Chapter 31 Electromagnetic Oscillations and Alternating Current

LC Oscillations, Qualitatively

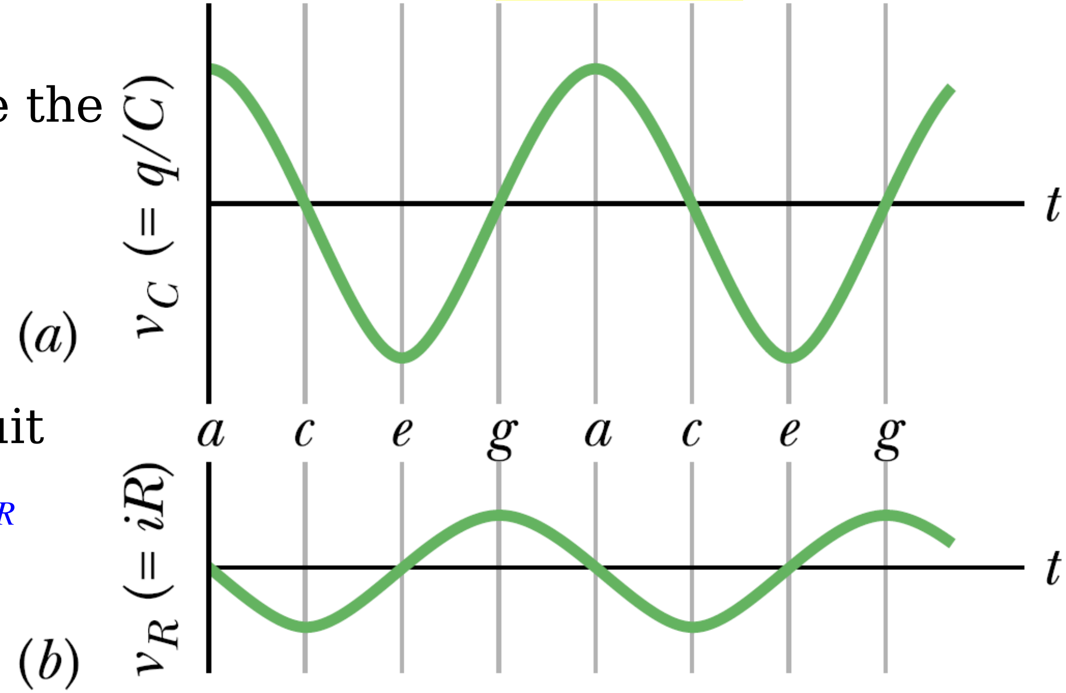
- In the *LC* circuit the charge, current, and potential difference vary sinusoidally (with period T and angular frequency ω). The resulting oscillations of the capacitor's electric field and the inductor's magnetic field are said to be **electromagnetic oscillations**.

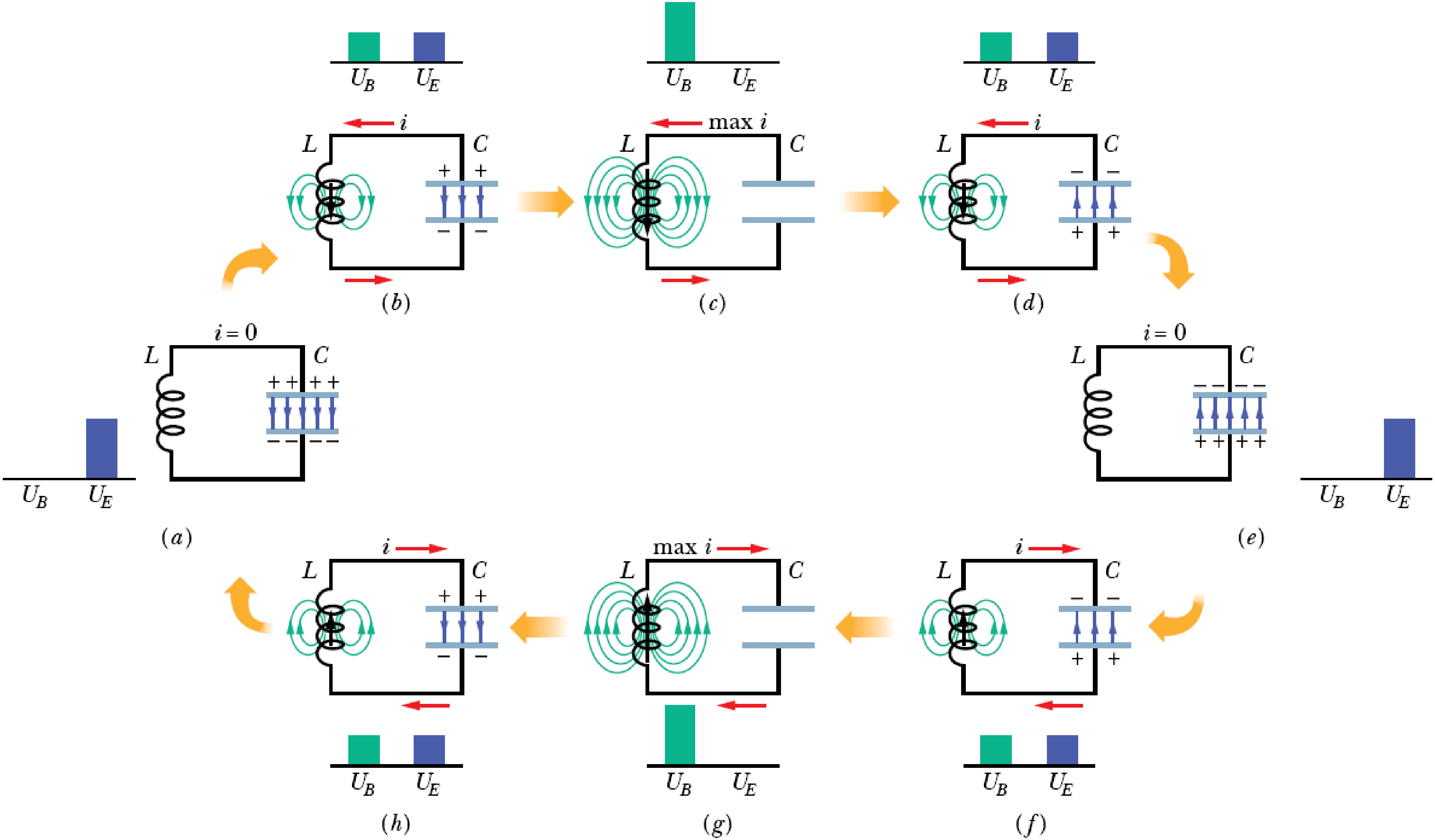
- The energy stored in the electric field of the capacitor is $U_E = \frac{q^2}{2C}$

- The energy stored in the magnetic field of the inductor is $U_B = \frac{Li^2}{2}$

- To determine the charge $q(t)$ on the capacitor,put in a voltmeter to measure the potential difference (or *voltage*) v_C that exists across the capacitor C: $v_C = \frac{q}{C}$

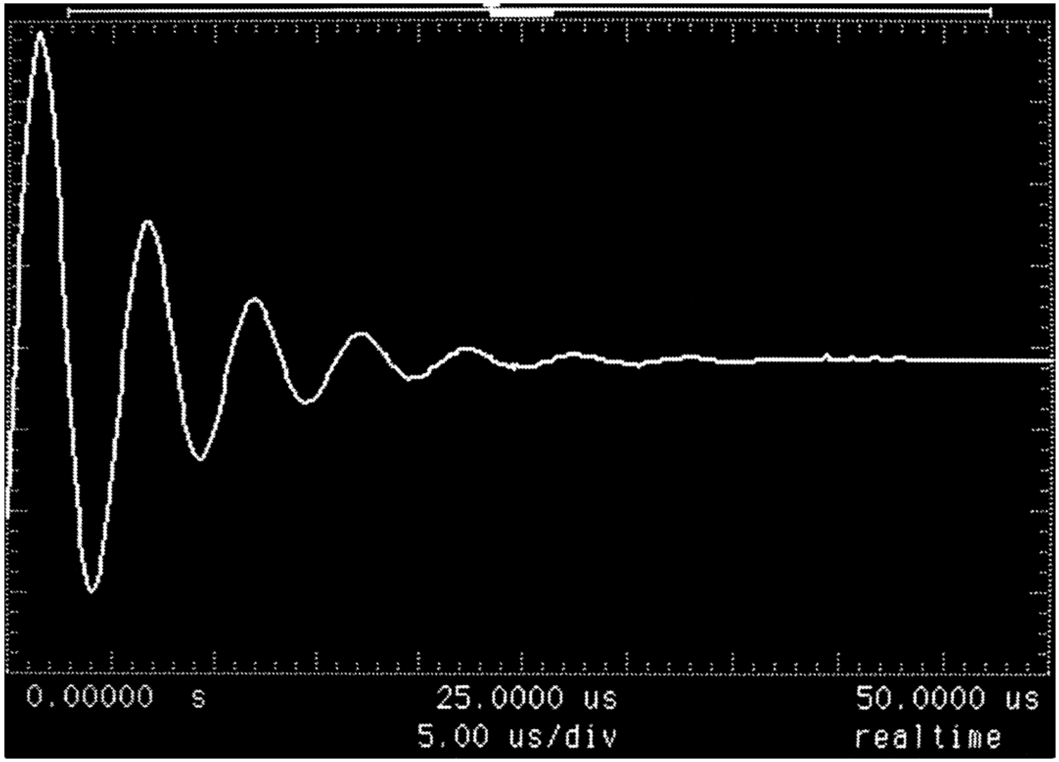
- To measure the current, connect a small resistance R in series in the circuit and measure the potential difference v_R across it: $v_R = i\,R$





- In an actual LC circuit, the oscillations will not continue indefinitely because there is always some resistance present that will drain energy from the electric and magnetic fields and dissipate it as thermal energy (the circuit may become warmer).

