Chapter 26 Current and Resistance

Electric Current

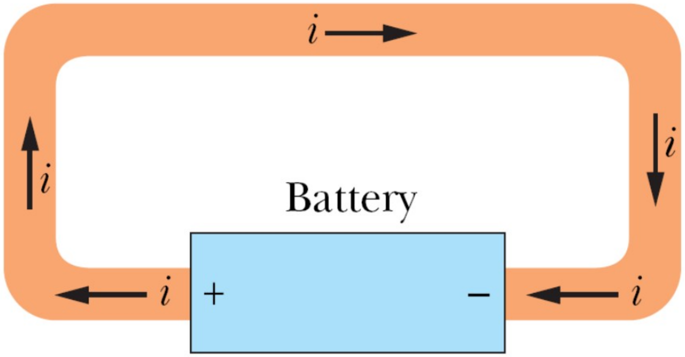
- Although an electric current is a stream of moving charges, not all moving charges constitute an electric current. If there is to be an electric current through a given surface, there must be a net flow of charge through that surface.

(a)

- 2 examples for clarification:

  1. The free electrons (conduction electrons) in an isolated length of copper wire;

  2. The flow of water through a garden hose.

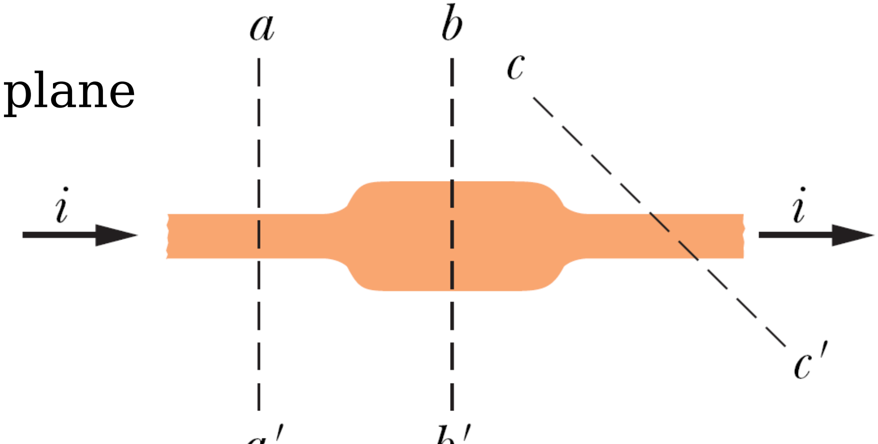


(b)

- study *steady* currents (not vary with time) of *conduction electrons* moving through *metallic conductors*.

- If charge dq passes through a hypothetical plane in time dt, then the current i through that plane is

$$i = \frac{dq}{dt} \quad \text{definition of current}$$



- The charge that passes through the plane in a time interval extending from 0 to t

$$q = \int dq = \int_0^t i\,dt$$

- Under steady-state conditions, the current is the same for all planes that pass completely through the conductor, no matter what their location or orientation.

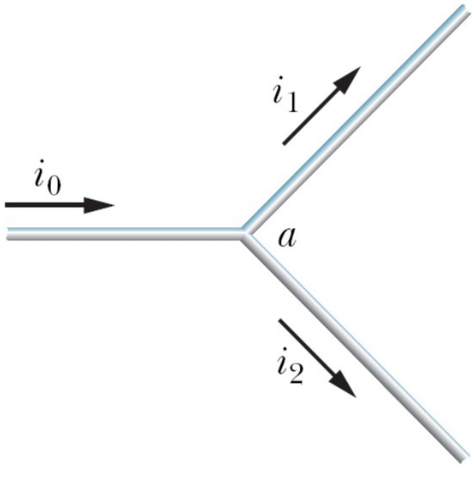
- The SI unit for current is the coulomb per second, or the ampere (A), $1 \text{ Ampere} = 1 \text{ A} = 1 \text{ coulomb per second} = 1 \text{ C/s}$

- Current is a scalar because both charge and time in that definition are scalars.

