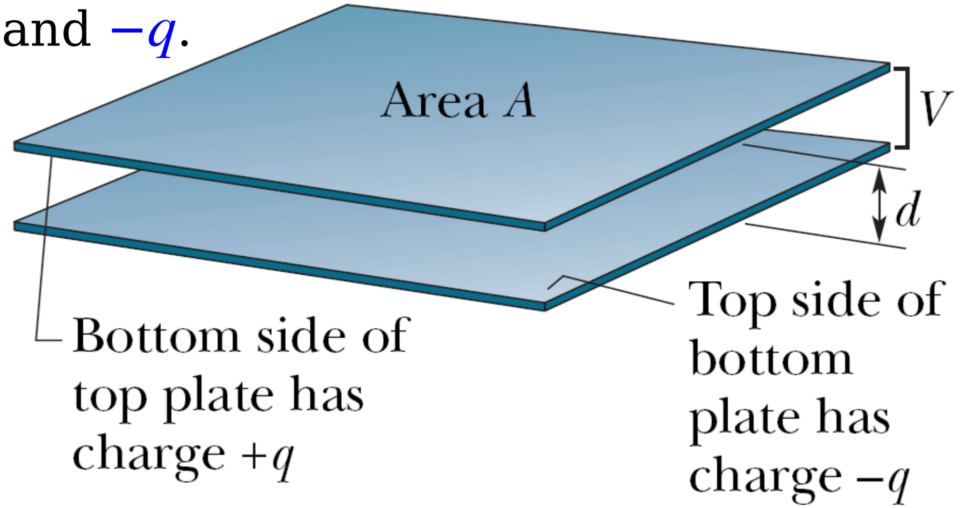
Chapter 25 Capacitance

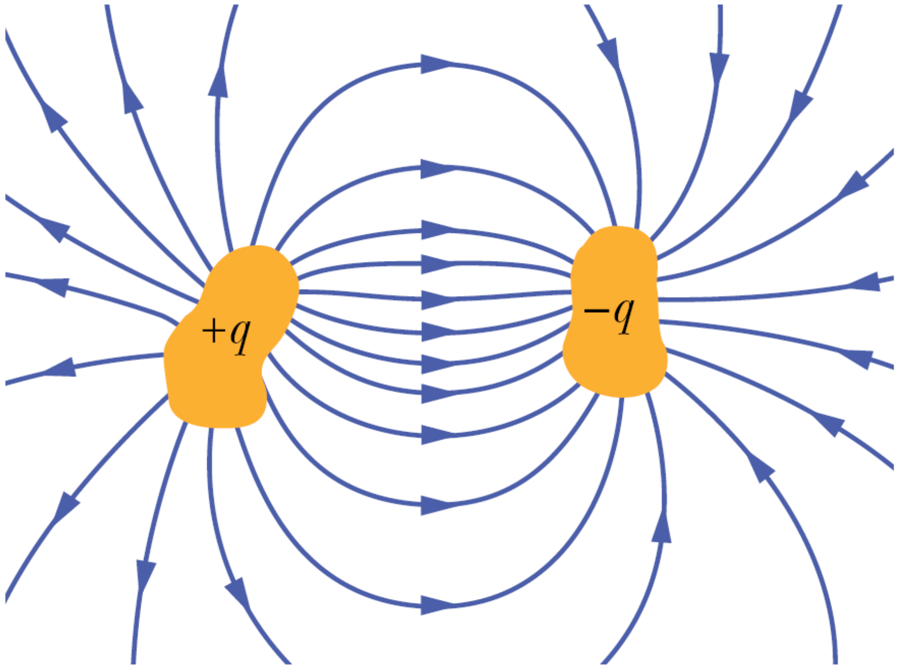
- the basic elements of *any* capacitor: 2 isolated conductors of any shape.

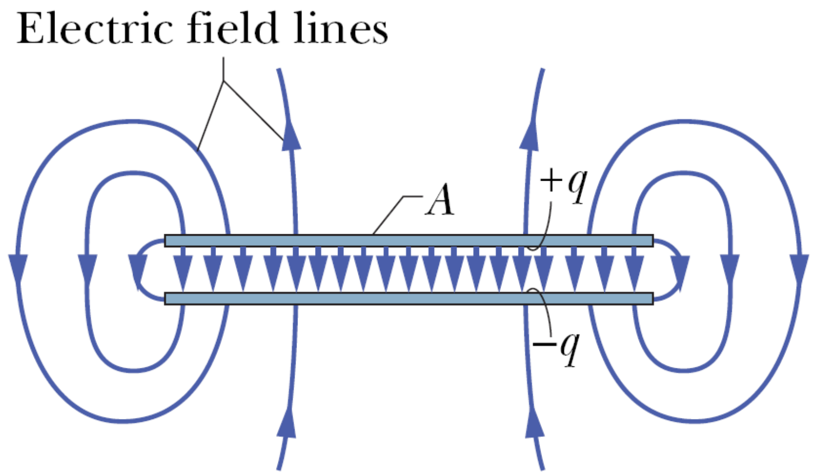
- *parallel-plate capacitor*: 2 parallel conducting plates of area A separated by a distance d.

- When a capacitor is *charged,* its plates have charges of equal magnitudes but opposite signs: $+q$ and $-q$.

- we refer to the charge of a capacitor as being q.