The Electrical– Mechanical Analogy



- The analogy between the oscillating LC system and an oscillating block–spring system:

q corresponds to x , $1/C$ correspond to k

i corresponds to v , L correspond to m

- These correspondences suggest that in an LC oscillator, the capacitor is mathematically like the spring in a block– spring system and the inductor is like the block.

- In a block– spring system: $\omega = \sqrt{\frac{k}{m}}$

- The correspondences suggest that to find the angular frequency of oscillation for an ideal LC circuit, k should be replaced by $1/C$ and m by L,

$$\omega = \frac{1}{\sqrt{LC}} \quad LC \text{ circuit}$$

Block-Spring System		LC Oscillator	
Element	Energy	Element	Energy
Spring	Potential , $k x^2/2$	Capacitor	Electrical , $q^2/2C$
Block	Kinetic , $m v^2/2$	Inductor	Magnetic , $L i^2/2$
$v = dx/dt$		$i = dq/dt$	

LC Oscillations, Quantitatively

The Block–Spring Oscillator

- The total energy of a block– spring oscillator: $U = U_b + U_s = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$

- Energy conservation, no friction:

$$0 \Leftarrow \frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2} m v^2 + \frac{1}{2} k x^2\right) = m v \frac{dv}{dt} + k x \frac{dx}{dt} \quad \Rightarrow \quad m \frac{d^2 x}{dt^2} + k x = 0$$

$$\Rightarrow \quad x(t) = X \cos(\omega t + \phi) \quad \text{displacement}$$

X is the amplitude of the mechanical oscillations, ω is the angular frequency of the oscillations, and ϕ is a phase constant.

The *LC* Oscillator

- The total energy in an oscillating *LC* circuit: $U = U_B + U_E = \frac{1}{2} L i^2 + \frac{1}{2} \frac{q^2}{C}$

U_B is the energy stored in the magnetic field of the inductor and U_E is the energy stored in the electric field of the capacitor.

- Energy conservation, no resistance:

$$0 \Leftarrow \frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2} L i^2 + \frac{1}{2} \frac{q^2}{C}\right) = L i \frac{di}{dt} + \frac{q}{C} \frac{dq}{dt} \quad \Rightarrow \quad L \frac{d^2 q}{dt^2} + \frac{1}{C} q = 0 \quad \text{LC oscillation}$$

$$\Rightarrow \quad q(t) = Q \cos(\omega t + \phi) \quad \text{charge}$$

- The current of the LC oscillator:

$$i(t)=\frac{dq}{dt}=-\omega Q\sin(\omega t+\phi)=-I\sin(\omega t+\phi)\quad \text{current} \quad \Leftarrow \quad I=\omega Q$$

- The angular frequency of the LC oscillator:

$$\frac{d^2q}{dt^2}=-\omega^2 Q\cos(\omega t+\phi) \quad \Rightarrow \quad -L\omega^2 Q\cos(\omega t+\phi)+\frac{Q}{C}\cos(\omega t+\phi)=0$$

$$\text{since } L\frac{d^2q}{dt^2}+\frac{q}{C}=0 \quad \Rightarrow \quad \omega=\frac{1}{\sqrt{LC}}$$

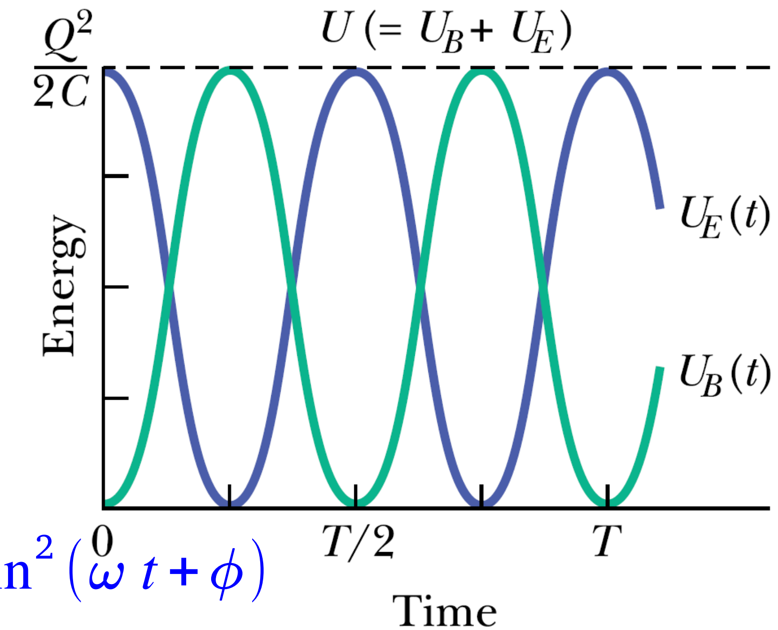
- ϕ is determined by the initial conditions.

- The electrical energy stored in the LC circuit

$$U_E=\frac{q^2}{2C}=\frac{Q^2}{2C}\cos^2(\omega t+\phi)$$

- The magnetic energy stored in the LC circuit

$$U_B=\frac{Q^2}{2C}\sin^2(\omega t+\phi) \quad \Leftarrow \quad U_B=\frac{Li^2}{2}=\frac{L\omega^2Q^2}{2}\sin^2(\omega t+\phi)$$



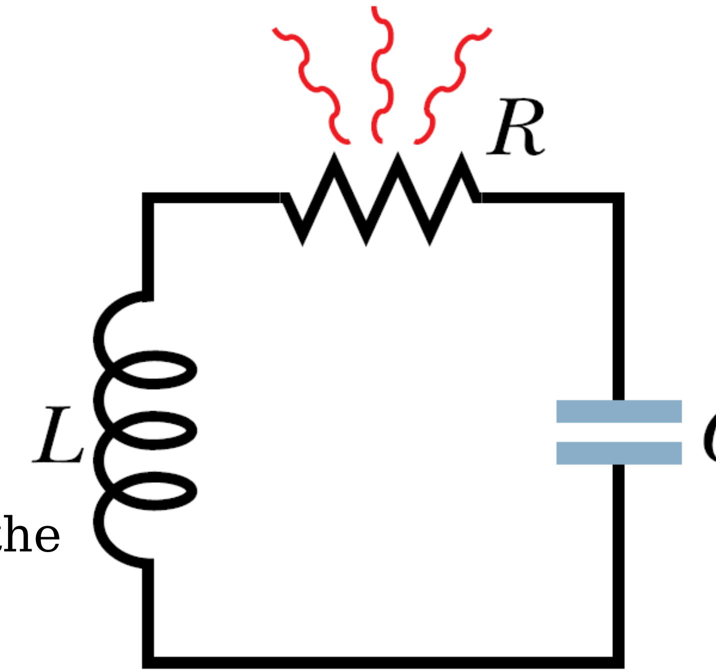
Note: (1) $(U_E)_{max}=(U_B)_{max}=\dfrac{Q^2}{2C}$

problem 31-1

(2) $U_E+U_B=\dfrac{Q^2}{2C}=\text{constant}$

(3) $U_E=(U_E)_{max}$ when $U_B=0$; $U_B=(U_B)_{max}$ when $U_E=0$

Damped Oscillations in an *RLC* Circuit



- A circuit containing resistance, inductance, and capacitance is called an *RLC circuit*. We shall here discuss only *series RLC circuits*

- With a resistance present, the total EM energy of the circuit is no longer constant; it decreases with time as energy is transferred to thermal energy in the resistance.