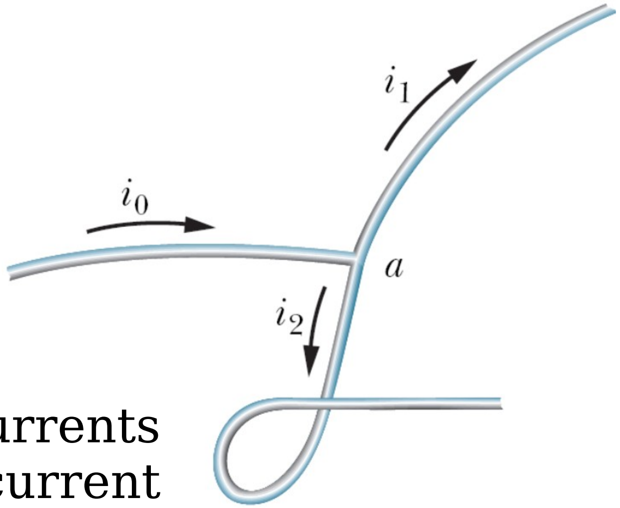
- We represent a current with an arrow to indicate that charge is moving. Such arrows are not vectors and they do not require vector addition.

- Because charge is conserved, the magnitudes of the currents in the branches must add to yield the magnitude of the current in the original conductor, $i_0 = i_1 + i_2$

- Current arrows show only a direction (or sense) of flow along a conductor, not a direction in space.



(a)



(b)

The Directions of Currents

- The current arrows are drawn in the direction in which positively charged particles would be forced to move through the loop by the electric field.

- Actually, the charge carriers in a conductor loop are electrons and thus are negatively charged. The charge carriers in the conductor loop are electrons and thus are negatively charged.

A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

problem 26-1

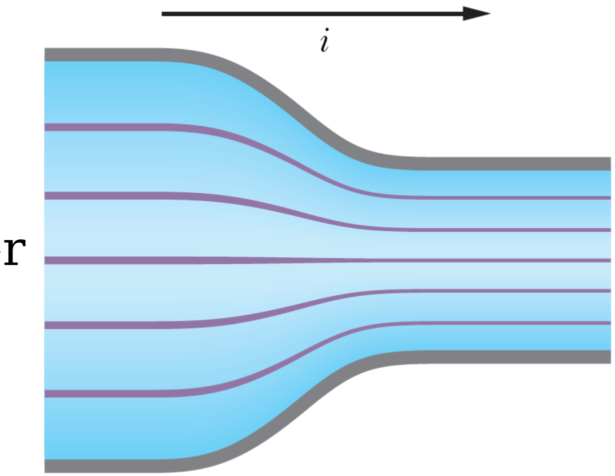
Current Density

- We define the **current density** to have the same direction as the velocity of the moving charges if they are positive and the opposite direction if they are negative. For each element of the cross section, the magnitude J is equal to the current per unit area through that element.

- The total current through a surface is $i = \int \vec{J} \cdot \mathrm{d}\vec{A}$

- If the current is uniform across and parallel to the surface,

$$i = \int J \, \mathrm{d}A = J \int \mathrm{d}A = J A \quad \Rightarrow \quad J = \frac{i}{A}$$



- The SI unit for current density is the ampere per square meter (A/m^2).

- In a transition from the wider conductor to the narrower conductor, the amount of current doesn't change because of the charge conservation, however, the current density does change.

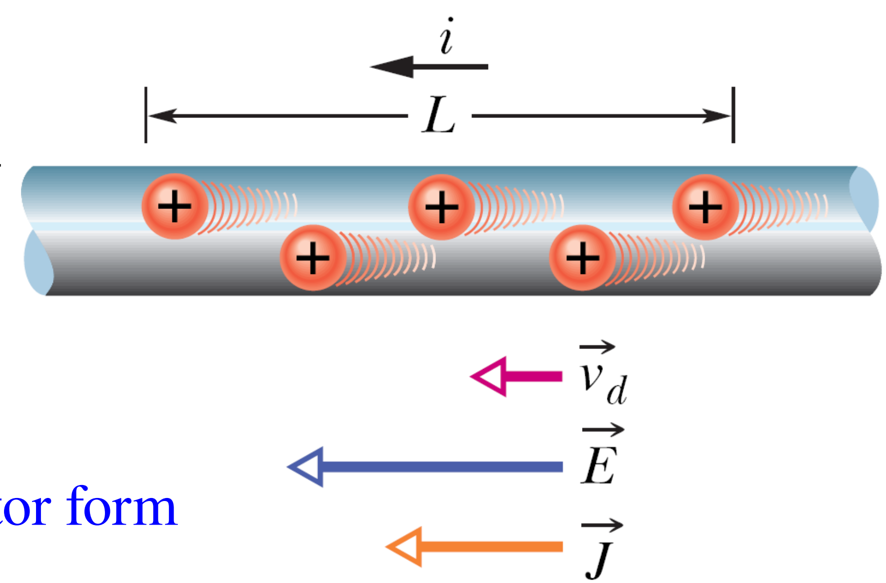
Drift Speed

- When a conductor have a current through it, the electrons actually move randomly, but they tend to drift with a **drift speed** v_d in the direction opposite that of the applied electric field that causes the current.

