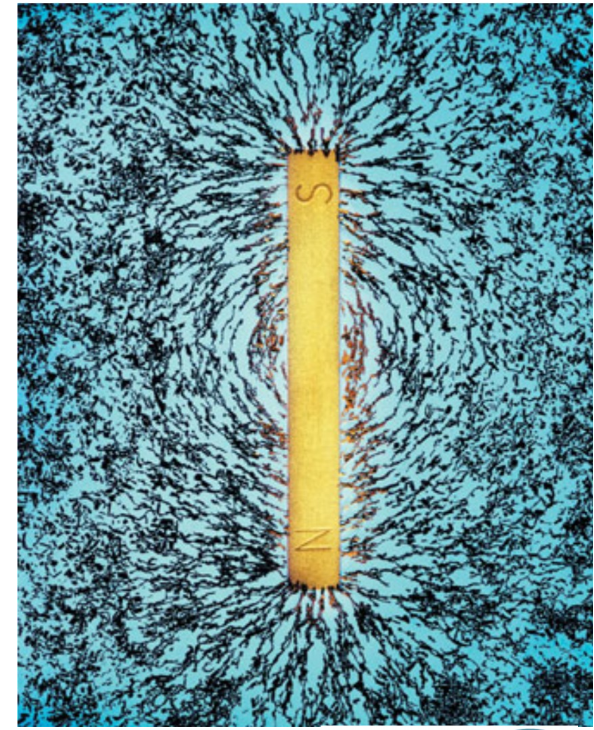
Chapter 32 Maxwell's Equations; Magnetism of Matter

Gauss' Law for Magnetic Fields

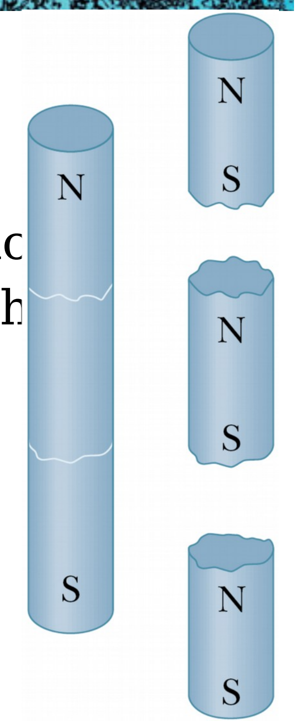


- One end of the magnet is a *source* of the field (the field lines diverge from it) and the other end is a *sink* of the field (the field lines converge toward it).

- We call the source the *north pole* of the magnet and the sink the *south pole*, and we say that the magnet, is an example of a **magnetic dipole**.

- Suppose we break a bar magnet into pieces, each fragment has a north pole and a south pole:

> The simplest magnetic structure that can exist is a magnetic dipole.
> Magnetic monopoles do not exist (as far as we know).

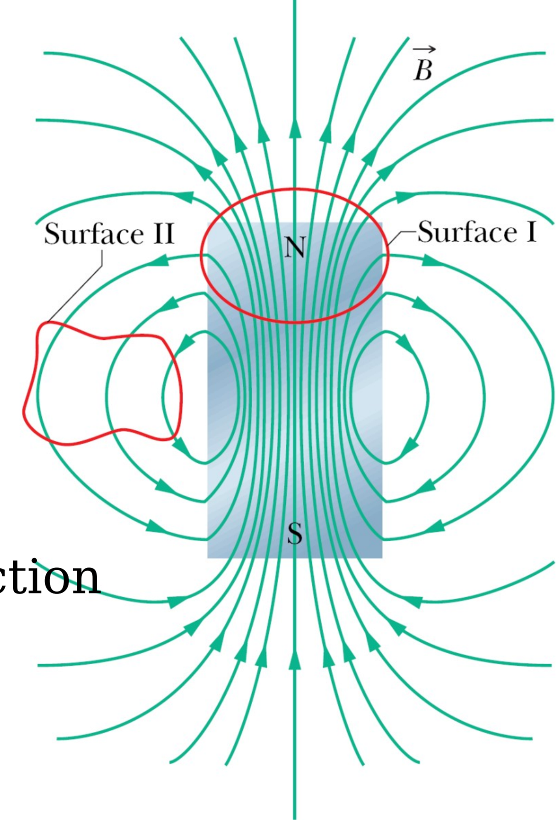


- Gauss' law for magnetic fields is a formal way of saying that magnetic monopoles do not exist. It asserts that the net magnetic flux Φ_B through any closed Gaussian surface is 0:

$$\Phi_B = \oint \vec{B} \cdot \mathrm{d}\vec{A} = 0 \qquad \text{Gauss' law for magnetic fields}$$

compare: $$\Phi_E = \oint \vec{E} \cdot \mathrm{d}\vec{A} = \frac{q_{\mathrm{enc}}}{\epsilon_0} \qquad \text{Gauss' law for electric fields}$$

- Gauss' law for magnetic fields says that there is no net magnetic flux through the surface because there is no net "magnetic charge" enclosed by the surface. The simplest magnetic structure that can exist and thus be enclosed by a Gaussian surface is a dipole.



Induced Magnetic Fields

- $\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ Faraday's law of induction

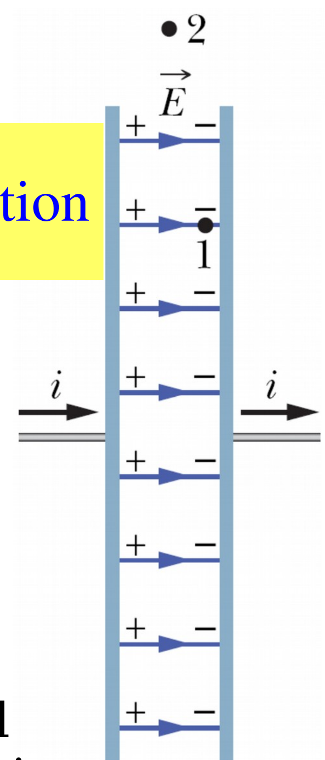
- Because of symmetry issue, we want to ask whether induction can occur in the opposite sense; that is, can a changing electric flux induce a magnetic field?

