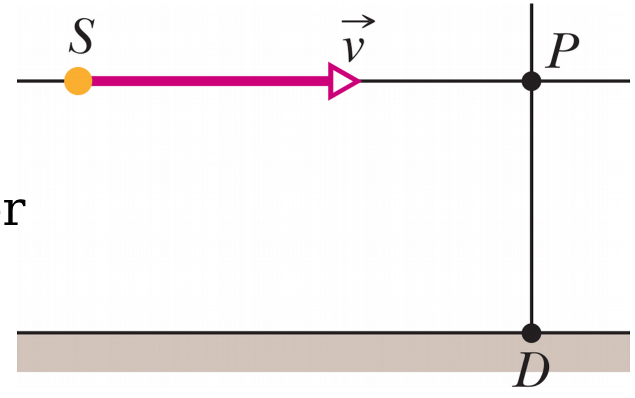
- We can determine how fast stars/galaxies are moving, either away from/toward us, by measuring the Doppler shift of the known frequency light that reaches us.

- This Doppler shift is due only to the radial motion of the star, and the speed we can determine by measuring this Doppler shift is only the radial speed of the star —only the radial component of the star's velocity relative to us.

- If a star moves away from us with a slow radial speed

$$f \approx f_0(1-\beta) \;\Rightarrow\; \lambda \approx \frac{\lambda_0}{1-\beta} \approx \lambda_0(1+\beta) \;\Rightarrow\; \beta \approx \frac{\lambda-\lambda_0}{\lambda_0}$$

$$\Rightarrow \boxed{v = \frac{\Delta\lambda}{\lambda_0}c \quad \text{radial speed of light source } v \ll c} \Leftarrow \quad \Delta\lambda \;\text{ wavelength Doppler shift}$$



Transverse Doppler Effect

- When S reaches point P, S is moving neither toward nor away from D.

- However, if the source is emitting light waves,there is still a Doppler effect, called the **transverse Doppler effect**.

- No longitudinal motion, so only time dilation

$$\Rightarrow \quad f = \frac{f_0}{\gamma} = f_0\sqrt{1-\beta^2} \;\text{ transverse Doppler effect } \Rightarrow \; f \approx f_0\left(1-\frac{\beta^2}{2}\right) \;\text{ low speed}$$

- The 1st term is expected for sound waves, and the relativistic effect for low-speed light sources and detectors appears with the β^2 term.

- For the speed being too slow like a car, the relativistic term $\beta^2/2$ in the transverse Doppler effect is extremely small. Thus, $f \approx f_0$ and the radar unit computes a speed of 0.

- The transverse Doppler effect is really another test of time dilation:

$$T = \frac{1}{f} \quad \Rightarrow \quad T = \frac{T_0}{\sqrt{1-\beta^2}} = \gamma T_0 \quad \Leftarrow \quad T_0 \text{ proper period}$$

Momentum and Energy

A New Look at Momentum

- $p = m\,v = m\,\dfrac{\mathrm{d}\,x}{\mathrm{d}\,t}$ classical momentum
- To find a relativistic expression for momentum, we start with the new definition

$$p = m\,\frac{\mathrm{d}\,x}{\mathrm{d}\,t_0}$$

$\mathrm{d}x$: the distance measured by an observer watching that particle;

$\mathrm{d}t_0$: the time measured not by the observer watching the moving particle but by an observer moving with the particle — comoving observer & proper time.

- $\mathrm{d}\,t = \gamma\,\mathrm{d}\,t_0 \;\Rightarrow\; p = m\,\dfrac{\mathrm{d}\,x}{\mathrm{d}\,t_0} = m\,\dfrac{\mathrm{d}\,x}{\mathrm{d}\,t}\,\dfrac{\mathrm{d}\,t}{\mathrm{d}\,t_0} = m\,\gamma\,\dfrac{\mathrm{d}\,x}{\mathrm{d}\,t} \;\Rightarrow\; p = \gamma\,m\,v$ momentum
- Relativistic momentum approaches ∞ as v approaches c.
- $\vec{p} = \gamma\,m\,\vec{v}$ momentum
- For $v \ll c$, it reduces to the classical definition of momentum $\vec{p} = m\,\vec{v}$

Mass Energy

- In 1905, Einstein showed that mass can be considered to be another form of energy. Thus, the law of conservation of energy is really the law of conservation of mass – energy.