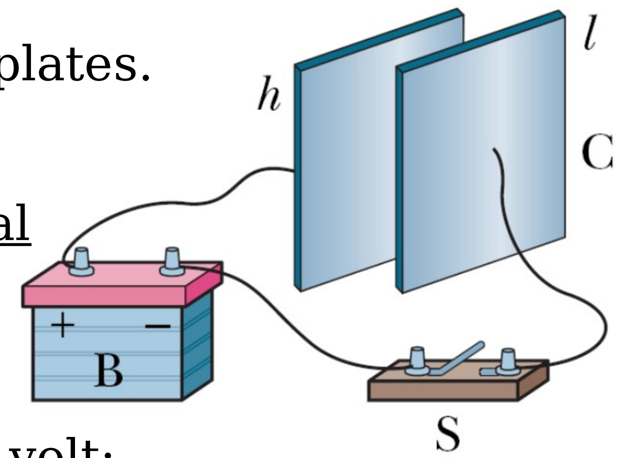


- The charge q and the potential difference V for a capacitor are proportional to each other;

$$q = C\,V$$ C is called the **capacitance** of the capacitor.

- The value of C depends only on the geometry of the plates.

- The capacitance is a measure of <u>how much charge must be put on the plates to produce a certain potential difference between them</u>: *The greater the capacitance, the more charge is required.*



(a)

- The SI unit of capacitance, farad, is the coulomb per volt:

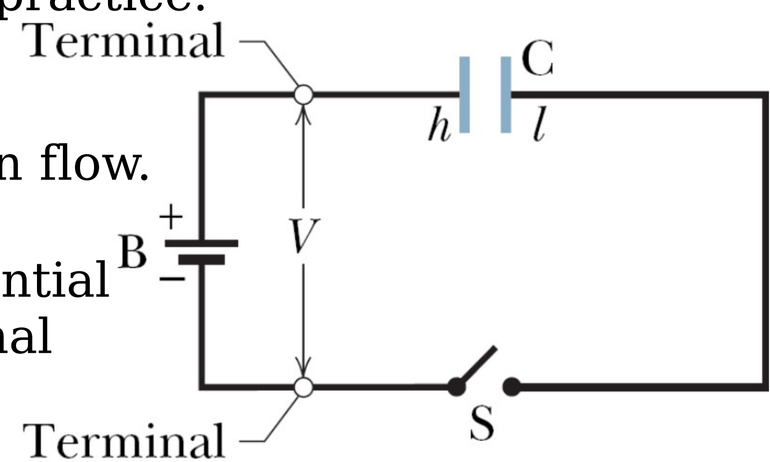
$$1 \text{ farad} = 1 \text{ F} = 1 \text{ coulomb per volt} = 1 \text{ C/V}$$

- Usually we use μF ($=10^{-6}$ F) and pF ($=10^{-12}$ F) in practice.

Charging a Capacitor

- *Electric circuit* is a path through which charge can flow.

- A *battery* is a device that maintains a certain potential difference between its terminals by means of internal electrochemical reactions.



(b)

- The circuit is said to be incomplete because switch S is *open*. When the switch is *closed*, electrically connecting those wires, the circuit is complete.

- When the circuit is completed, electrons are driven through the wires by an electric field that the battery sets up in the wires.

- The field drives electrons from capacitor plate h to the positive terminal of the battery; thus, plate h, becomes positively charged.

- The field drives just as many electrons from the negative terminal of the battery to capacitor plate ℓ; thus, plate ℓ, gaining electrons, becomes negatively charged just as much as plate h.

- As the plates become oppositely charged, that potential difference increases until it equals the potential difference V between the terminals of the battery.

Calculating the Capacitance

- The plan of calculating the capacitance:

(1) Assume a charge q on the plates;

(2) Calculate the electric field between the plates in terms of this charge, using Gauss's law;

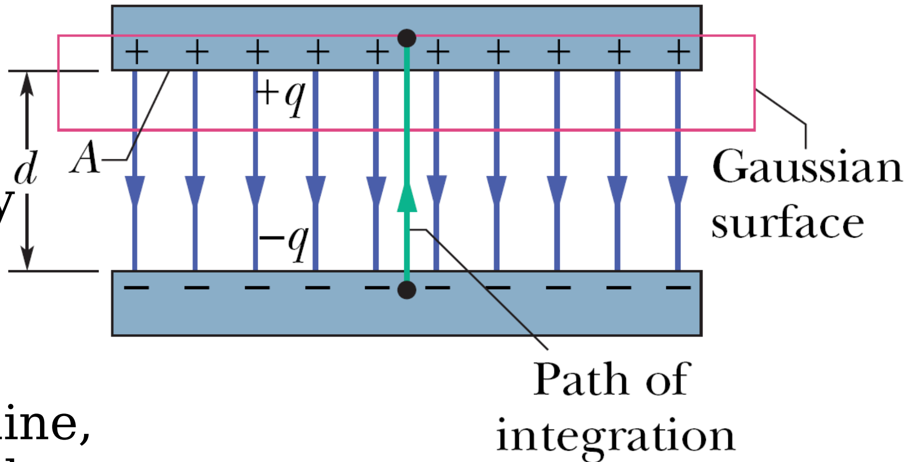
(3) Knowing the electric field, calculate the potential difference V between the plates;

(4) Calculate C from $q = CV$.