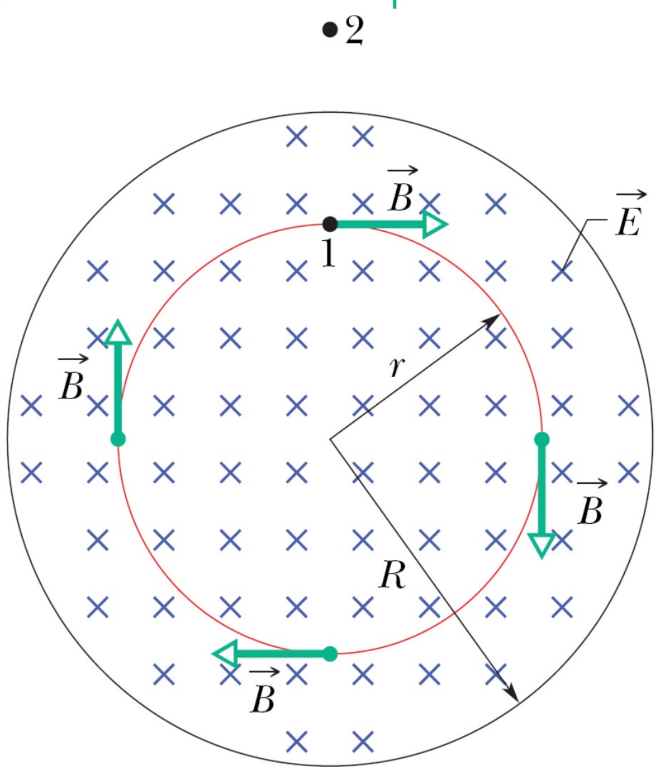
- The answer is yes:

$$\oint \vec{B} \cdot d\vec{s} = \epsilon_0 \mu_0 \frac{d\Phi_E}{dt} \quad \text{Maxwell's law of induction}$$

1 Maxwell's law has 2 extra symbol, ϵ_0, μ_0.

2 Maxwell's law lacks of the minus sign.





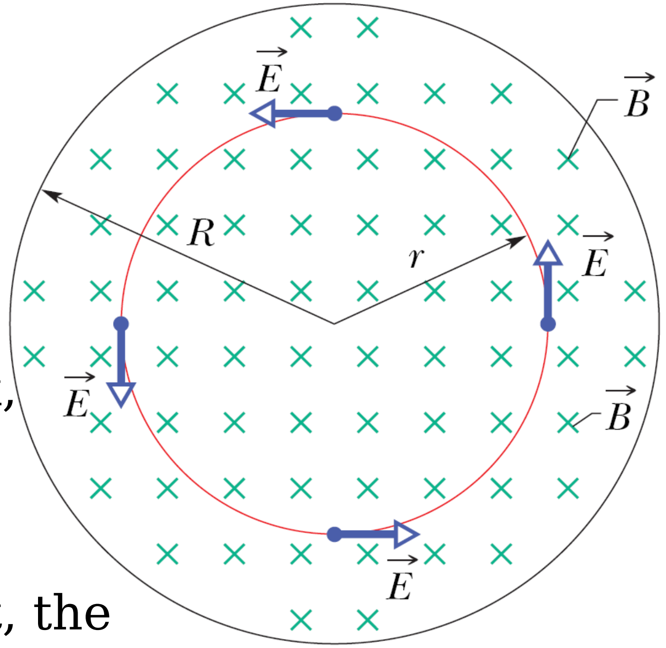
Ampere–Maxwell Law

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \quad \text{Ampere's law}$$

- 2 equations specifying the magnetic field produced by either a current or by a changing electric field give the field in exactly the same form.

- Combine the 2 equations into the single equation

$$\oint \vec{B} \cdot d\vec{s} = \epsilon_0 \mu_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc} \quad \text{Ampere-Maxwell law}$$

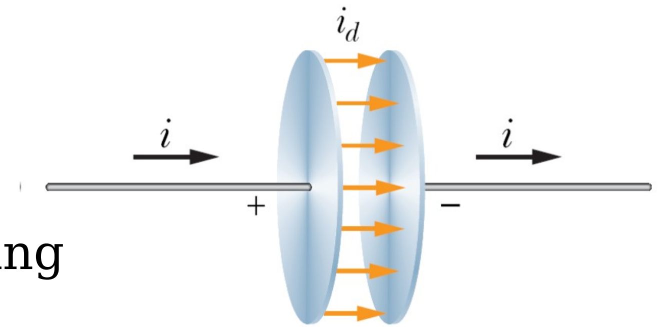


- When there is a current but no change in electric flux, the 1st term on the rhs is 0, and the eqn reduces to Ampere's law.

- When there is a change in electric flux but no current, the 2nd term on the rhs is 0, and the eqn reduces to Maxwell's law of induction.

Problem 32-1

Displacement Current



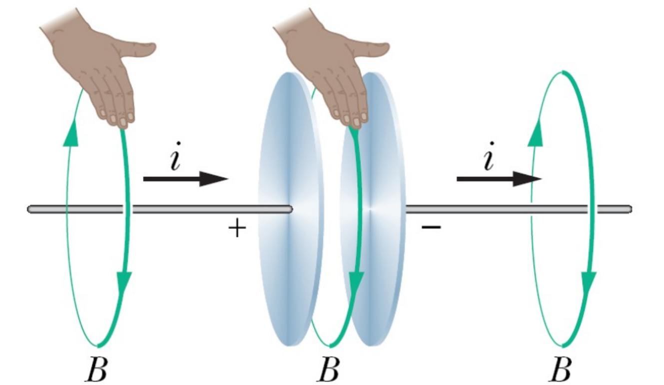
- The 1st term of Ampere-Maxwell eqn is treated as being a fictitious current called the displacement current i_d :

$$i_d \equiv \epsilon_0 \frac{d\Phi_E}{dt} \quad \text{displacement current}$$

$$\Rightarrow \quad \oint \vec{B} \cdot d\vec{s} = \mu_0 \left(i_{d,\,enc} + i_{enc} \right) \quad \text{Ampere-Maxwell law}$$

- The charge on the plates at any time is

$$q = \epsilon_0 A E \quad \Rightarrow \quad i = \frac{dq}{dt} = \epsilon_0 A \frac{dE}{dt}$$



- For the displacement current, assuming that the electric field between the 2 plates is uniform, we can replace the electric flux in that equation with EA

$$i_d \equiv \epsilon_0 \frac{d\Phi_E}{dt} = \epsilon_0 \frac{d}{dt}\int \vec{E}\cdot d\vec{A} = \epsilon_0 \frac{d\,E\,A}{dt} = \epsilon_0 A \frac{dE}{dt}$$

- The real current charging the capacitor and the fictitious displacement current between the plates have the same magnitude:

$$i_d = i \quad \text{displacement current in a capacitor}$$

Finding the Induced Magnetic Field

- The direction of the magnetic field produced by a real current by using the right-hand rule. Use the same rule to find the direction of an induced magnetic field produced by a fictitious displacement current.