- In a *chemical reaction*, the amount of mass transferred into other forms of energy is tiny. So mass and energy *seem* to be separately conserved. In a *nuclear reaction*, the energy released is often about a million times greater than in a chemical reaction, and the change in mass can easily be measured.

- $E_0 = m c^2$

- This energy associated with the mass of an object is called **mass energy/rest energy**. E_0 is an energy that the object has even when it is at rest, simply because it has mass.

Object	Mass(kg)	Energy Equivalent	
Electron	$\approx 9.11 \times 10^{-31}$	$\approx 8.19 \times 10^{-14}$ J	$(\approx 511$ keV$)$
Proton	$\approx 1.67 \times 10^{-27}$	$\approx 1.50 \times 10^{-10}$ J	$(\approx 938$ MeV$)$
Uranium atom	$\approx 3.95 \times 10^{-25}$	$\approx 3.55 \times 10^{-8}$ J	$(\approx 225$ GeV$)$
Dust particle	$\approx 1 \times 10^{-13}$	$\approx 1 \times 10^{4}$ J	$(\approx 2$ kcal$)$
U.S. penny	$\approx 3.1 \times 10^{-3}$	$\approx 2.8 \times 10^{14}$ J	$(\approx 78$ GW$\cdot$h$)$

- In this aspect, masses are usually measured in atomic mass units, and energies are usually measured in electron-volts,

$$1\ \text{u} = 1.66053886 \times 10^{-27}\ \text{kg}\ , \quad 1\ \text{eV} = 1.602176462 \times 10^{-19}\ \text{J}$$

$$\Rightarrow \quad c^2 = 9.31494013 \times 10^{8}\ \text{eV/u} = 9.31494013 \times 10^{5}\ \text{keV/u} = 931.494013\ \text{MeV/u}$$

Total Energy

- If the object is moving, it has additional energy in the form of kinetic energy. Assuming its potential energy is 0, then its total energy is

$$E = E_0 + K = mc^2 + K \quad (1)$$

- Given without proof, the total energy can also be expressed as

$$E = \gamma mc^2 \quad (2)$$

- The law of conservation of total energy still applies when the mass energy is included and the changes in mass energy are significant:

The total energy E of an *isolated system* cannot change.

- **Q Value**: When undergoing a chemical/nuclear reaction, a change in the total mass energy of the system due to the reaction is often given as a Q value

$$\begin{pmatrix}\text{system's initial} \\ \text{total mass energy}\end{pmatrix} = \begin{pmatrix}\text{system's final} \\ \text{total mass energy}\end{pmatrix} + Q$$

$$\Rightarrow \quad E_{0i} = E_{0f} + Q \quad \Rightarrow \quad M_i c^2 = M_f c^2 + Q \quad \Rightarrow \quad Q = M_i c^2 - M_f c^2 = -\Delta M c^2$$

- If a reaction results in the transfer of energy from mass energy to other form of energy, the system's total mass energy E_0 (and total mass M) decreases and Q is positive. If a reaction requires that energy be transferred to mass energy, the system's total mass energy E_0 (and its total mass M) increases and Q is negative.

- For 2 hydrogen nuclei undergoing a *fusion reaction* (as occurred in Sun)

$$^{1}H + {}^{1}H \rightarrow {}^{2}H + e^{+} + \nu_{e} + \text{energy} \quad \Leftarrow \quad {}^{2}H : \text{Deuterium}$$

- The total mass energy (and total mass) of deuterium, positron, and neutrino is less than the total mass energy (and total mass) of the initial hydrogen nuclei. Thus, $Q > 0$, and energy is said to be *released* by the reaction.