Calculating the Electric Field

- Use Gauss's law to relate the electric field between the plates of a capacitor to the charge q on either plate, $\epsilon_0 \oint \vec{E} \cdot d\vec{A} = q$
- If the electric field is uniform and parallel to the surface vector, then

$$q = \epsilon_0 E A \quad \text{special case}$$



Calculating the Potential Difference

- The potential difference between the plates of a capacitor is related to the electric field by

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

- Choose a path that follows an electric field line, from the negative plate to the positive plate, then

$$V \equiv V_f - V_i = \int_-^+ E\, ds \quad \text{special case of the above equation}$$

A Parallel-Plate Capacitor

- Follow the plan:

$$(1)\quad \text{charge } q \qquad (2)\quad q = \epsilon_0 E A$$

$$(3)\quad V = \int_-^+ E\, ds = E \int_0^d ds = E d \qquad (4)\quad q = C V$$

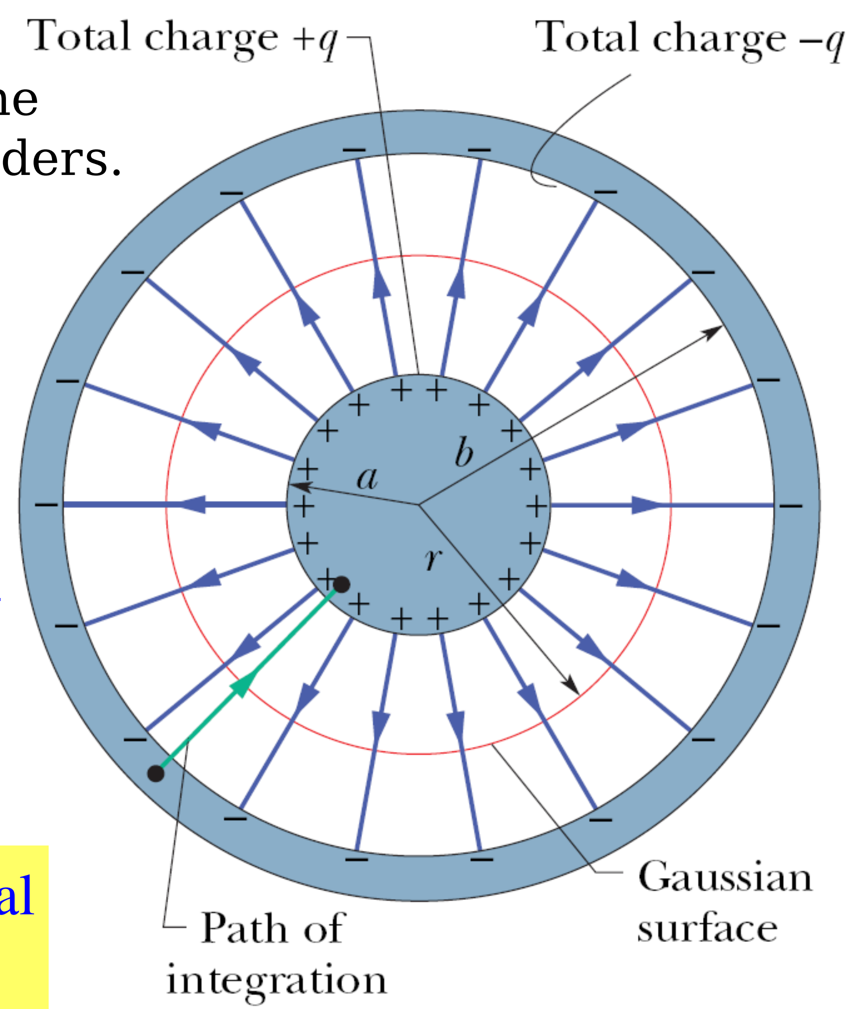
we find $C = \epsilon_0 \dfrac{A}{d}$ parallel-plate capacitor

- The capacitor C increases with the area A and decrease with the separation d.

- We can use the capacitance to express the permittivity constant ϵ_0

$$\epsilon_0 = 8.85\times 10^{-12}\ \mathrm{F/m} = 8.85\ \mathrm{pF/m}\,[\,= 8.85\times 10^{-12}\ \mathrm{C^2/(N\cdot m^2)}]$$

A Cylindrical Capacitor



- Assume that $L \gg b$ to neglect the fringing of the electric field that occurs at the ends of the cylinders.

(1) charge q

(2) find $\vec{E}$: $q = \epsilon_0 E A = \epsilon_0 E (2\pi r L)$

$$\Rightarrow \quad E = \frac{q}{2\pi\epsilon_0 L r}$$

(3) find V : $V = \int_-^+ E\,ds = -\frac{q}{2\pi\epsilon_0 L}\int_b^a \frac{dr}{r}$

$$= \frac{q}{2\pi\epsilon_0 L}\ln\frac{b}{a}$$

(4) $q = CV \Rightarrow$ $C = 2\pi\epsilon_0 \frac{L}{\ln(b/a)}$ cylindrical capacitor

A Spherical Capacitor

- With the charge q, find $\vec{E}$: $q = \epsilon_0 E A = \epsilon_0 E (4\pi r^2) \Rightarrow E = \frac{q}{4\pi\epsilon_0 r^2}$

find V : $V = \int_-^+ E\,ds = -\frac{q}{4\pi\epsilon_0}\int_b^a \frac{dr}{r^2} = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{q}{4\pi\epsilon_0}\frac{b-a}{ab}$

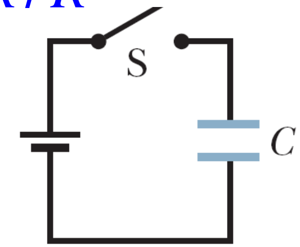
$$q = C\,V \quad \Rightarrow \quad \boxed{C = 4\,\pi\,\epsilon_0 \frac{a\,b}{b-a} \quad \text{spherical capacitor}}$$