- Drift speed ~ 10^{-5} or 10^{-4} m/s, but random-motion speeds ~ 10^{6} m/s.

- The total charge of the carriers in the length L, each with charge e, is

$$q = (n\ A\ L)\ e$$



- If the carriers all move along the wire with speed v_d, this total charge moves through any cross section of the wire in the time interval

$$t = \frac{L}{v_d} \quad \Rightarrow \quad i = \frac{q}{t} = \frac{n\ A\ L\ e}{L / v_d} = n\ A\ e\ v_d$$

$$\Rightarrow \quad v_d = \frac{i}{n\ A\ e} = \frac{J}{n\ e} \quad \Rightarrow \quad \boxed{\vec{J} = n\ e\ \vec{v}_d} \quad \text{vector form}$$

- The product *ne*, whose SI unit is the coulomb per cubic meter (C/m^3), is the *carrier charge density*.

- For positive carriers, *ne* is positive, thus the current density and the drift velocity have the same direction. For negative carriers, *ne* is negative, thus the current density and the drift velocity and have opposite directions.

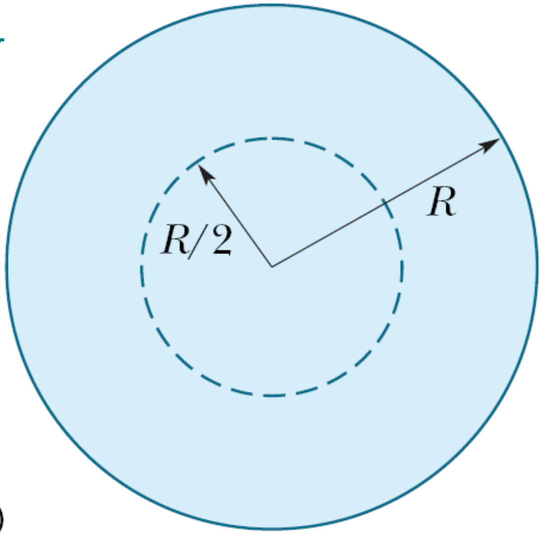
problem 26-3

problem 26-2

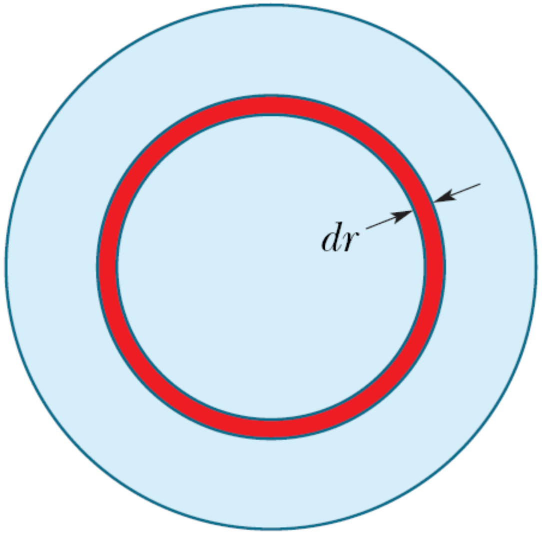
Resistance and Resistivity

- We determine the resistance between any two points of a conductor by applying a potential difference V between those points and measuring the current i that results,

$$R = \frac{V}{i} \quad \text{definition of } R$$



(a)