- Because of this loss of energy, the oscillations of charge, current, and potential difference decrease in amplitude, and the oscillations are *damped*.

- The rate of energy transferred to thermal energy: $\frac{dU}{dt} = -i^2 R$

$$\Rightarrow \frac{d}{dt}\left(\frac{L i^2}{2} + \frac{q^2}{2C}\right) = L i \frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^2 R \Rightarrow L\frac{d^2 q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0 \quad RLC \text{ curcuit}$$

$$\Rightarrow q = Q e^{-Rt/2L}\cos(\omega' t + \phi) \Leftarrow \omega' = \sqrt{\omega^2 - (R/2L)^2} \le \omega \Leftarrow \omega = 1/\sqrt{LC}$$

- The electrical energy: $U_E = \frac{q^2}{2C} = \frac{Q^2}{2C} e^{-Rt/L}\cos^2(\omega' t + \phi)$

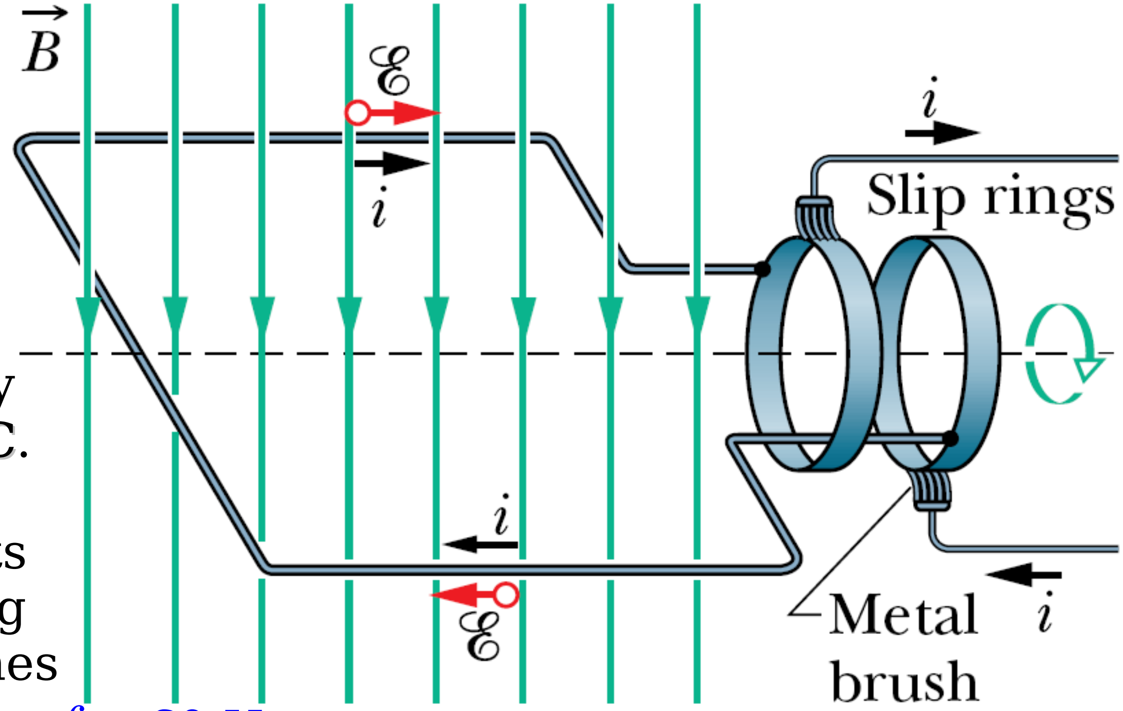
  the magnetic energy: $U_B = \frac{L i^2}{2} = \frac{Q^2}{2C} e^{-Rt/L}\sin^2(\omega' t + \phi)$

$$\Rightarrow U = \frac{Q^2}{2C} e^{-Rt/L}$$

problem 31-2

Alternating Current

- If the energy is supplied via oscillating emfs and currents, the current is said to be an **alternating current**, or **AC** for short. The nonoscillating current from a battery is said to be a **direct current**, or **DC**.



- These oscillating emfs and currents vary sinusoidally with time, reversing direction (in North America) 120 times per second and thus having frequency $f = 60$ Hz.

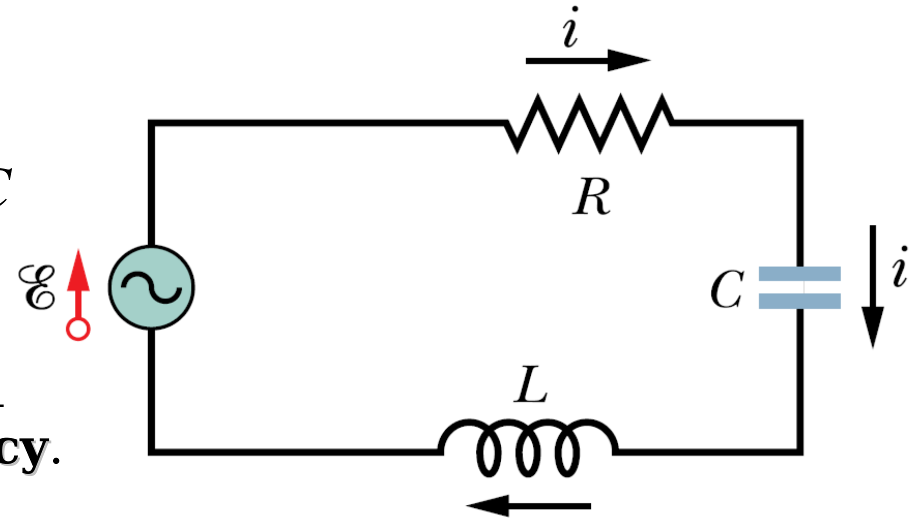
- The advantage of alternating current: *As the current alternates, so does the magnetic field that surrounds the conductor*. This makes possible the use of Faraday's law of induction.

- In a generator: $\mathscr{E} = \mathscr{E}_m \sin \omega_d t$, $i = I \sin\left(\omega_d t - \phi\right)$ where ω_d is called the **driving angular frequency**.

- The current may not be in phase with the emf.

- The **driving frequency** $f_d = \dfrac{\omega_d}{2\pi}$

Forced Oscillations



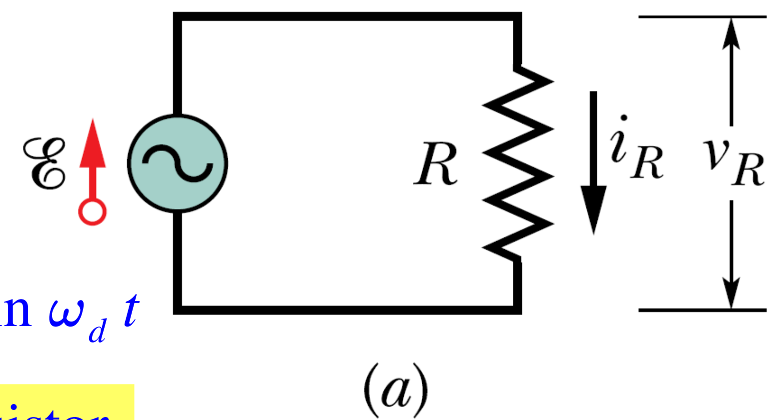
- An undamped *LC* circuits or a damped *RLC* circuits (with small enough *R*) without any external emf are said to be *free oscillations*, and the angular frequency $\omega = 1/\sqrt{LC}$ is said to be the circuit's **natural angular frequency**.

- When the external alternating emf is connected to an *RLC* circuit, the oscillations of charge, potential difference, and current are said to be *driven oscillations* or *forced oscillations*., with the driving angular frequency ω_d:

> Whatever the natural angular frequency ω of a circuit may be, forced oscillations of charge, current, and potential difference in the circuit always occur at the driving angular frequency ω_d.