An Isolated Sphere

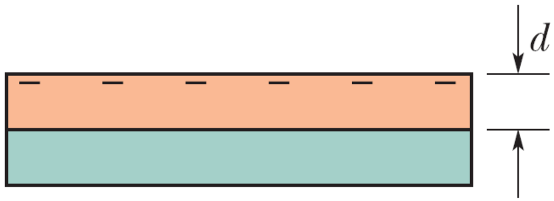
- We can assign a capacitance to a single isolated spherical conductor of radius R by assuming that the missing plate is a conducting sphere of infinite radius:

$$C = \lim_{R' \to \infty} 4\,\pi\,\epsilon_0 \frac{R\,R'}{R' - R} = \lim_{R' \to \infty} 4\,\pi\,\epsilon_0 \frac{R}{1 - R/R'} = 4\,\pi\,\epsilon_0\,R$$

$$\Rightarrow \quad \boxed{C = 4\,\pi\,\epsilon_0\,R \quad \text{isolated sphere}}$$

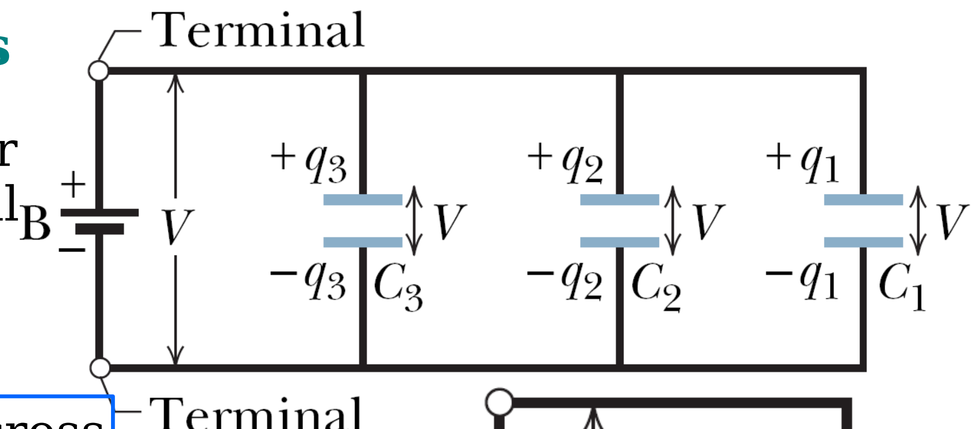


problem 25-1



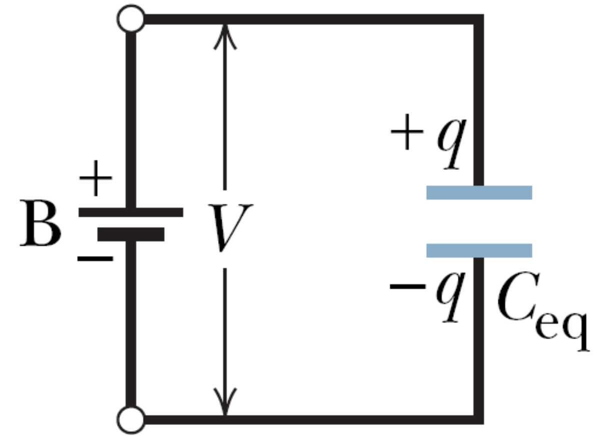
Capacitors in Parallel and in Series

- **Equivalent capacitor**: a single capacitor that has the same capacitance as the actual combination of capacitors.



Capacitors in Parallel

When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor. The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.



- To analyze a circuit of capacitors in parallel, we can simplify it with this mental replacement:

> Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same *total* charge q and the same potential difference V as the actual capacitors.

- Find the charge on each actual capacitor: $q_1 = C_1 V$, $q_2 = C_2 V$, $q_3 = C_3 V$
- The total charge is $q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3) V$
- The equivalent capacitance, with the same total charge q and applied potential difference V as the combination, is $C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3$
- Generalization: $C_{eq} = \sum_{j=1}^{n} C_j \quad n$ capacitors in parallel

Capacitors in Series

> When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q. The sum of the potential differences across all the capacitors is equal to the applied potential difference V.

- To analyze a circuit of capacitors in series, we can simplify it with this mental replacement:

Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same total potential difference V as the actual series capacitors.

- Find the potential difference of each actual capacitor:

$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad V_3 = \frac{q}{C_3}$$

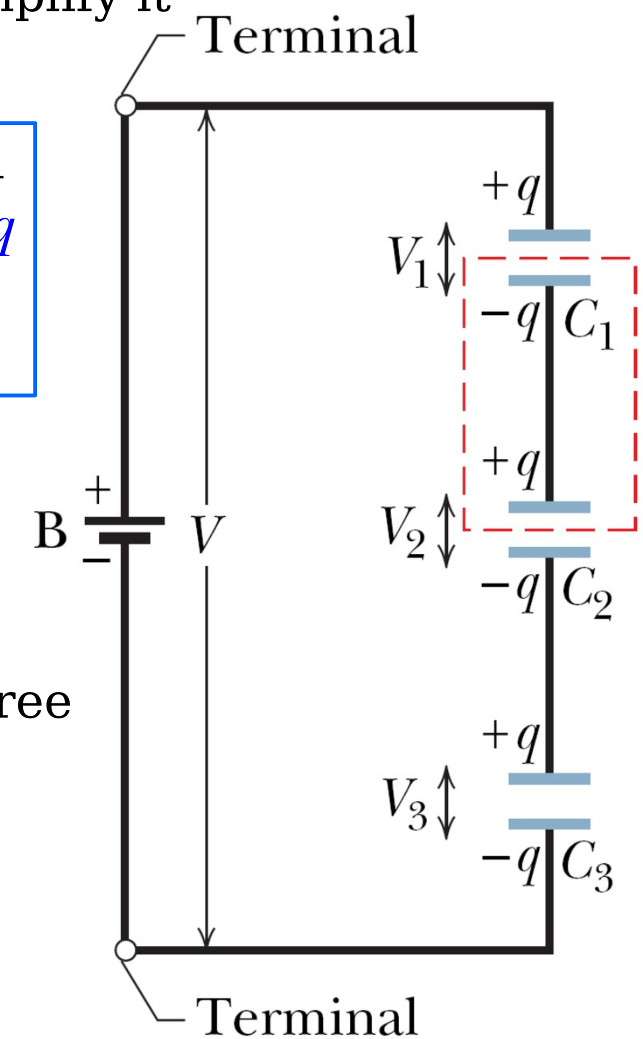
- The total potential difference V is the sum of these three potential differences:

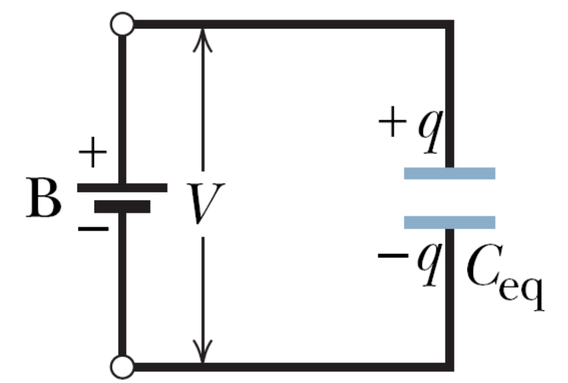
$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

- The equivalent capacitance is then

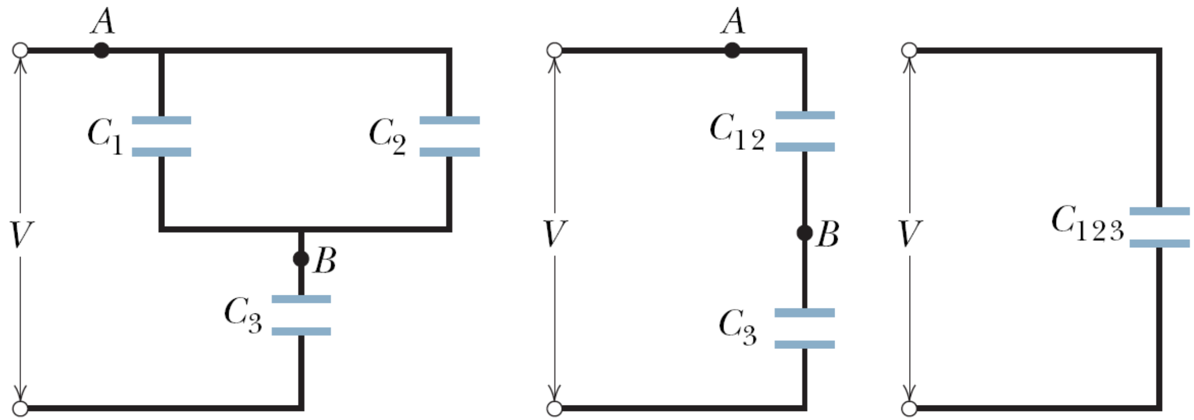
$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3} \quad \Rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- Generalization: $\displaystyle \frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j}$ n capacitors in series



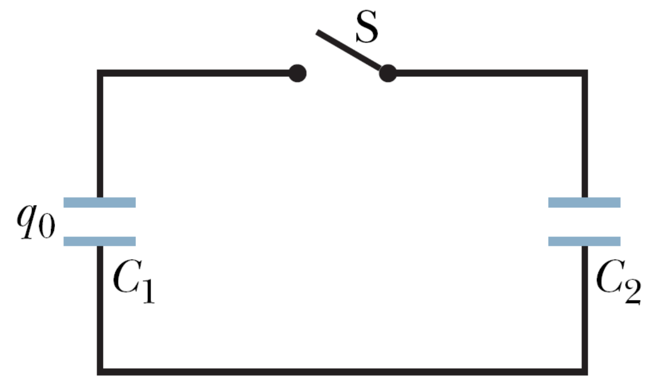


problem 25-2



(a) (b) (c)

problem 25-3



Energy Stored in an Electric Field

- Suppose a charge q' has been transferred from one plate of a capacitor to the other, then the potential difference V' between the plates will be q'/C.

- If an extra increment of charge dq' is then transferred, the increment of work required will be,

$$dW = V'\,dq' = \frac{q'}{C}\,dq'$$

- The work required to bring the total capacitor charge up to a final value q is

$$W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C} = \frac{1}{2}CV^2$$

- This work is stored as potential energy U in the capacitor,

$$\boxed{U = \frac{q^2}{2C} = \frac{1}{2}CV^2 \quad \text{potential energy}}$$

- If there are 2 parallel-plate capacitors, the area $A_1=A_2$ and the seperations $d_1=2d_2$, then $C_1=C_2/2$, $E_1=E_2$, therefore $U_1=2U_2$, thus

> The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

The Medical Defibrillator

- If in a defibrillator C=70μF, V=5000V, then the energy in the capacitor is

$$U = \frac{1}{2} C V^2 = \frac{1}{2}\left(70\times 10^{-6}\ \mathrm{F}\right)\left(5000\ \mathrm{V}\right)^2 = 857\ \mathrm{J}$$

- About 200 J is sent through the victim in 2.0 ms, then the power is

$$P = \frac{U}{t} = \frac{200\ \mathrm{J}}{2.0\times 10^{-3}\ \mathrm{s}} = 100\ \mathrm{kW}$$

Explosions in Airborne Dust

- In many industries involving the production and transport of powder, such as in the cosmetic and food industries, a spark can be disastrous.