- The magnitude of the magnetic field at a point inside the capacitor

$$B = \frac{\mu_0 i_d}{2\pi R^2} r \quad \text{inside a circular capacitor}$$

- The magnitude of the magnetic field at a point outside the capacitor

$$B = \frac{\mu_0 i_d}{2\pi r} \quad \text{outside a circular capacitor}$$

problem 32-2

Maxwell's Equations

- *Maxwell's Equations*: 4 fundamental equations of electromagnetism

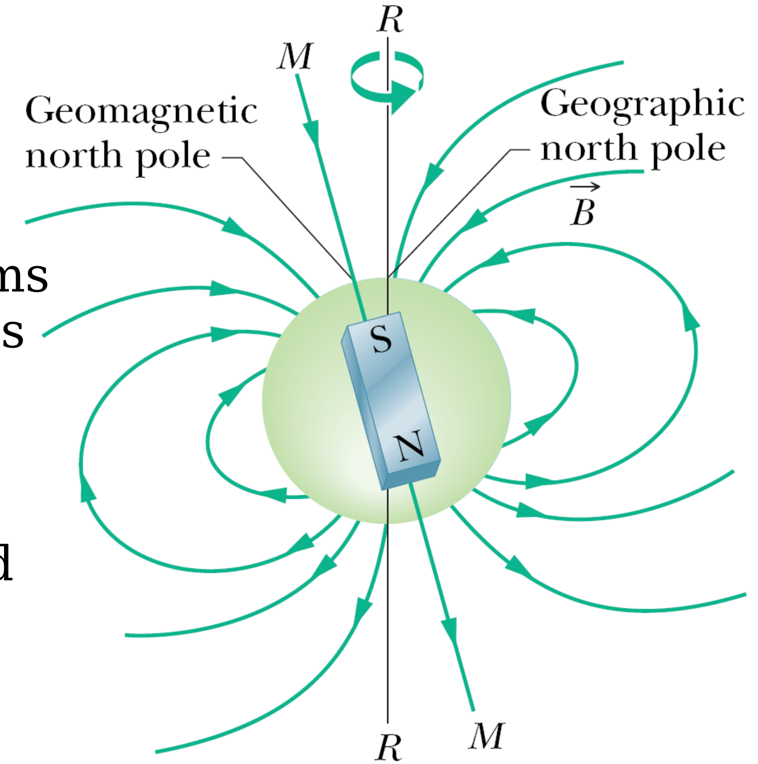
Name	Equation	
Gauss' law for electricity	$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$	Relates net electric flux to net enclosed electric charge
Gauss' law for magnetism	$\oint \vec{B} \cdot d\vec{A} = 0$	Relates net magnetic flux to net enclosed magnetic charge
Faraday's law	$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$	Relates induced electric field to changing magnetic flux
Ampere–Maxwell law	$\oint \vec{B} \cdot d\vec{s} = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} + \mu_0 i_{enc}$	Relates induced magnetic field to changing electric flux and to current

- Written on the assumption that no dielectric or magnetic materials are present.

Magnets

- The 1st known magnets were *lodestones*.

- The magnetic properties can be traced to the atoms and electrons, a direct result of the quantum physics taking place in the atomic and subatomic material.

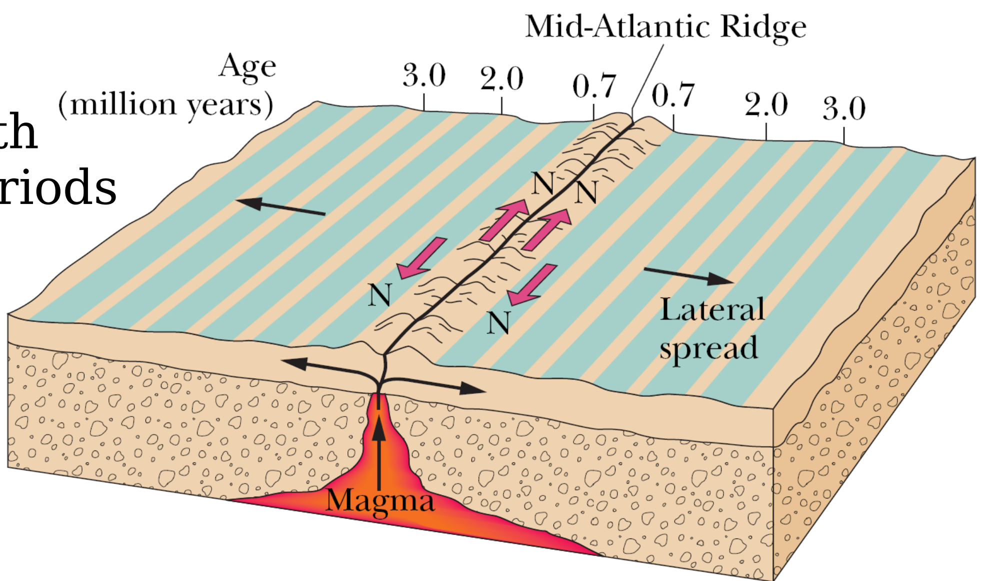


The Magnetism of Earth

- Earth's magnetic dipole moment is 8×10^{22} J/T and the direction of makes an angle of 11.5° with the rotation axis of Earth.