Three Simple Circuits

A Resistive Load

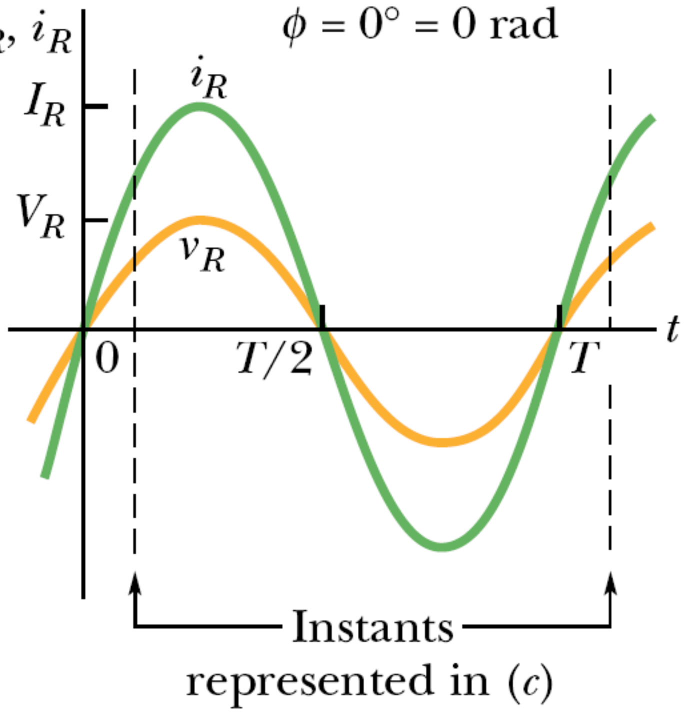


- By the loop rule: $\mathscr{E} - v_R = 0 \;\Rightarrow\; v_R = \mathscr{E}_m \sin\omega_d t$

$$\Rightarrow\; v_R = V_R \sin\omega_d t \;\Leftarrow\; V_R = \mathscr{E}_m \;\Rightarrow\; i_R = \frac{v_R}{R} = \frac{V_R}{R}\sin\omega_d t$$

$$i_R = I_R \sin(\omega_d t - \phi) \;\Rightarrow\; \phi = 0\,, \quad V_R = I_R R \quad \text{resistor}$$

- v_R and i_R are *in phase*, which means that their corresponding maxima (and minima) occur at the same times.



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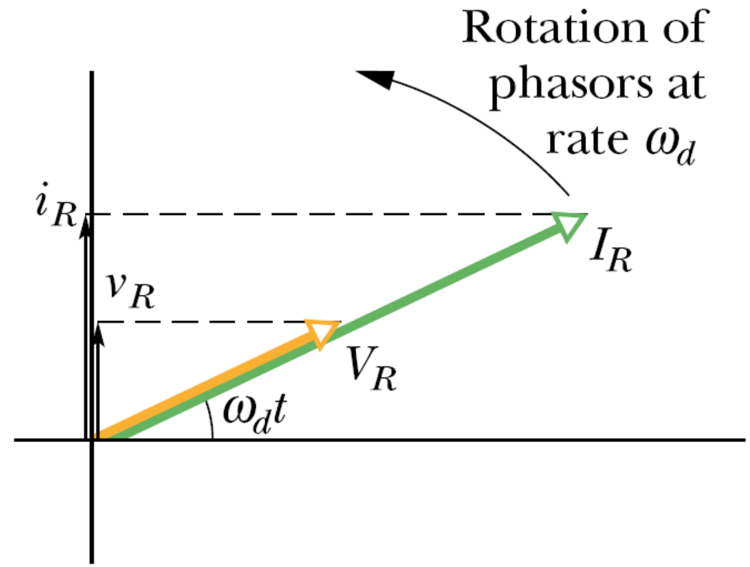


(c)

problem 31-3

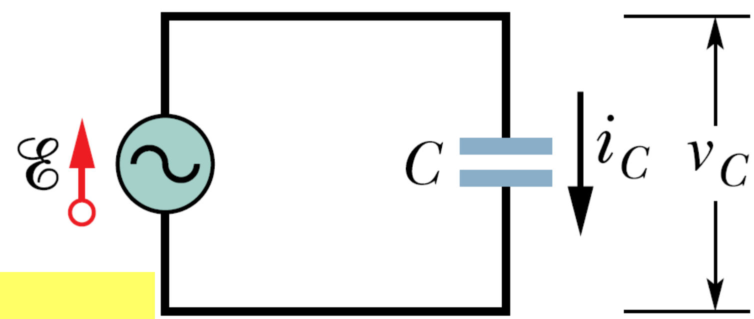
A Capacitive Load

- The potential difference across the capacitor

$$v_C = V_C \sin \omega_d t \quad \Rightarrow \quad q_C = C\, v_C = C\, V_C \sin \omega_d t$$

- The current: $i_C = \dfrac{d\, q_C}{d\, t} = \omega_d\, C\, V_C \cos \omega_d t$

- **Capacitive reactance**: $X_C = \dfrac{1}{\omega_d C}$ capacitive reactance

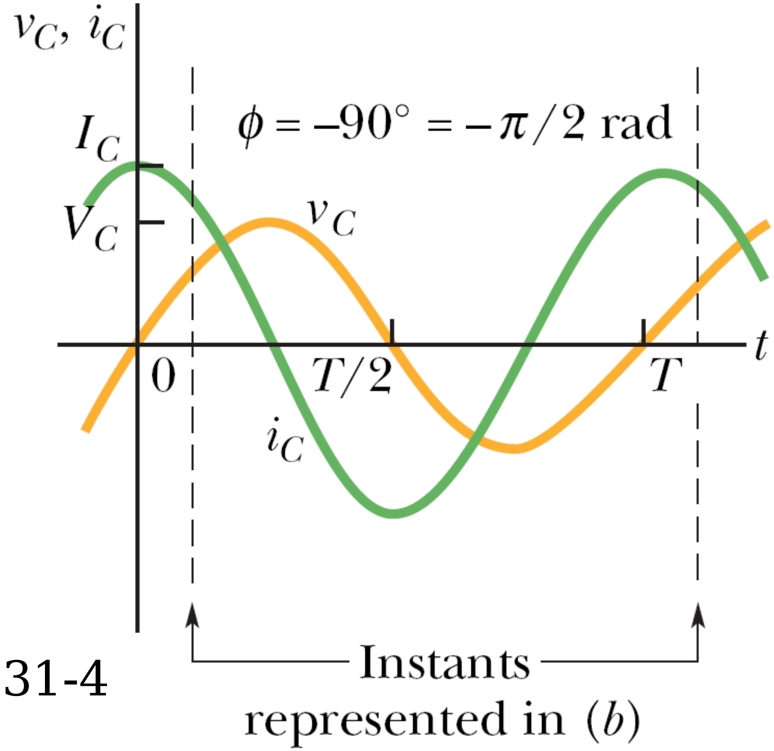


(a)

- The SI unit of X_C is the *ohm*, just as for resistance R.

- $\cos \omega_d t = \sin\left(\omega_d t + \dfrac{\pi}{2}\right) \quad \Rightarrow \quad i_C = I_C \sin(\omega_d t - \phi) = \dfrac{V_C}{X_C} \sin\left(\omega_d t + \dfrac{\pi}{2}\right)$

$\Rightarrow$ $V_C = I_C\, X_C$ capacitor $\Leftarrow$ true for any capacitance in any circuit



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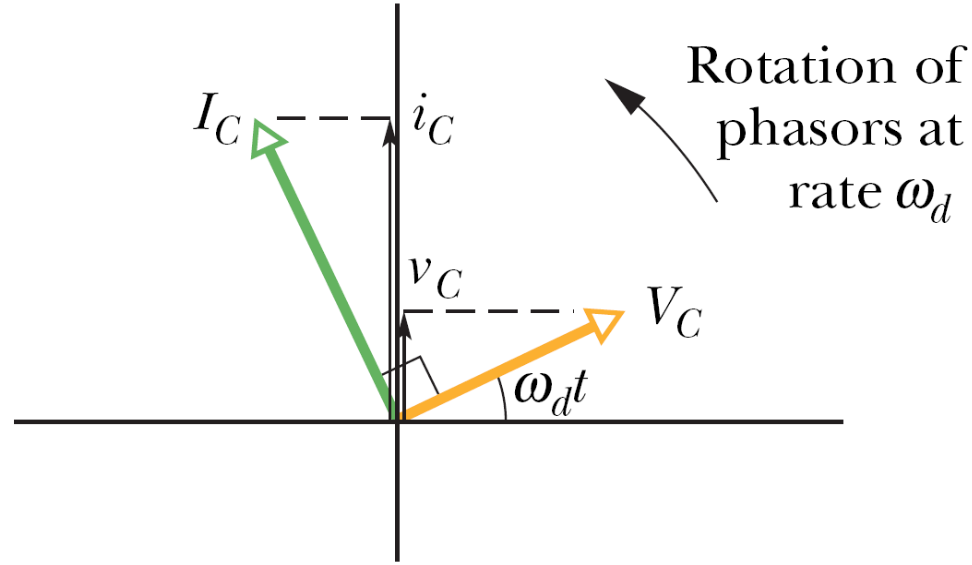