- When individual powder grains are airborne and thus surrounded by oxygen, they can burn so fiercely that a cloud of the grains burns as an explosion.

- Engineers cannot eliminate all possible sources of sparks. So they keep the energy available in the sparks below the threshold value U_t (~150 mJ) typically required to ignite airborne grains.

- Suppose a person becomes charged by contact with various surfaces as he walks through an airborne powder.

- Roughly model the person as a spherical capacitor of radius $R = 1.8$ m,

$$\Rightarrow \quad C = 4\pi\epsilon_0 R \quad \Rightarrow \quad U = \frac{C}{2}V^2 = 2\pi\epsilon_0 R V^2$$

$$\Rightarrow \quad \text{threshold potential} \;\; V = \sqrt{\frac{U_t}{2\pi\epsilon_0 R}} = \sqrt{\frac{0.15\ \text{J}}{2\pi\left(8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.8\ \text{m}\right)}}$$

$$= 3.9\times 10^4\ \text{V}$$

- Safety engineers attempt to keep the potential of the personnel below this level by "bleeding" off the charge through, say, a conducting floor.

Energy Density

- the **energy density** u: the potential energy per unit volume

- In a parallel-plate capacitor, neglecting fringing, the electric field is uniform, and the energy density is also uniform,

$$u = \frac{U}{A\,d} = \frac{C\,V^2}{2\,A\,d} = \frac{1}{2}\epsilon_0\left(\frac{V}{d}\right)^2 \quad \Leftarrow \quad C = \epsilon_0\frac{A}{d}$$

$$E = \frac{V}{d} \quad \Rightarrow \quad \boxed{u = \frac{1}{2}\epsilon_0 E^2 \quad \text{energy density}}$$

- The above equation stands for general cases.

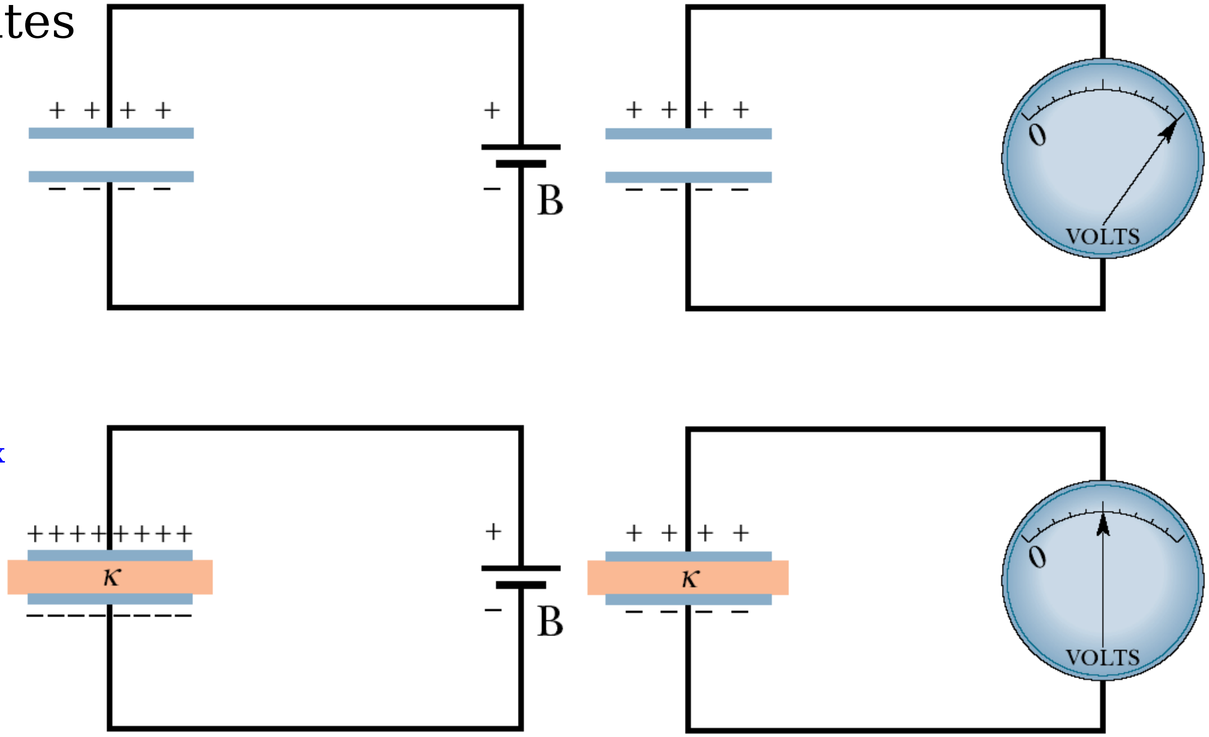
problem 25-4

Capacitor with a Dielectric

- Fill the space between the plates of a capacitor with a *dielectric*, the capacitance increased by a numerical factor κ, which is called the **dielectric constant** of the insulating material.