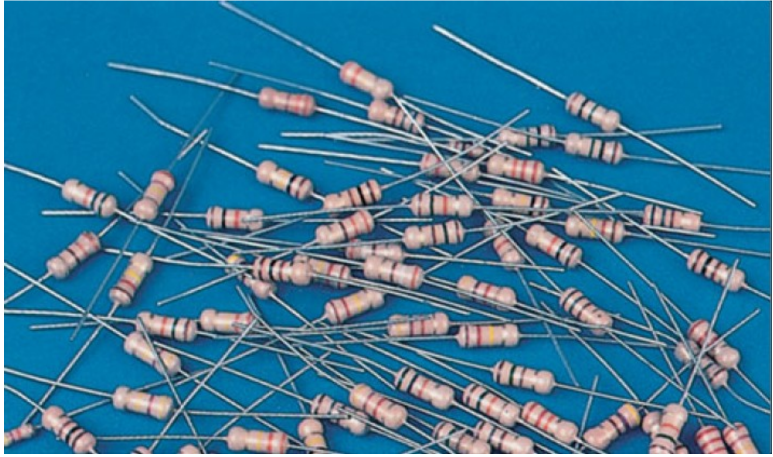


(b)

- The SI unit for resistance is the volt per ampere, or the ohm (symbol Ω),

$$1 \text{ ohm} = 1\ \Omega = 1 \text{ volt per ampere} = 1\ \text{V/A}$$

- A conductor whose function in a circuit is to provide a specified resistance is called a **resistor** (with the symbol $-\!\wedge\!\!\!\wedge\!\!\!\wedge\!-$).

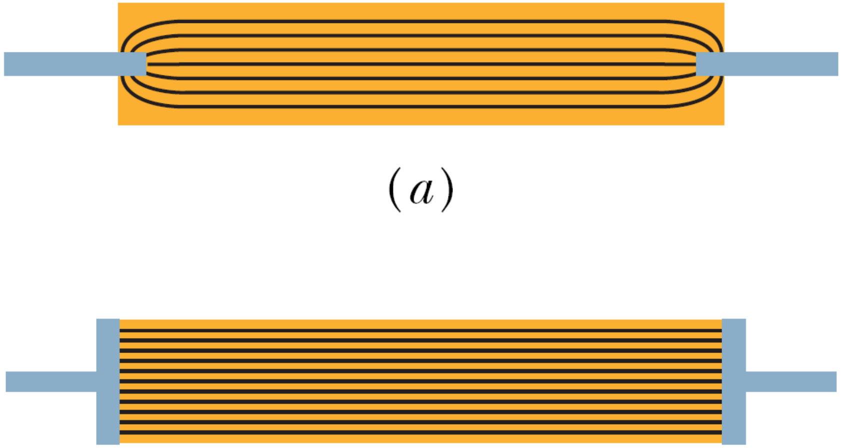


- From $i = V / R$, for a given potential difference, the greater the resistance, the smaller the current.

- we assume that any given potential difference is applied uniformly.

- Instead of the resistance R of an object, we can deal with the **resistivity** of the *material*:

$$\rho = \frac{E}{J} \text{ definition of } \rho \quad \Rightarrow \quad \vec{E} = \rho\, \vec{J}$$



(a)

(b)

- The unit of the resistivity is ohm-meter ($\Omega \cdot \text{m}$):

$$\frac{\text{unit}\,(E)}{\text{unit}\,(J)} = \frac{\text{V/m}}{\text{A/m}^2} = \frac{\text{V}}{\text{A}}\,\text{m} = \Omega \cdot \text{m}$$

- The equations above for the resistivity hold only for *isotropic* materials – materials whose electrical properties are the same in all directions.

- The **conductivity** of a material is simply the reciprocal of its resistivity,

$$\sigma = \frac{1}{\rho} \text{ definition of } \sigma$$

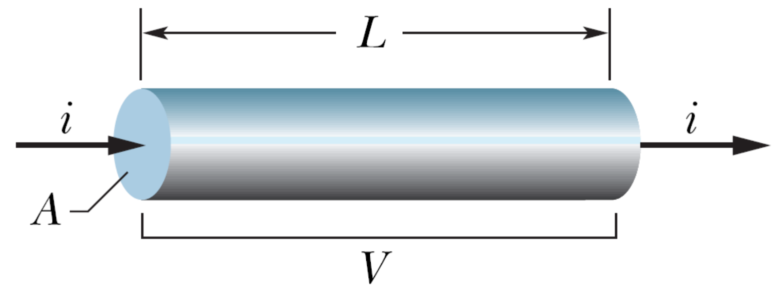
- The definition of conductivity allows us to write $\vec{J} = \sigma\, \vec{E}$

Calculating Resistance from Resistivity

Resistance is a property of an object. Resistivity is a property of a material.