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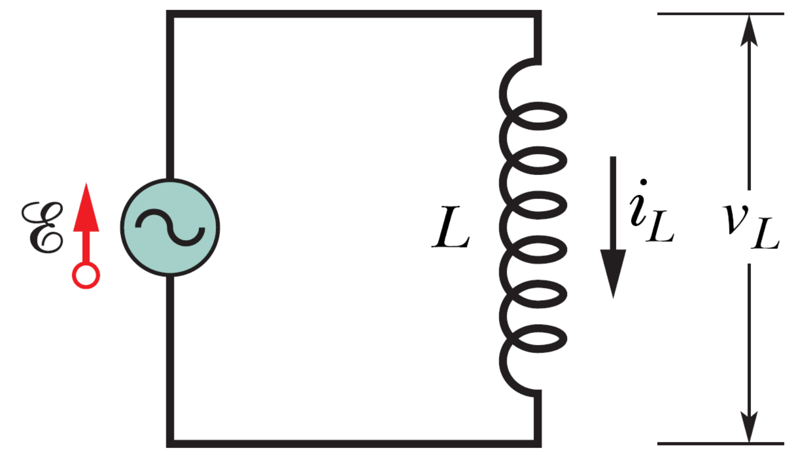
Problem 31-4

An Inductive Load



- The potential difference across the inductance

$$v_L = V_L \sin \omega_d t = L \frac{d i_L}{d t} \quad \Rightarrow \quad \frac{d i_L}{d t} = \frac{V_L}{L} \sin \omega_d t$$

- $i_L = \int d i_L = \frac{V_L}{L} \int \sin \omega_d t \, d t = -\frac{V_L}{\omega_d L} \cos \omega_d t$

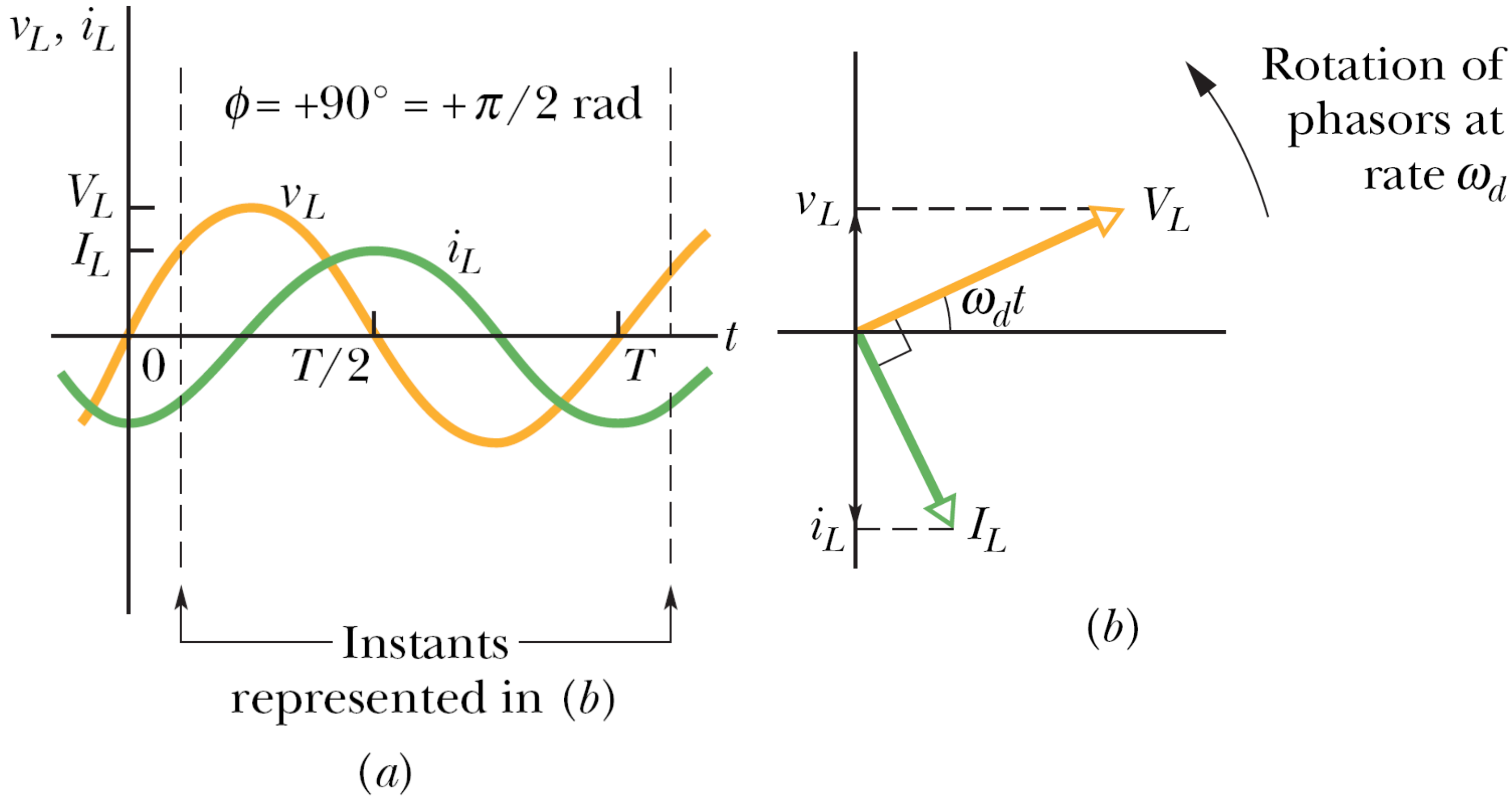
- **Inductive reactance**: $X_L = \omega_d L$ inductive reactance

- The SI unit of X_L is the *ohm*, just as for X_C an for R.

- $-\cos \omega_d t = \sin\left(\omega_d t - \frac{\pi}{2}\right) \quad \Rightarrow \quad i_L = I_L \sin(\omega_d t - \phi) = \frac{V_L}{X_L} \sin\left(\omega_d t - \frac{\pi}{2}\right)$

$\Rightarrow$ $V_L = I_L X_L$ inductor $\Leftarrow$ true for any inductance in any circuit

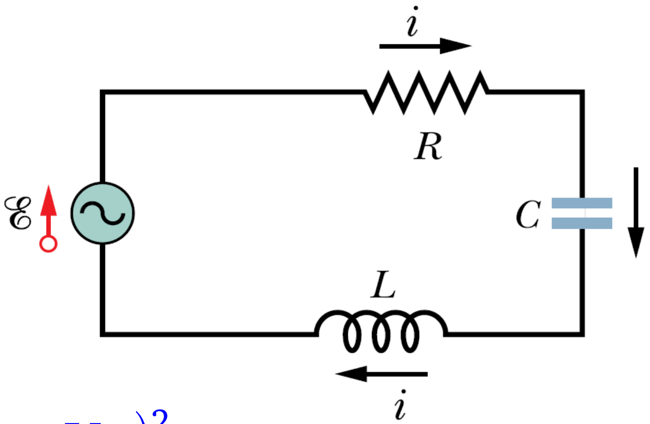
Problem 31-5



Phase and Amplitude Relations for Alternating Currents and Voltages

Circuit Element	Symbol	Resistance or Recactance	Phase of the Current	Phase Constant (or Angle) ϕ	Amplitude Relation
Resistor	R	R	In phase with v_R	0	$V_R = I_R R$
Capacitor	C	$X_C = \frac{1}{\omega_d C}$	Leads v_C by $\frac{\pi}{2}$	$-\frac{\pi}{2}$	$V_C = I_C X_C$
Inductor	L	$X_L = \omega_d L$	Lags v_L by $\frac{\pi}{2}$	$\frac{\pi}{2}$	$V_L = I_L X_L$

The Series *RLC* Circuit



- Apply a *RLC* circuit the alternating emf

$$\mathscr{E} = \mathscr{E}_m \sin \omega_d t \quad \text{applied emf} \quad \Rightarrow \quad i = I \sin(\omega_d t - \phi)$$