- If the *breakdown potential* V_{max} is substantially exceeded, the dielectric material will break down and form a conducting path between the plates.

- The *dielectric strength* is the maximum value of the electric field that it can tolerate without breakdown.



V = a constant (a)

q = a constant (b)

- The capacitance of any capacitor can be written in the form $C = \epsilon_0 \mathscr{L}$, where $\mathscr{L}$ has the dimension of length.

- With a dielectric *completely* filling the space between the plates,

$$C = \kappa \, \epsilon_0 \mathscr{L} = \kappa \, C_{\text{air}}$$

In a region completely filled by a dielectric material of dielectric constant κ, all electrostatic equations containing the permittivity constant ϵ_0 are to be modified by replacing ϵ_0 with $\kappa\epsilon_0$.

- The magnitude of the electric field produced by a point charge inside a dielectric is given by this modified form

$$E = \frac{1}{4\pi\kappa\epsilon_0}\frac{q}{r^2}$$

- The electric field just outside an isolated conductor immersed in a dielectric becomes

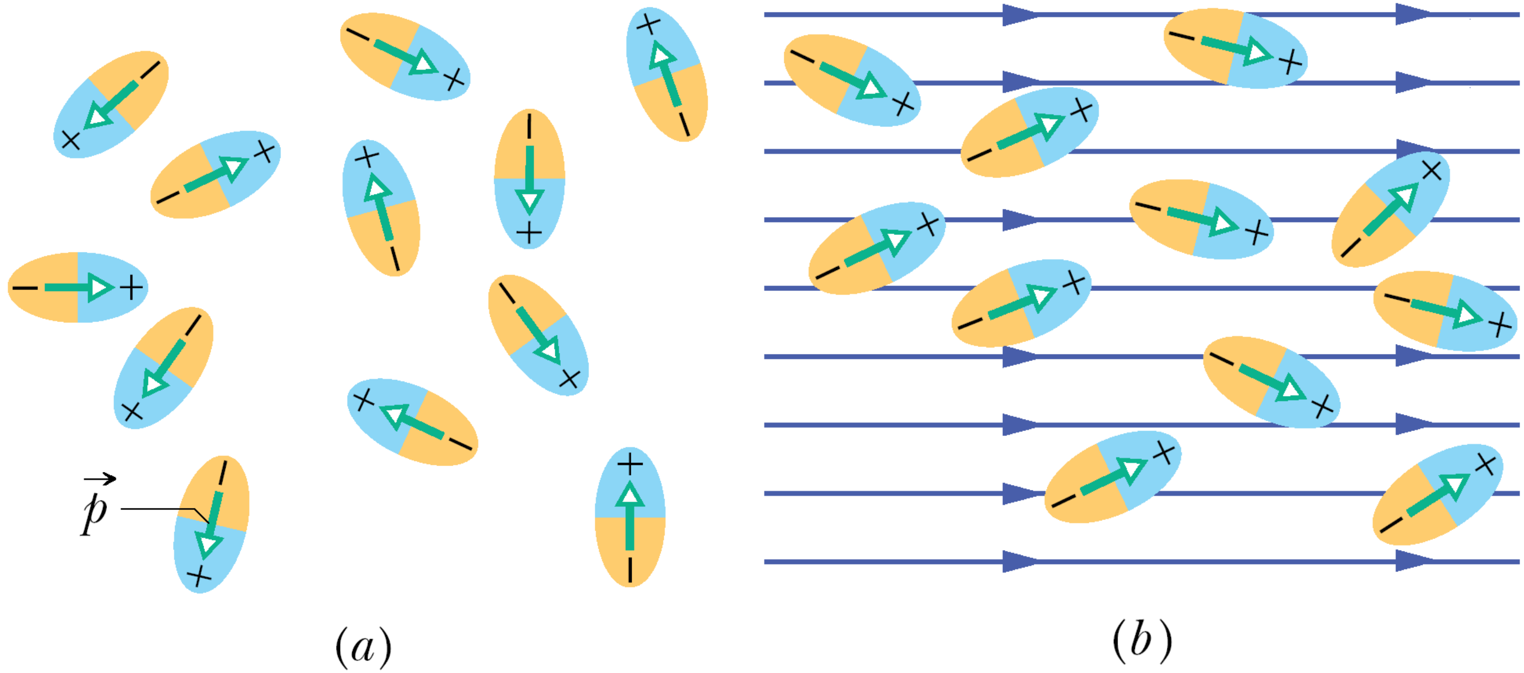
$$E = \frac{\sigma}{\kappa\,\epsilon_0}$$