- Since $E = \dfrac{V}{L}$ and $J = \dfrac{i}{A}$ and $R = \dfrac{V}{i}$

$$\Rightarrow \quad \rho = \frac{E}{J} = \frac{V/L}{i/A} = \frac{R\ A}{L} \quad \Rightarrow \quad R = \rho\,\frac{L}{A}$$



- The equation can be applied only to a homogeneous isotropic conductor of uniform cross section.

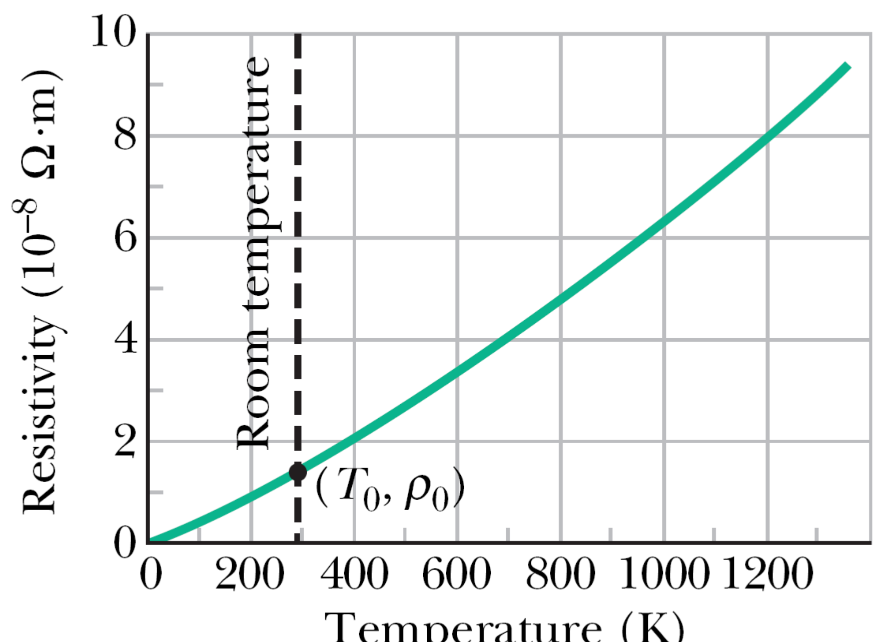
- The macroscopic quantities V, i, and R are of greatest interest when we are making electrical measurements on specific conductors. We turn to the microscopic quantities E, J, and ρ when we are interested in the fundamental electrical properties of materials.

Variation with Temperature

- The relation between temperature and resistivity for metals is fairly linear over a rather broad temperature range.

- An empirical approximation

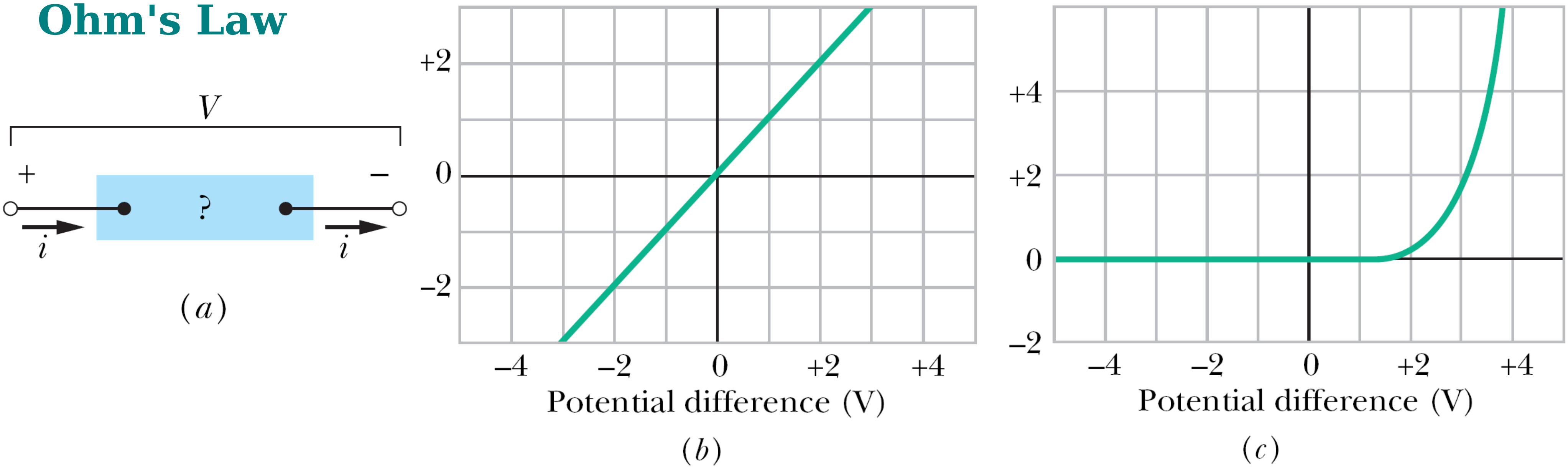
$$\rho - \rho_0 = \rho_0\,\alpha\,(T - T_0)$$