- The direction of the magnetic field on Earth's surface is specified in terms of:
1. **field declination**: the angle (left or right) between geographic north (which is toward 90° latitude) and the horizontal component of the field ⇐ compass.
2. **field inclination**: the angle (up or down) between a horizontal plane and the field's direction ⇐ dip meter.

- The magnetic field on the surface of Earth varies with time. Variations over longer periods can be studied by measuring the weak magnetism of the ocean floor on either side of the Mid-Atlantic Ridge.



- Earth's magnetic field has reversed its *polarity* about every million years.

Magnetism and Electrons

Spin Magnetic Dipole Moment

- An electron has an intrinsic angular momentum called its **spin angular momentum** (or just **spin**) $\vec{S}$; associated with this spin is an **intrinsic spin magnetic dipole moment** $\vec{\mu}_S$

- Spin is different from the angular momenta in 2 respects: $\vec{\mu}_S = -\frac{e}{m}\vec{S}$

1. Spin itself cannot be measured, but its component along any axis can.

2. A measured component of the electron's spin is quantized, and only 2 values.

- Assume the component of spin is measured along the z-axis

$$S_z = m_S \frac{h}{2\pi} \quad \text{for } m_S = \pm\frac{1}{2} \quad \text{spin magnetic quantum number}, \quad h = 6.63 \times 10^{-34}\ J \cdot \text{s} \quad \text{Planck constant}$$

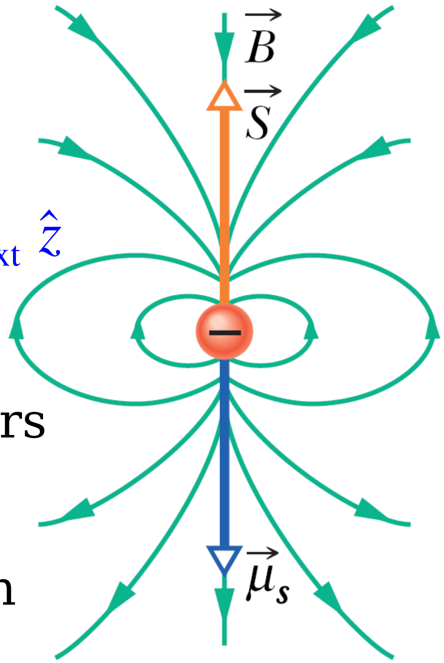
- When S_z is parallel to the z-axis (m_s=+1/2), the electron is said to be *spin up*.

When S_z is antiparallel to the z-axis (m_s=-1/2), the electron is said to be *spin down*.

- The spin magnetic dipole moment of an electron also cannot be measured; only its component along any axis can be measured, and that component too is quantized, with 2 possible values of the same magnitude but different signs.

$$\mu_{S,z} = -\frac{e}{m} S_z = \pm \mu_B \quad \Leftarrow \quad \mu_B = \frac{e\,h}{4\pi m} = 9.27 \times 10^{-24}\ J/T \quad \text{Bohr magneton}$$

- Spin magnetic dipole moments of electrons and other elementary particles can be expressed in terms of μ_B.



- When an electron is placed in an external magnetic field, the potential energy is $U = -\vec{\mu}_S \cdot \vec{B}_{ext} = -\mu_{S,z} B_{ext}$ $\Leftarrow$ $\vec{B}_{ext} = B_{ext}\,\hat{z}$

- Protons and neutrons also have a spin and an associated intrinsic spin magnetic dipole moment. For a proton the 2 vectors have the same direction, and for a neutron they have opposite directions. The contributions of these dipole moments to the magnetic fields of atom are about a thousand times smaller than that due to an electron.

Orbital Magnetic Dipole Moment

- In an atom, an electron has an additional angular momentum called its **orbital angular momentum** $\vec{L}_{orb}$. Associated with $\vec{L}_{orb}$ is an **orbital magnetic dipole moment** $\vec{\mu}_{orb}$,

$$\vec{\mu}_{orb} = -\frac{e}{2m}\vec{L}_{orb}$$

- Orbital angular momentum cannot be measured; only its component along any axis can, and that component is quantized.

$$L_{orb,z} = m_\ell \frac{h}{2\pi} \quad \text{for } m_\ell = 0, \pm 1, \pm 2, \cdots, \pm(\text{limit})$$

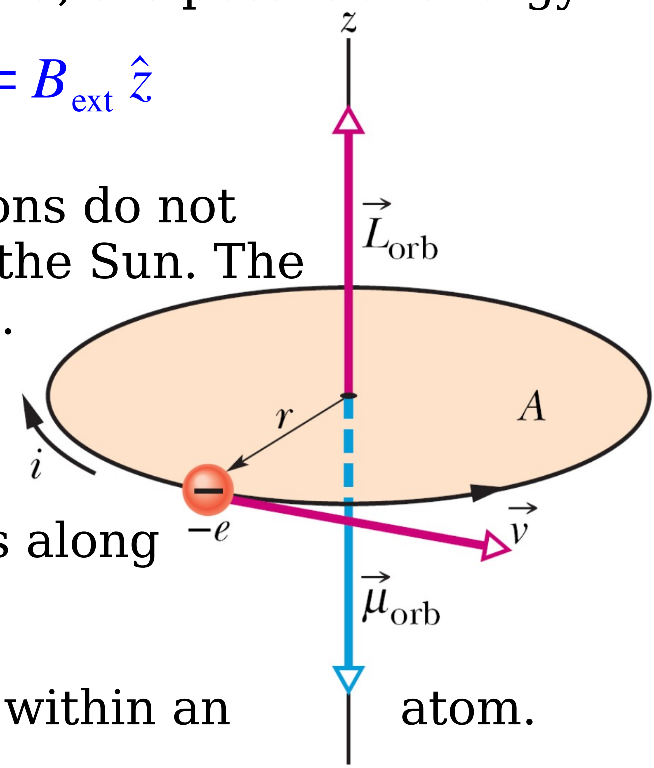
orbital magnetic quantum number

- The orbital magnetic dipole moment of an electron cannot itself be measured; only its component along an axis can, and that component is quantized.

$$\mu_{\text{orb},z} = -m_\ell\,\mu_B = -m_\ell\,\frac{e\,h}{4\pi m}$$

- When an electron is placed in an external magnetic field, the potential energy

$$U = -\vec{\mu}_{\text{orb}} \cdot \vec{B}_{\text{ext}} = -\mu_{\text{orb},z}\,B_{\text{ext}} \quad \Leftarrow \quad \vec{B}_{\text{ext}} = B_{\text{ext}}\,\hat{z}$$



- Although we have used the words "orbit" here, electrons do not really orbit the nucleus of an atom like planets orbiting the Sun. The real reason can be explained only with quantum physics.