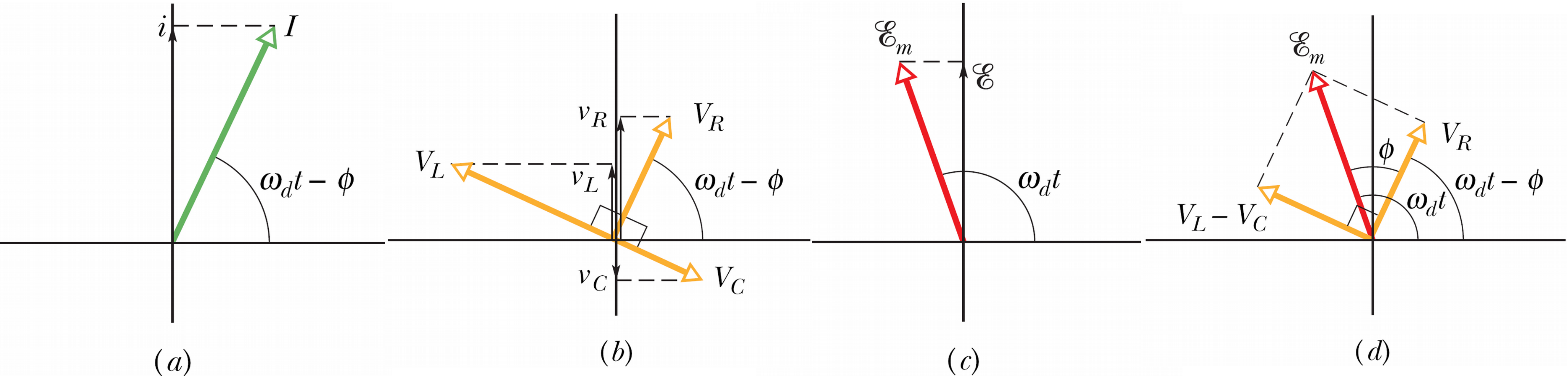
The Current Amplitude

- For the loop rule: $\mathscr{E} = v_R + v_C + v_L \quad \Rightarrow \quad \mathscr{E}_m^2 = V_R^2 + (V_L - V_C)^2$

$$\Rightarrow \quad \mathscr{E}_m^2 = (I R)^2 + (I X_L - I X_C)^2 \quad \Rightarrow \quad I = \frac{\mathscr{E}_m}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{\mathscr{E}_m}{Z} \quad \text{where}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \quad \text{impedance} \quad \Rightarrow \quad I = \frac{\mathscr{E}_m}{\sqrt{R^2 + [\omega_d L - 1/(\omega_d C)]^2}} \quad \text{current amplitude}$$

- The value of I depends on the difference between $\omega_d L$ and $1/\omega_d C$ or, equivalently, the difference between X_L and X_C.



$$\mathscr{E} = \mathscr{E}_m \sin \omega_d t \quad \Rightarrow \quad i = I \sin(\omega_d t - \phi)$$

$$\Rightarrow \quad \begin{aligned} v_R &= i\,R = V_R \sin(\omega_d t - \phi) \\ v_C &= \quad -V_C \cos(\omega_d t - \phi) \\ v_L &= \quad\ \ V_L \cos(\omega_d t - \phi) \end{aligned} \quad \Leftarrow \quad \begin{aligned} V_R &= I\,R \\ V_C &= I\,X_C \\ V_L &= I\,X_L \end{aligned} \quad \Rightarrow \quad \text{Define } V_X = V_L - V_C$$

$$\mathscr{E}_m \sin \omega_d t \Leftarrow \mathscr{E} = v_R + v_C + v_L = V_R \sin(\omega_d t - \phi) + V_X \cos(\omega_d t - \phi)$$

$$= (V_R \cos\phi + V_X \sin\phi) \sin \omega_d t + (V_X \cos\phi - V_R \sin\phi) \cos \omega_d t$$

Coefficient comparison gives $\begin{aligned} V_R \cos\phi + V_X \sin\phi &= \mathscr{E}_m \\ V_X \cos\phi - V_R \sin\phi &= 0 \end{aligned}$

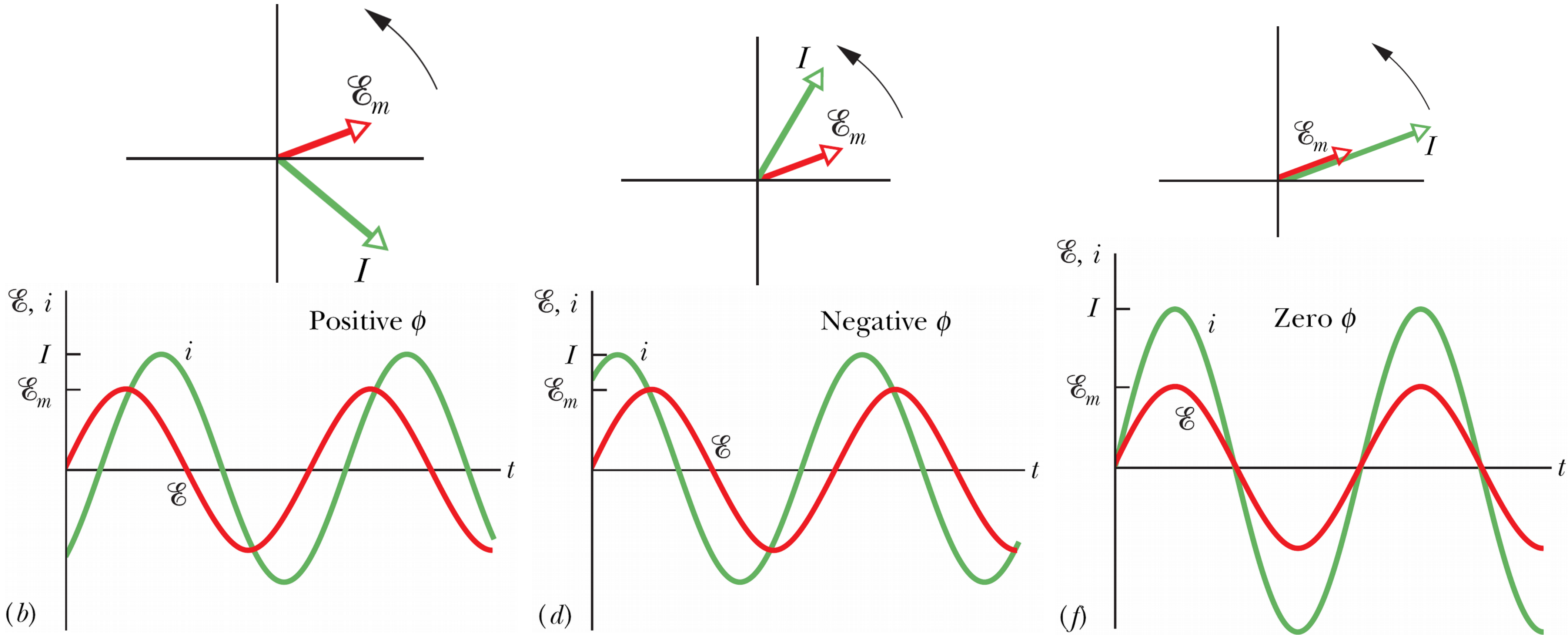
$$\Rightarrow \quad \begin{aligned} &\tan\phi = \frac{V_X}{V_R} = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} \\ &V_R = \mathscr{E}_m \cos\phi\,, \quad V_X = \mathscr{E}_m \sin\phi \quad \Rightarrow \quad \mathscr{E}_m^2 = V_R^2 + V_X^2 = V_R^2 + (V_L - V_C)^2 \end{aligned}$$

- The current that we have been describing in this section is the *steady-state current* that occurs after the alternating emf has been applied for some time.

- When the emf is first applied to a circuit, a brief *transient current* occurs. Its duration is determined by the time constants $\tau_L = L/R$ and $\tau_C = RC$ as the inductive and capacitive elements "turn on."

The Phase Constant

- From the plot: $\tan\phi = \dfrac{V_L - V_C}{V_R} = \dfrac{I\,X_L - I\,X_C}{I\,R} \Rightarrow$ $\tan\phi = \dfrac{X_L - X_C}{R}$ phase constant



- 3 different results for the phase constant

  $X_L > X_C$: The circuit is said to be *more inductive than capacitive*.

  $X_C > X_L$: The circuit is said to be *more capacitive than inductive*.

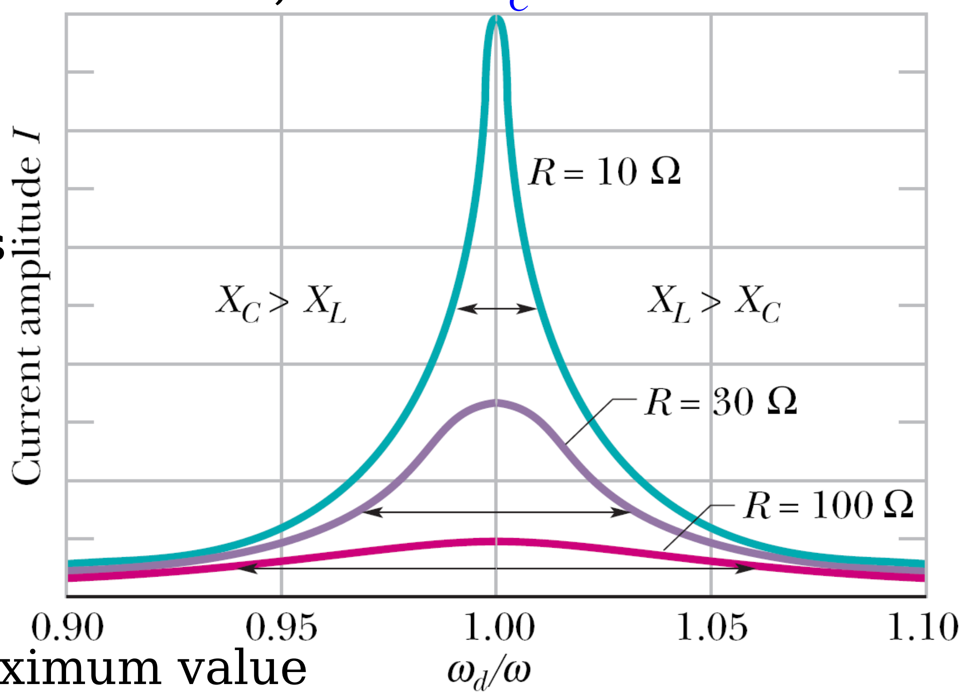
  $X_C = X_L$: The circuit is said to be in *resonance*.