Here T_0 is a selected reference temperature and ρ_0 is the resistivity at that temperature. The quantity α called the temperature coefficient of resistivity, is chosen so that the equation gives good agreement with experiment for temperatures in the chosen range.

problem 26-4

Ohm's Law



Ohm's law is an assertion that the current through a device is always directly proportional to the potential difference applied to the device.

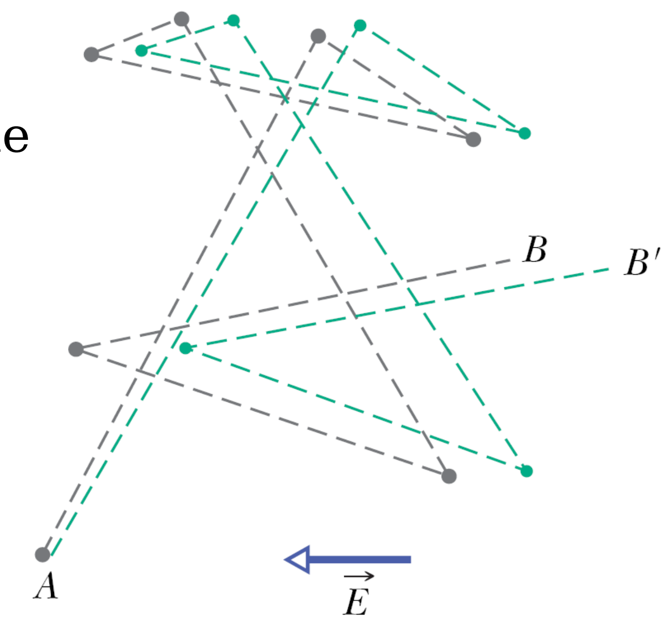
A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

- $V = i\,R$ is not a statement of Ohm's law. This equation is the defining equation for resistance, and it applies to all conducting devices, whether they obey Ohm's law or not.

- If we focus on conducting materials, then

A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

A Microscopic View of Ohm's Law



- The *free-electron model*, in which we assume that the conduction electrons in the metal are free to move throughout the volume of a sample.

- Conduction electrons in a metal move with a single effective speed v_{eff}, and this speed is essentially independent of the temperature.

- When we apply an electric field to a metal sample, the electrons modify their random motions slightly and drift very slowly in a direction opposite that of the field with an average drift speed v_d.

- If an electron of mass m is placed in an electric field of magnitude E, then

$$a = \frac{F}{m} = \frac{e\ E}{m}$$

- In the average time τ between collisions, the average electron will acquire a drift speed of

$$J = n\, e\, v_d \quad \Rightarrow \quad \frac{J}{n\, e} \Leftarrow v_d = a\, \tau = \frac{e\, E\, \tau}{m} \quad \Rightarrow \quad E = \frac{m}{e^2\, n\, \tau}\, J$$

- For $\vec{E} = \rho\, \vec{J} \quad \Rightarrow \quad \rho = \frac{m}{e^2\, n\, \tau}$

- Because n, m, and e are constant, and τ, the average time (or *mean free time*) between collisions, is a constant, therefore, the resistivity ρ is a constant, independent of the strength of the applied electric field. Thus metals obey Ohm's law.