Loop Model for Electron Orbits

- We can obtain $\vec{\mu}_{\text{orb}}$ with the nonquantum derivation that follows, in which we assume that an electron moves along a circular path with a large radius.

- However, the derivation does not apply to an electron within an atom.

- $\vec{\mu} = N\,i\,\vec{A} \quad \Rightarrow \quad \mu_{\text{orb}} = i\,A = \dfrac{e}{2\pi r/v}\,\pi r^2 = \dfrac{e\,v\,r}{2} \quad \Leftarrow \quad A = \pi r^2, \quad i = \dfrac{\text{charge}}{\text{time}} = \dfrac{e}{2\pi r/v}$

$$\vec{\ell} = \vec{r} \times m\,\vec{v} \quad \Rightarrow \quad L_{\text{orb}} = m\,r\,v\,\sin\frac{\pi}{2} = m\,r\,v \quad \Rightarrow \quad \vec{\mu}_{\text{orb}} = -\frac{e}{2m}\,\vec{L}_{\text{orb}}$$

Loop Model in a Nonuniform Field

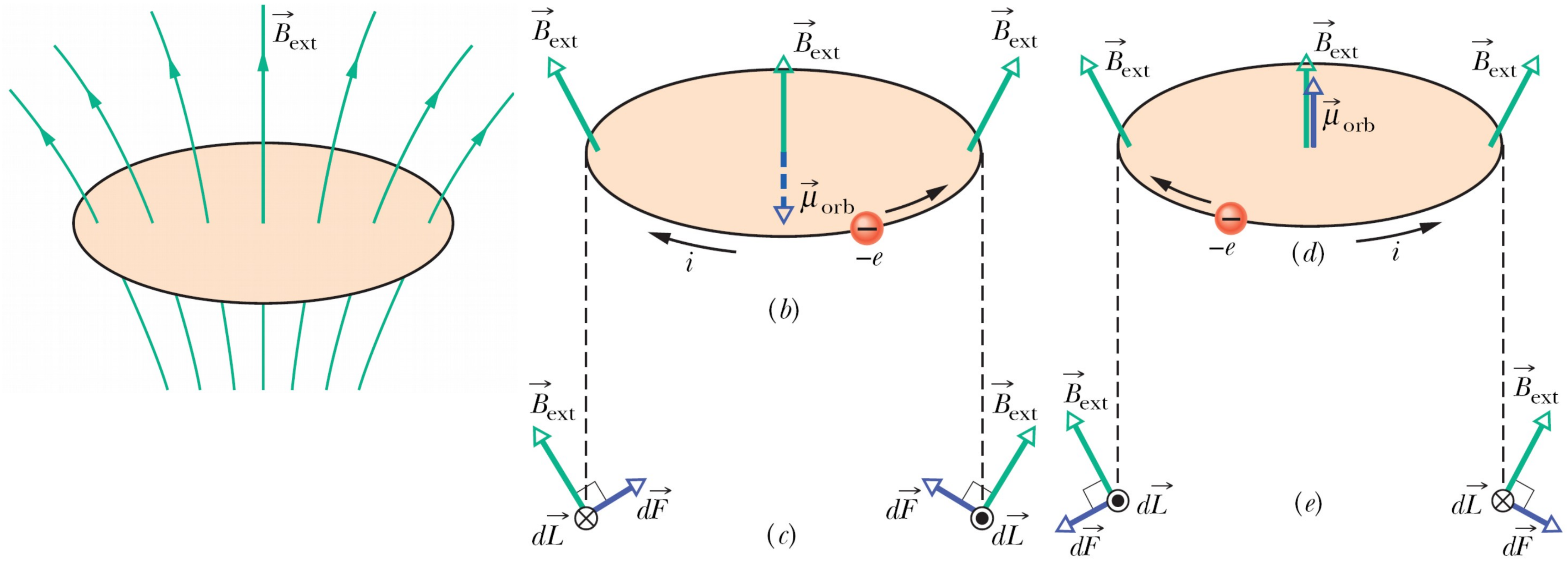
- Assume the magnetic field vectors all around the electron's circular path have the same magnitude and form the same angle with the vertical, Also assume all the electrons in an atom move either counterclockwise or clockwise.

- A current along an element in a magnetic field experiences a magnetic force

$$d\vec{F} = i\, d\vec{L} \times \vec{B}_{ext}$$

- The horizontal components of these forces cancel & the vertical components add.

- The net force on the current loop of (b) must be upward. The same reasoning leads to a downward net force on the loop in (d).



Magnetic Materials

- Each electron in an atom has an orbital and a spin magnetic dipole moment that combine vectorially. The resultant of the 2 vector quantities combines with similar resultants for all other electrons in the atom, and the resultant for each atom combines with those for all the other atoms in a material. If the combination produces a magnetic field, then the material is magnetic.

- 3 general types of magnetism:

1. **Diamagnetism** exists in all common materials. Weak magnetic dipole moments are induced when the material is placed in an external field; the combination of all the induced dipole moments gives a feeble net magnetic field. The dipole moments and the net field disappear when the external field is removed.

2. **Paramagnetism** exists in materials containing transition elements, rare earth elements, and actinide elements. Each atom of such a material has a permanent resultant magnetic dipole moment, but randomly oriented and the material as a whole lacks a net magnetic field. An external field can partially align the atomic magnetic dipole moments to give a net magnetic field. The alignment and its field disappear when the external field is removed.

3. **Ferromagnetism** is a property of iron, nickel. Some of the electrons in the materials have their resultant magnetic dipole moments aligned, which produces regions with strong magnetic dipole moments. An external field can align the magnetic moments of such regions, producing a strong magnetic field; the field partially persists when the external field is removed.