Kinetic Energy

- Classically, $K \approx \frac{1}{2} m v^2$

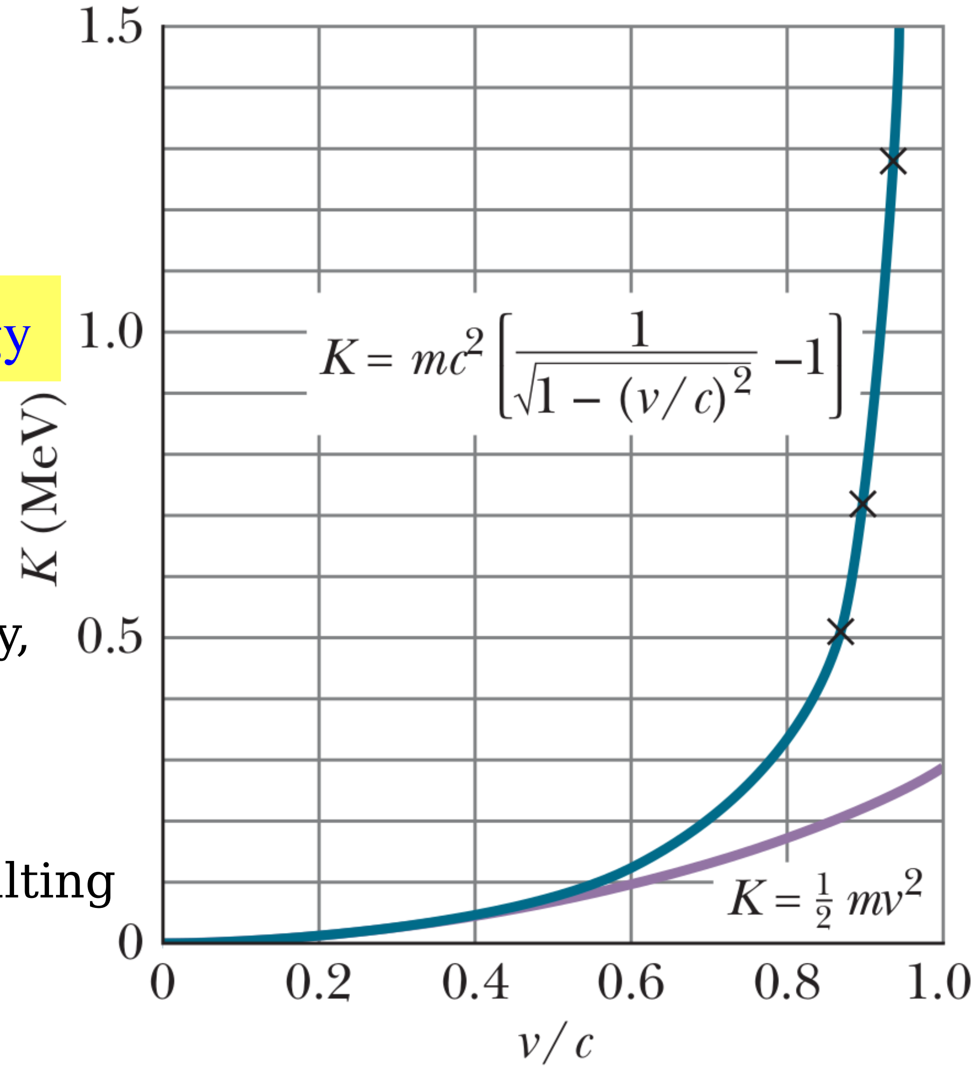
- From (1) & (2)

$$\Rightarrow K = E - m c^2 = m c^2 (\gamma - 1) \quad \text{kinetic energy}$$

$$= m c^2 \left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right) \approx \frac{1}{2} m v^2$$

for $v \ll c$

- The kinetic energy increases dramatically, approaching ∞ as v/c approaches 1.



Work

- The required work W is equal to the resulting change ΔK in the object's kinetic energy.

- If the change is to occur on the high-speed, the required work could be enormous because the kinetic energy increases so rapidly.

- To increase an object's speed to c would require an infinite amount of energy; thus, doing so is impossible.