- In the *purely inductive circuit*, where X_L is nonzero and $X_C = R = 0$, then $\phi = \pi/2$ (the greatest value of ϕ). In the *purely capacitive circuit*, where X_C is nonzero and $X_L = R = 0$, then $\phi = -\pi/2$ (the least value of ϕ).

Resonance

- For a given resistance R, that amplitude is a maximum when the quantity $\omega_d L - 1/\omega_d C$ in the denominator is 0

$$\omega_d L = \frac{1}{\omega_d C} \quad \Rightarrow \quad \omega_d = \frac{1}{\sqrt{LC}} \quad \text{maximum } I$$



- The natural angular frequency ω of the *RLC* circuit is also equal to $1/\sqrt{LC}$, the maximum value of I occurs when the driving angular frequency matches the natural angular frequency—that is, at resonance.

$$\omega_d = \omega = \frac{1}{\sqrt{LC}} \quad \text{resonance}$$

- The *resonance curves* peak at their maximum current amplitude I $(=\mathscr{E}_m/R)$ when $\omega_d=\omega$, but the maximum value of I decreases with increasing R. The curves also increase in width (measuring at half the maximum value of I) with increasing R.

- For small ω_d, $X_L(=\omega_d L)$ is small and $X_C(=1/\omega_d C)$ is large. Thus, the circuit is mainly capacitive and the impedance is dominated by the large X_C, which keeps the current low.

- As ω_d increases, X_C remains dominant but decreases while X_L increases. The decrease in X_C decreases the impedance, allowing the current I to increase. When the increasing X_L and the decreasing X_C reach equal values, the current I is greatest and the circuit is in resonance, with $\omega_d=\omega$.

- As ω_d continue to increase, the increasing X_L becomes more dominant over the decreasing X_C. The impedance increases because of X_L and the current decreases.

- In summary: The low-angular-frequency side of a resonance curve is dominated by the capacitor's reactance, the high-angular frequency side is dominated by the inductor's reactance, and resonance occurs in the middle.

Problem 31-6

Power in Alternating-Current Circuits

- In steady-state operation the average energy stored in the capacitor and inductor together remains constant. The net transfer of energy is thus from the generator to the resistor, where EM energy is dissipated as thermal energy.

- The instantaneous rate at which energy is dissipated in the resistor

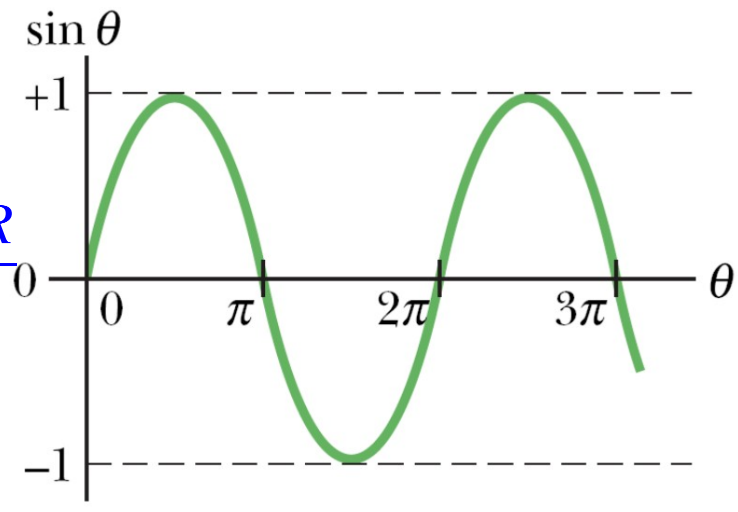
$$P = i^2 R = [I \sin(\omega_d t - \phi)]^2 R = I^2 R \sin^2(\omega_d t - \phi)$$

- The *average* rate at which energy is dissipated

$$P_{\text{avg}} = \frac{1}{T}\int_0^T P\, dt = \frac{I^2 R}{T}\int_0^T \sin^2(\omega_d t - \phi)\, dt = \frac{I^2 R}{2}$$

$$= \left(\frac{I}{\sqrt{2}}\right)^2 R \Rightarrow I_{\text{rms}} \equiv \frac{I}{\sqrt{2}} \quad \text{rms current}$$

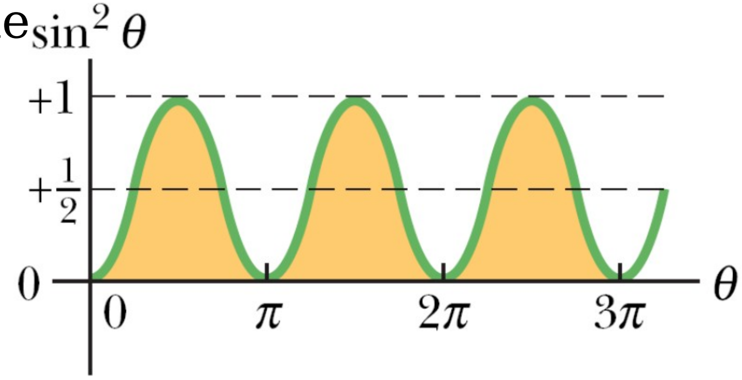
$$\Rightarrow P_{\text{avg}} = I_{\text{rms}}^2 R \quad \text{average power}$$



(a)

- If we switch to the rms current, we can compute the average rate of energy dissipation for alternating-current circuits just as for direct-current circuits.

- $$V_{\text{rms}} = \frac{V}{\sqrt{2}}, \quad \mathscr{E}_{\text{rms}} = \frac{\mathscr{E}}{\sqrt{2}} \quad \text{rms voltage; rms emf}$$



(b)

- Alternating-current instruments, such as ammeters and voltmeters, are usually calibrated to read I_{rms}, V_{rms}, and $\mathscr{E}_{rms}$.

- Plug an alternating-current voltmeter into a electrical outlet and it reads 120 V, that is an rms voltage. The maximum value of the potential difference at the outlet is $\sqrt{2}\times(120\text{ V})$ or 170 V.

- $$I_{rms}=\frac{\mathscr{E}_{rms}}{Z}=\frac{\mathscr{E}_{rms}}{\sqrt{R^2+(X_L-X_C)^2}} \quad \Rightarrow \quad P_{avg}=\frac{\mathscr{E}_{rms}}{Z}I_{rms}R=\mathscr{E}_{rms}I_{rms}\frac{R}{Z}$$

$$\Rightarrow \quad P_{avg}=\mathscr{E}_{rms}I_{rms}\cos\phi \quad \text{average power} \quad \Leftarrow \quad \cos\phi=\frac{V_R}{\mathscr{E}_m}=\frac{I\,R}{I\,Z}=\frac{R}{Z} \quad \text{power factor}$$

- The equation is independent of the sign of the phase constant $\phi \Leftarrow \cos\phi=\cos(-\phi)$.

- To maximize the rate at which energy is supplied to a resistive load in an *RLC* circuit, we should keep the power factor cosϕ as close to 1 as possible $\Leftarrow$ $\phi=0$.

Problem 31-7

Transformers

Energy Transmission Requirements

- An ac circuit with only a resistive load, the power factor $\cos 0 = 1$, $P_{\text{avg}} = \mathscr{E}\, I = I\, V$

- A range of choices of I and of V provided only that the product IV is as required.

- In the transmission of electrical energy from the generating plant to the consumer, we want the lowest practical current (hence the largest practical voltage) to minimize I^2R losses (often called ohmic losses) in the transmission line.