Diamagnetism

- Assume that in an atom of a diamagnetic material each electron can orbit only clockwise or counterclockwise, and the atom lacks a net magnetic dipole moment. It implies the number of electrons orbiting in one direction is the same as that orbiting in the opposite direction.

- Now turn on the nonuniform field. As the magnitude increases, a clockwise electric field is induced around each electron's orbital loop according to Faraday's law and Lenz's law.

- The counterclockwise electron is accelerated by the clockwise electric field. It means that the associated current & the downward magnetic dipole moment also increase. The clockwise electron is decelerated by the clockwise electric field. The associated current & the upward magnetic dipole moment all decrease.

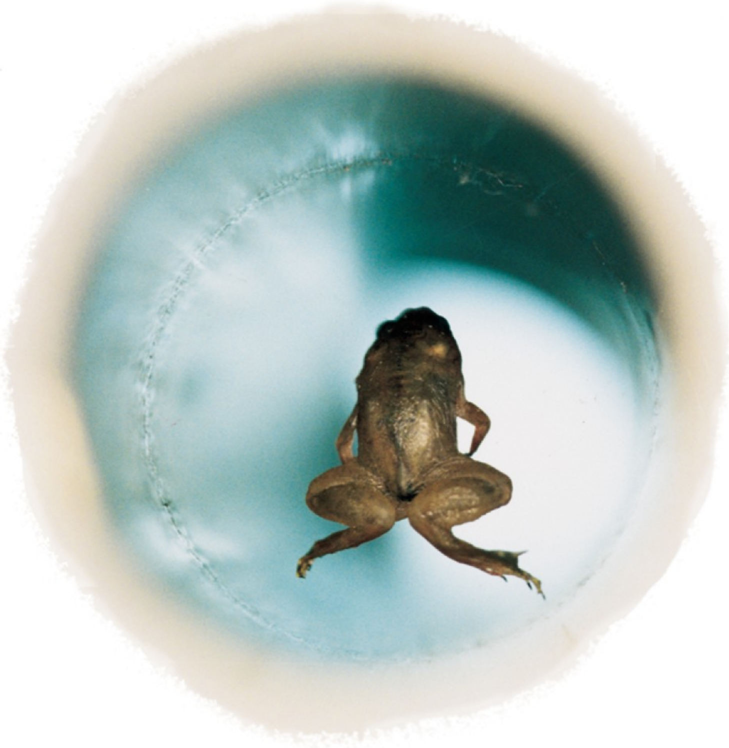
- By turning on field, we have given the atom a net magnetic dipole moment that is downward. This would also be so if the magnetic field were uniform.

- The nonuniformity of field also affects the atom. Because the current in (b) increases, the upward magnetic forces increase, as does the net upward force on the current loop. Because current in (d) decreases, the downward magnetic forces also decrease, as does the net downward force on the current loop.

- By turning on the nonuniform field, we have produced a net force on the atom; and that force is directed away from the region of greater magnetic field.

- If we apply the magnetic field, the material develops a downward reverse dipole moment and experiences an upward force. When the field is removed, both the dipole moment and the force disappear.

A diamagnetic material placed in an external magnetic field develops a magnetic dipole moment directed opposite . If the field is nonuniform, the diamagnetic material is repelled from a region of greater magnetic field toward a region of lesser field.



Paramagnetism

- In paramagnetic materials, the spin & orbital magnetic dipole moments of the electrons add vectorially to give the atom a (permanent) magnetic dipole moment.

- Without an external magnetic field, these atomic dipole moments are randomly oriented, and the net magnetic dipole moment of the material is 0.

- With an external magnetic field , the magnetic dipole moments tend to line up with the field, which gives a net magnetic dipole moment.

A paramagnetic material placed in an external magnetic field $\vec{B}_{\text{ext}}$ develops a magnetic dipole moment in the direction of $\vec{B}_{\text{ext}}$. If the field is nonuniform, the material is attracted toward a region of greater magnetic field from a region of lesser field.

- Random collisions of atoms due to their thermal agitation disrupt the alignment and thus reduce the sample's magnetic dipole moment.

- The mean translational kinetic energy $K=3kT/2$ is usually much bigger than the energy difference ΔU_B $(=2\mu B_{\text{ext}})$ between parallel alignment and antiparallel alignment of the magnetic dipole moment of an atom $K \gg \Delta U_B$, even for ordinary temperatures and field magnitudes.

- Energy transfers during collisions among atoms can significantly disrupt the alignment of the atomic dipole moments, keeping the magnetic dipole moment of a sample much less than $N\mu$.

- Magnetization $\vec{M}$: the ratio of its magnetic dipole moment to its volume

$$M = \frac{\text{measured magnetic moment}}{V} \quad \Leftarrow \quad \text{unit} = \text{A/m}$$

- Complete alignment of the atomic dipole moments is called *saturation*

$$M_{\max} = \frac{N\,\mu}{V}$$

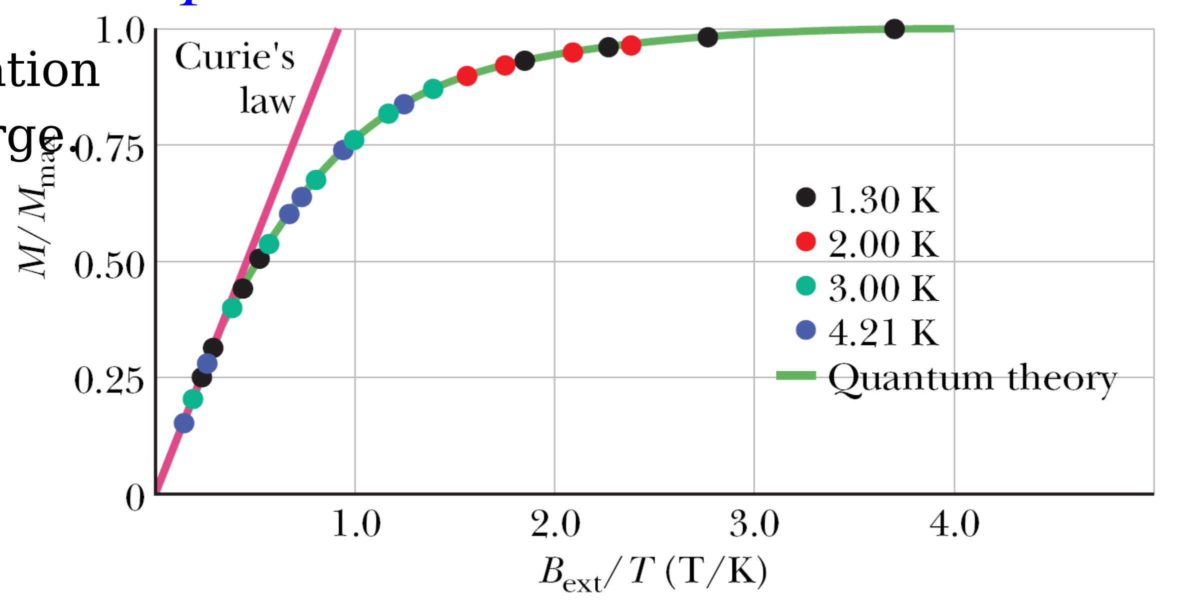
- Curie discovered that the magnetization of a paramagnetic sample is directly proportional to the magnitude of the external magnetic field and inversely proportional to the temperature