problem 26-5

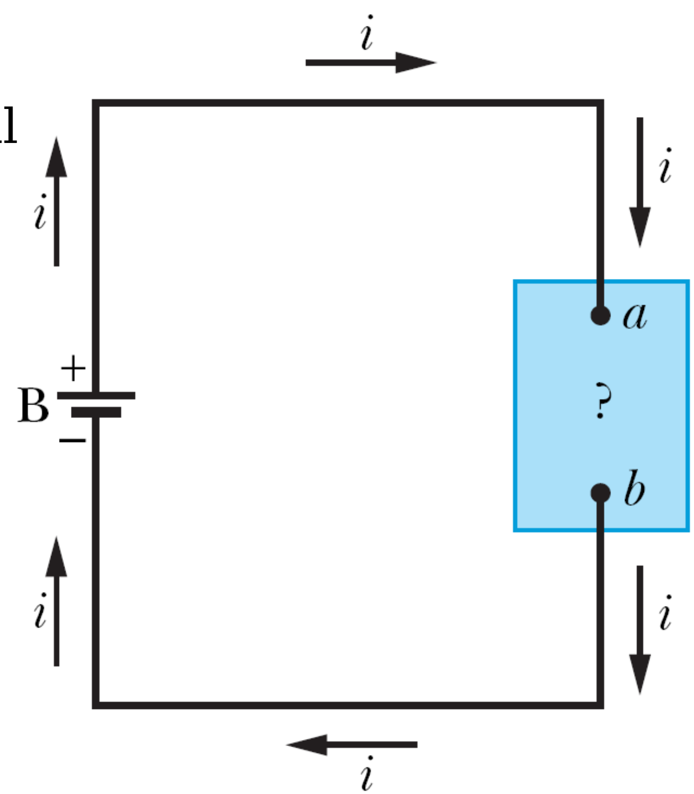
Power in Electric Circuits

- This charge dq moves through a decrease in potential of magnitude V, and thus its electric potential energy decreases in magnitude by the amount

$$dU = V\, dq = V\, i\, dt$$

- The power P associated with that transfer is the rate of transfer dU/dt, which is

$$P = V\, i \quad \text{rate of electrical energy transfer}$$



- This power P is also the rate at which energy is transferred from the battery to the unspecified device.

- The unit of power is the volt-ampere ($V \cdot A$): $1\ \mathrm{V \cdot A} = 1\ \frac{\mathrm{J}}{\mathrm{C}} \cdot 1\ \frac{\mathrm{C}}{\mathrm{s}} = 1\ \frac{\mathrm{J}}{\mathrm{s}} = 1\ \mathrm{W}$

- The lost electric potential energy of an electron in the wire appears as thermal energy in the resistor and the surroundings. The energy transfer is due to collisions between the electron and the molecules of the resistor, which leads to an increase in the temperature of the resistor lattice. The mechanical energy thus transferred to thermal energy is *dissipated* (lost) because the transfer cannot be reversed.

- For the rate of electrical energy dissipation due to a resistance,

$$P = i^2 R \quad \text{resistive dissipation}$$

$$\text{or} \quad P = \frac{V^2}{R} \quad \text{resistive dissipation}$$

- $P = V i$ applies to electrical energy transfers of all kinds; $P = i^2 R$ and $P = V^2/R$ apply only to the transfer of electric potential energy to thermal energy in a device with resistance.

problem 26-6

Semiconductors

- Pure silicon has such a high resistivity that it is effectively an insulator and thus not of much direct use in microelectronic circuits.

- Its resistivity can be greatly reduced in a controlled way by adding minute amounts of specific “impurity” atoms in a process called *doping*.

- In a metallic conductor, most of the electrons are firmly locked in place within the atoms. However, there are also some electrons that are only loosely held in place and that require only little energy to become free. Thermal energy or an electric field can supply that energy. The electric field would not only free these loosely held electrons but would also propel them along the wire; thus, the field would drive a current through the conductor.

- In an insulator, significantly greater energy is required to free electrons. Thus, no electrons are available to move through the insulator, and hence no current occurs even with an applied electric field.

- A semiconductor is like an insulator *except* that the energy required to free some electrons is not quite so great.