Momentum and Kinetic Energy

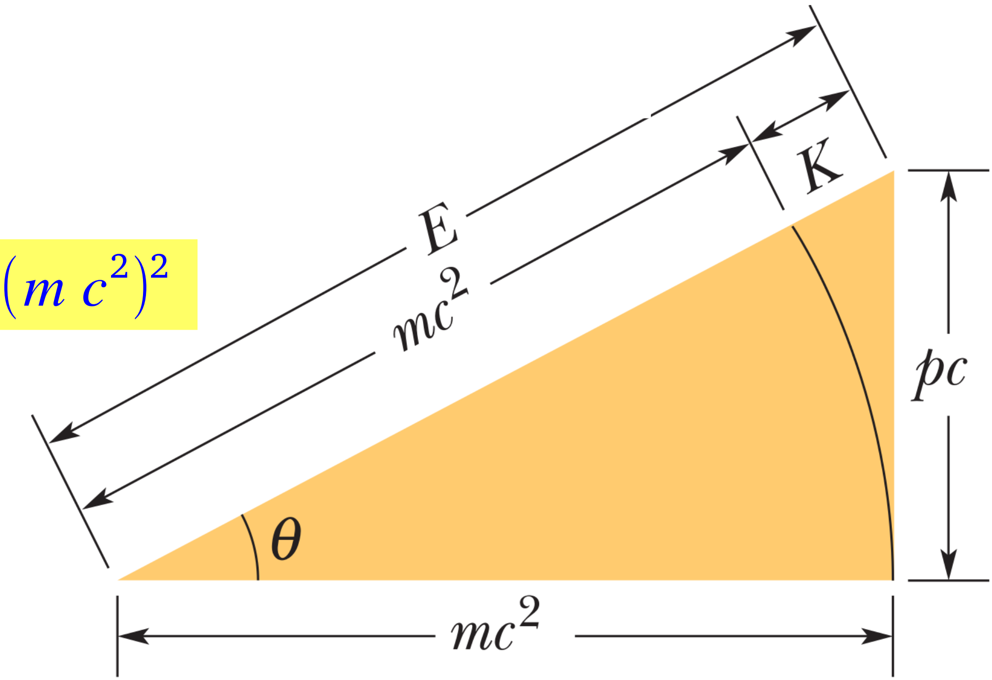
- $p^2 = 2Km$ classically

- With their relativistic definitions

$$(pc)^2 = K^2 + 2Kmc^2 \Rightarrow E^2 = (pc)^2 + (mc^2)^2$$

- The triangle can help you keep these useful relations in mind, where

$$\sin\theta = \beta, \quad \cos\theta = \frac{1}{\gamma}$$



Problem 37.6
Problem 37.7

Selected Problems: 10, 38, 50, 58

$$\vec{p} = \gamma\, m\, \vec{v} \quad \Rightarrow \quad (p\, c)^2 = \gamma^2\, m^2\, v^2\, c^2 \quad \Leftarrow \quad \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$K^2 + 2\, K\, m\, c^2 = [m\, c^2\, (\gamma - 1)]^2 + 2\, m\, c^2\, (\gamma - 1)\, m\, c^2$$

$$= m^2\, c^4\, (\gamma^2 - 2\,\gamma + 1) + 2\, m^2\, c^4\, (\gamma - 1)$$

$$= m^2\, c^4\, (\gamma^2 - 1) = m^2\, c^4 \left(\frac{1}{1 - \beta^2} - 1 \right) \quad \Leftarrow \quad \beta = \frac{v}{c}$$

$$= m^c\, c^4\, \frac{\beta^2}{1 - \beta^2} = m^2\, c^4\, \gamma^2\, \beta^2 = \gamma^2\, m^2\, v^2\, c^2 = (p\, c)^2$$

$$E = \gamma\, m\, c^2 \quad \Rightarrow \quad E^2 = \gamma^2\, m^2\, c^4$$

$$(p\, c)^2 + (m\, c^2)^2 = \gamma^2\, m^2\, c^2\, v^2 + m^2\, c^4 = m^2\, c^4\, (\gamma^2\, \beta^2 + 1)$$

$$= m^2\, c^4 \left(\frac{\beta^2}{1 - \beta^2} + 1 \right) = \gamma^2\, m^2\, c^4 = E^2$$