$$M = C\,\frac{B_{\text{ext}}}{T} \quad \text{Curie's law} \quad \Leftarrow \quad C : \text{Curie constant}$$

- The law is actually an approximation valid only when B_{ext}/T is not too large.

- *Magnetization curve*

Problem 32-3



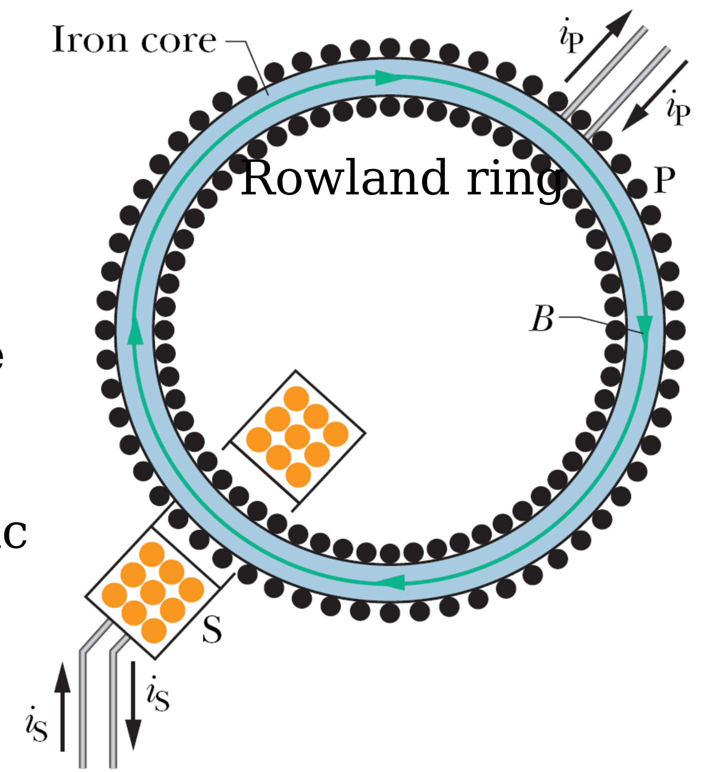
Ferromagnetism

- Iron, cobalt, nickel, gadolinium, dysprosium exhibit ferromagnetism because of the *exchange coupling* quantum physical effect.

- **exchange coupling**: the electron spins of one atom interact with those of neighboring atoms.

- The result is alignment of the magnetic dipole moments of the atoms. This persistent alignment gives ferromagnetic materials their permanent magnetism.

- If the temperature of a ferromagnetic material is raised above a certain critical value, i.e., the *Curie temperature*, the exchange coupling ceases to be effective. Materials then become simply paramagnetic.



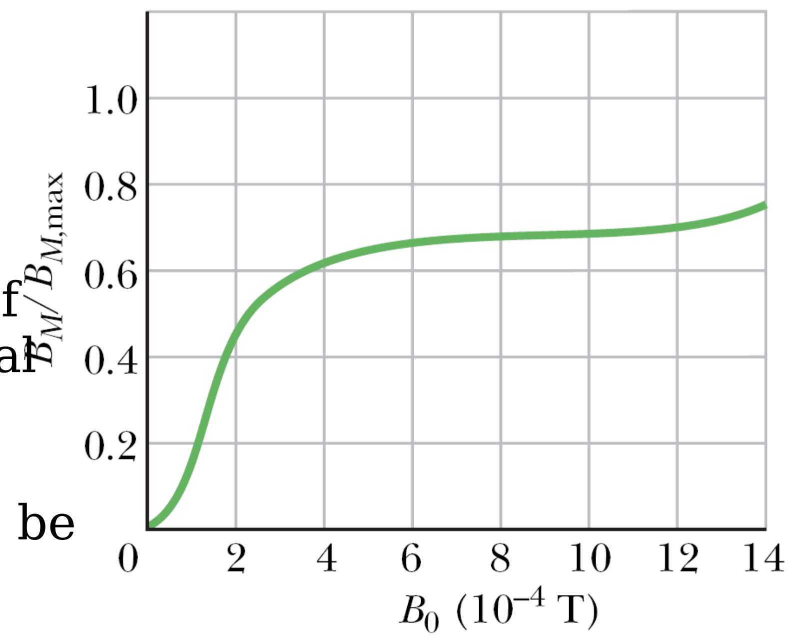
- The Curie temperature for iron is 1043K (=770°C).

- If the iron core were not present $B_0 = \mu_0 i_p n$

- With the iron core $B = B_0 + B_M > B_0$ where B_M is the the magnetic field contributed by the iron core.

- This contribution is from the alignment of the atomic dipole moments, due to exchange coupling and to the applied magnetic field, and is proportional to the magnetization of the iron.