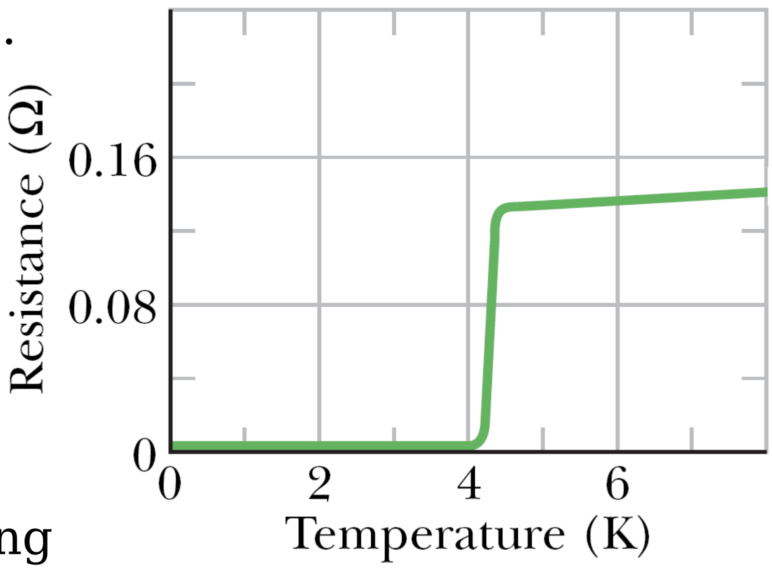
- Doping can supply electrons or positive charge carriers that are very loosely held within the material and thus are easy to get moving. Moreover, by controlling the doping of a semiconductor, we can control the density of charge carriers that can participate in a current and thereby can control some of its electrical properties.

- Most semiconducting devices are fabricated by the selective doping of different regions of the silicon with impurity atoms of different kinds.

- the resistivity expression can be also applied to semiconductor: $\rho = \dfrac{m}{e^2\, n\, \tau}$

- In a semiconductor, n is small but increases very rapidly with temperature as the increased thermal agitation makes more charge carriers available. This causes a *decrease* of resistivity with increasing temperature.

Superconductors



- In 1911, a Dutch physicist discovered that the resistivity of mercury absolutely disappears at temperatures below about $4K$, the phenomenon of **superconductivity**.

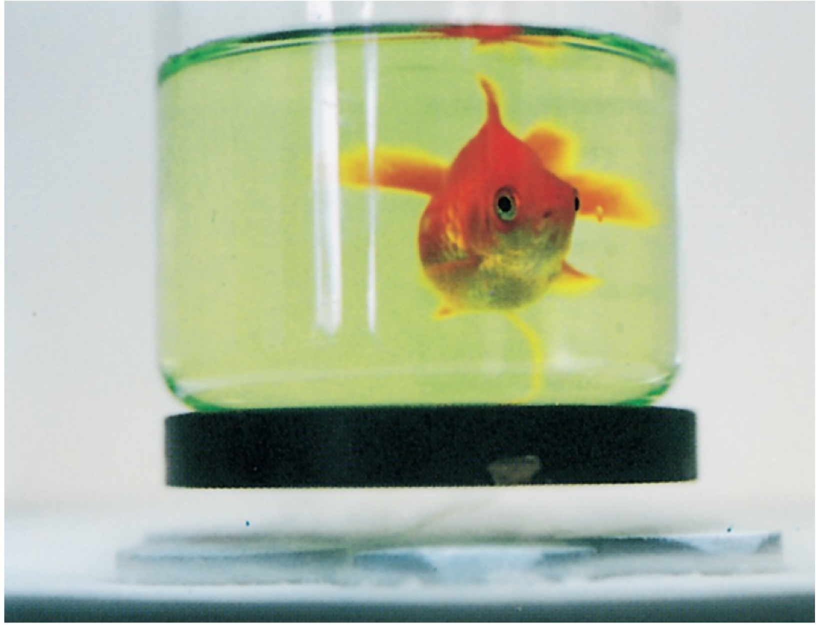
- Superconductivity means that charge can flow through a superconducting conductor without losing its energy to thermal energy.

- In 1986, new ceramic materials were discovered that become superconducting at considerably higher temperatures.

- One explanation for superconductivity is that the electrons that make up the current move in coordinated pairs, so-called **Cooper-pair**s. One of the electrons in a pair may electrically distort the molecular structure as it moves through, creating nearby a short-lived concentration of positive charge.

- The other electron in the pair may then be attracted toward this positive charge. such coordination between electrons would prevent them from colliding with the molecules and thus would eliminate electrical resistance.

- new theories are needed for the newer, high-Tc superconductors.



Selected problems: 10, 18, 24, 46, 48