- Consider a 735 kV line to transmit electrical energy for 1000 km. If the current is 500 A and the power factor ~ unity. Then

$$P_{\text{avg}}^{\text{supply}} = \mathscr{E}\, I = (735000\ \text{V})(500\ \text{A}) = 368\ \text{MW}$$

- The resistance of the transmission line is about 0.22Ω/km $\Rightarrow$ $R_{\text{total}} = 220\ \Omega$
Energy is dissipated due to that resistance at a rate

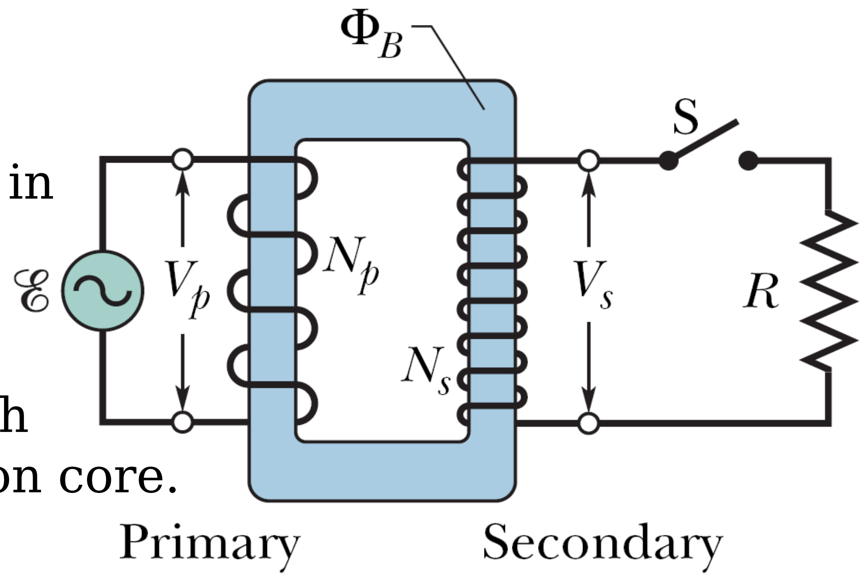
$$P_{\text{avg}}^{R} = I^2 R = (500\ \text{A})^2 (220\ \Omega) = 55\ \text{MW} \sim 15\ \% \times P_{\text{avg}}^{\text{supply}}$$

- In the other case: $I' = 2I,\ \mathscr{E}' = \dfrac{\mathscr{E}}{2} \quad \Rightarrow \quad P_{\text{avg}}^{\text{supply}} = \mathscr{E}'\, I' = \mathscr{E}\, I$

$$\Rightarrow \quad P_{\text{avg}}^{R} = I'^2 R = (1000\ \text{A})^2 (220\ \Omega) = 220\ \text{MW} \sim 65\ \% \times P_{\text{avg}}^{\text{supply}}$$

- The general energy transmission rule: Transmit at the highest possible voltage and the lowest possible current.

The Ideal Transformer



- Need a device with which we can raise (for transmission) and lower (for use) the ac voltage in a circuit, keeping the product current voltage essentially constant ⇒ the **transformer**.

- The *ideal transformer* consists of two coils, with different numbers of turns, wound around an iron core.

- The primary winding, of N_p turns, is connected to an AC generator whose emf is

$$\mathscr{E} = \mathscr{E}_m \sin \omega t$$

- The secondary winding, of N_s turns, is connected to load resistance R.

- The primary current, the *magnetizing current* I_{mag}, lags the primary voltage V_p by 90° (no power is delivered). The sinusoidally changing primary current I_{mag} produces a sinusoidally changing magnetic flux Φ_B in the iron core.

- The core strengthens the flux and to bring it through the secondary winding.

- Because Φ_B varies, it induces an emf $\mathscr{E}_{turn}$ ($d\Phi_B/dt$) in each turn of the secondary. the emf per turn $\mathscr{E}_{turn}$ is the same in the primary and the secondary

$$\mathscr{E}_{turn} = \frac{V_p}{N_p} = \frac{V_s}{N_s} \quad \Rightarrow \quad V_s = V_p \frac{N_s}{N_p} \quad \text{transformation of voltage}$$

- $N_s > N_p$: *step-up transformer* because $V_s > V_p$;
  $N_s < N_p$: *step-down transformer* because $V_s < V_p$.

- Connect the secondary to the resistive load R, now energy is transferred from the generator:

1 An AC I_s appears in a secondary circuit, with corresponding energy dissipation rate $I_s^2 R (= V_s^2/R)$ in the resistive load.

2 I_s produces its own alternating magnetic flux in the iron core, and this flux induces an opposing emf in the primary windings.

3 V_p of the primary cannot change in response to this opposing emf because it must always be equal to the emf that is provided by the generator.

4 To maintain V_p, the generator now produces (in addition to I_{mag}) an AC I_p in the primary circuit; the emf induced by I_p in the primary will exactly cancel the emf induced there by I_s. Because the phase constant of I_p is not $90°$ like that of I_{mag}, this current I_p can transfer energy to the primary.

- Assume no energy is lost, conservation of energy requires that

$$I_p V_p = I_s V_s \quad \Rightarrow \quad I_s = I_p \frac{N_p}{N_s} \quad \text{transformation of currents}$$

$$\Rightarrow \quad I_p = I_s \frac{N_s}{N_p} = \frac{N_s}{N_p}\frac{V_s}{R} = \left(\frac{N_s}{N_p}\right)^2 \frac{V_p}{R} \equiv \frac{V_p}{R_{\text{eq}}} \quad \Rightarrow \quad R_{\text{eq}} = \left(\frac{N_p}{N_s}\right)^2 R$$

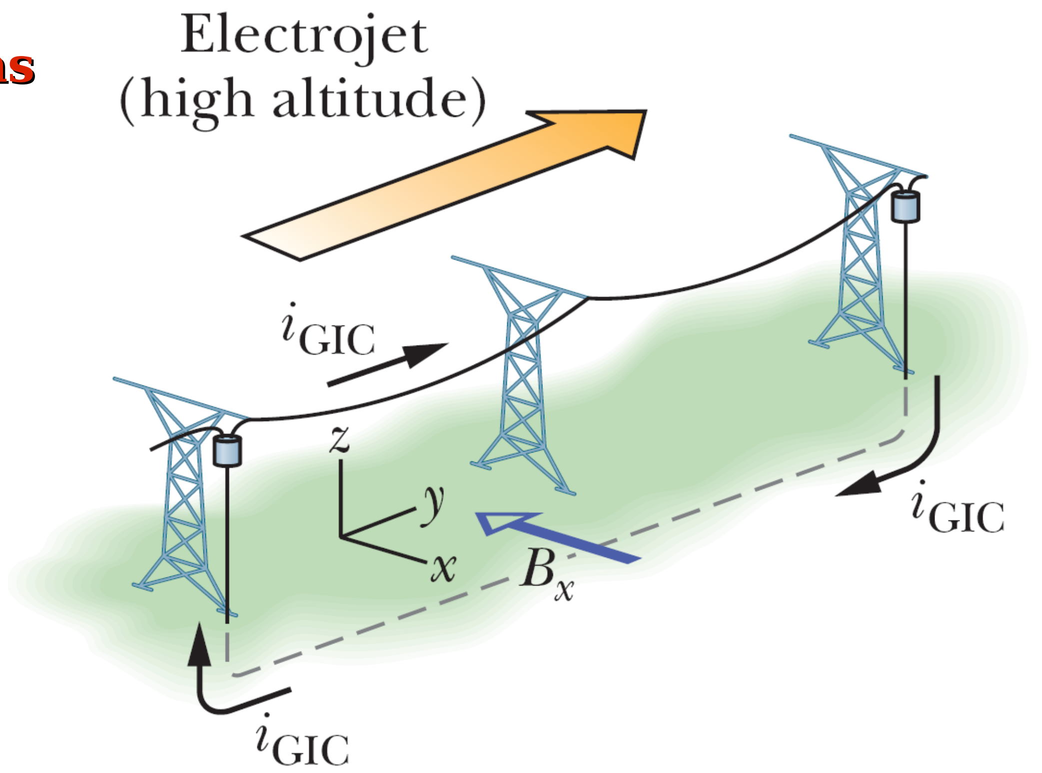
Impedance Matching

- For maximum transfer of energy from an emf device to a load, the impedance of the emf device must equal the impedance of the load. We can match the impedances of the two devices by coupling them through a transformer that has a suitable turns ratio.

Solar Activity and Power-Grid Systems

problem 31-8

Selected problems: 22, 26, 36,50, 58



Impedance Matching and Maximum Power Transfer

$$R_S \neq 0 \quad \Rightarrow \quad I = \frac{V_S}{R_S + R_L}$$

$$\Rightarrow \quad P_L \equiv I^2 R_L = \frac{V_S^2 R_L}{(R_S + R_L)^2}$$

To find $P_{L,\max}$ $\Rightarrow$ $\dfrac{d P_L}{d R_L} = 0$

$$\Rightarrow \quad R_L = R_S \quad \Rightarrow \quad P_{L,\max} = \frac{V_S^2}{4 R_S}$$