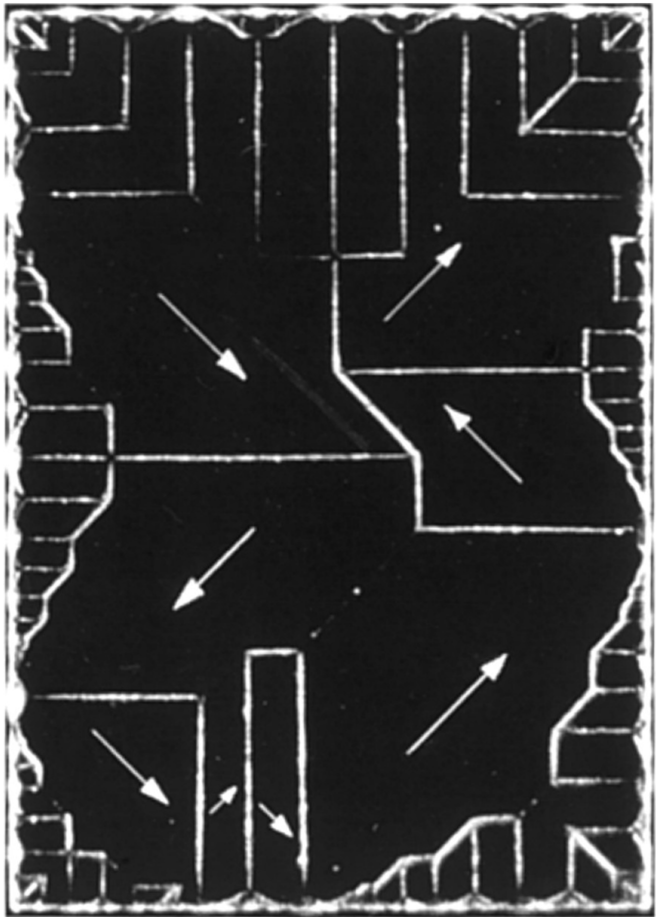
- To determine B_M we use a coil S to measure B.



Magnetic Domains

- Exchange coupling produces strong alignment of adjacent atomic dipoles in a ferromagnetic material below the Curie temperature.

- A crystal in the material will, in its normal state, be made up of a number of *magnetic domains*.



- The alignment of the atomic dipoles in a domain is essentially perfect. The domains, however, are not all Aligned. Thus, the crystal may have only a very small resultant magnetic moment.

- A piece of iron is an assembly of many tiny crystals, randomly arranged, so not a naturally strong magnet.

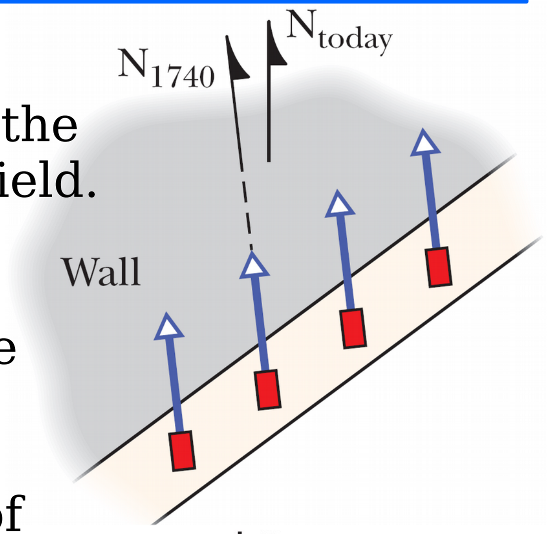
- Placing iron in an magnetic field produces 2 effects:

I: growth in size of the domains oriented along the external field;

II: shift of the orientation of the dipoles within a domain to become closer to the field direction.

A ferromagnetic material placed in an external magnetic field $\vec{B}_{ext}$ develops a strong magnetic dipole moment in the direction of $\vec{B}_{ext}$. If the field is nonuniform, the ferromagnetic material is attracted toward a region of greater magnetic field from a region of lesser field.

Mural Paintings Record Earth's Magnetic Field

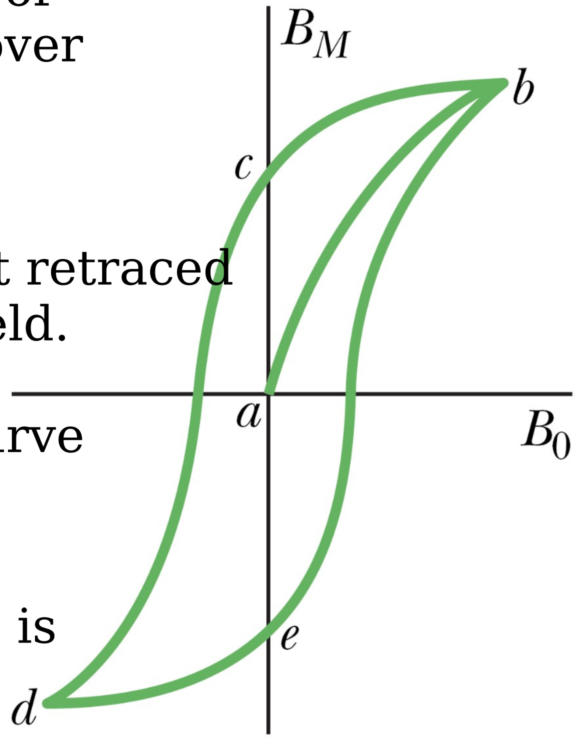


- When a pigment is applied to a wall, each grain rotates in the liquid until its dipole moment aligns with Earth's magnetic field.

- When the paint dries, the moments are locked into place and thus record the direction of Earth's magnetic field at the time of the painting.

- Evidence from mural paintings reveals that the direction of Geomagnetic north has varied gradually but continuously over recorded history.

Hysteresis



- Magnetization curves for ferromagnetic materials are not retraced as we increase and then decrease the external magnetic field.

- The lack of retraceability is called **hysteresis**, and the curve bcdeb is called a *hysteresis loop*.

- At points *c* & *e* the iron is magnetized, even though there is no current. It is the phenomenon of permanent magnetism.

- Hysteresis can be understood through the concept of magnetic domains.

- When the applied magnetic field is increased and then decreased back to its initial value, the domains do not return completely to their original configuration but retain some “memory” of their alignment.

- This memory of magnetic materials is essential for the magnetic storage of information, as on magnetic tapes and disks.

Problem 32-4

Selected problems: 14, 24, 38, 52