problem 25-5

Dielectrics: An Atomic View

- When we put a dielectric in an electric field,

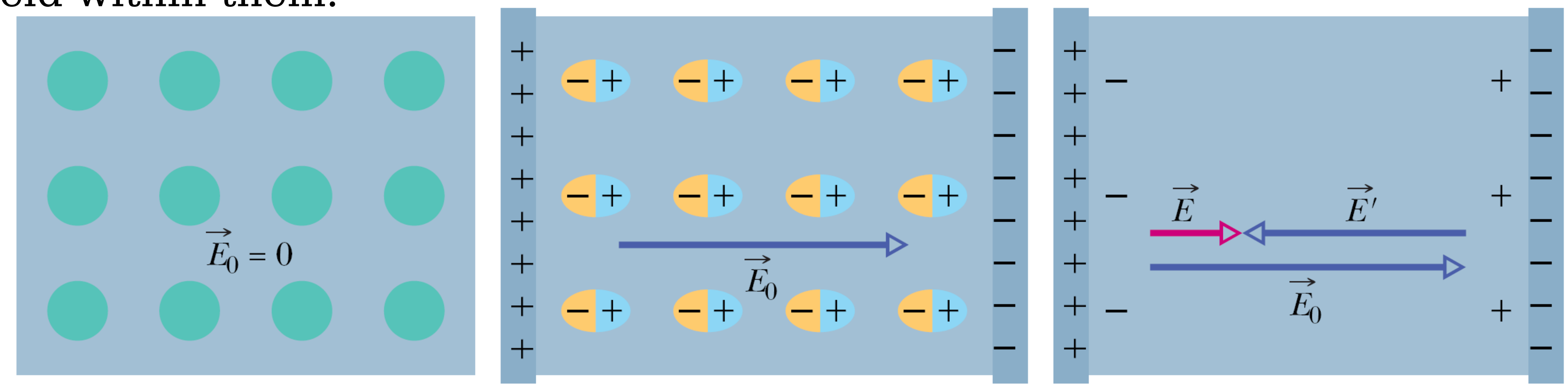
  (1) for *polar dielectrics*, the electric dipoles tend to line up with an external electric field. The alignment of the electric dipoles produces an electric field that is directed opposite the applied field and is smaller in magnitude;



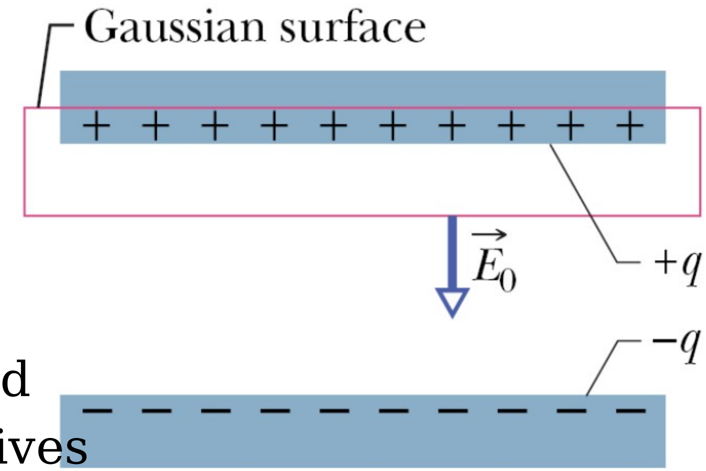
(2) for non-polar dielectrics, molecules acquire dipole moments by induction when placed in an external electric field.

- Both the electric field produced by the induced and by the permanent electric dipoles act in the same way — they oppose the applied field.

- Thus, the effect of both polar and non-polar dielectrics is to weaken any applied field within them.



Dielectrics and Gauss' Law



(a)

- Without a dielectric, Gauss' Law gives

$$\epsilon_0 \oint \vec{E} \cdot \mathrm{d}\vec{A} = \epsilon_0 E_0 A = q \quad \Rightarrow \quad E_0 = \frac{q}{\epsilon_0 A}$$

- With a dielectric, the induced charge is q', and the net enclosed charge is $q-q'$, Gauss's Law gives

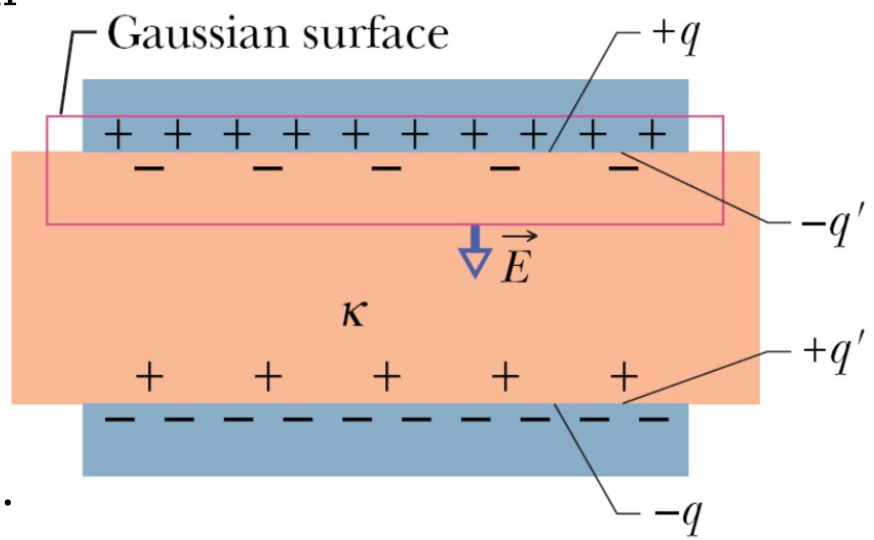
$$\epsilon_0 \oint \vec{E} \cdot \mathrm{d}\vec{A} = \epsilon_0 E A = q - q' \quad \Rightarrow \quad E = \frac{q-q'}{\epsilon_0 A}$$

- Since the effect of the dielectric is to weaken the original field E_0 by a factor of κ,

$$E = \frac{E_0}{\kappa} = \frac{q}{\kappa \epsilon_0 A} \quad \Rightarrow \quad q - q' = \frac{q}{\kappa}$$

- Thus the Gauss's Law is in the form

$$\epsilon_0 \oint \kappa \vec{E} \cdot \mathrm{d}\vec{A} = q \quad \text{Gauss' law with dielectric}$$



(b)

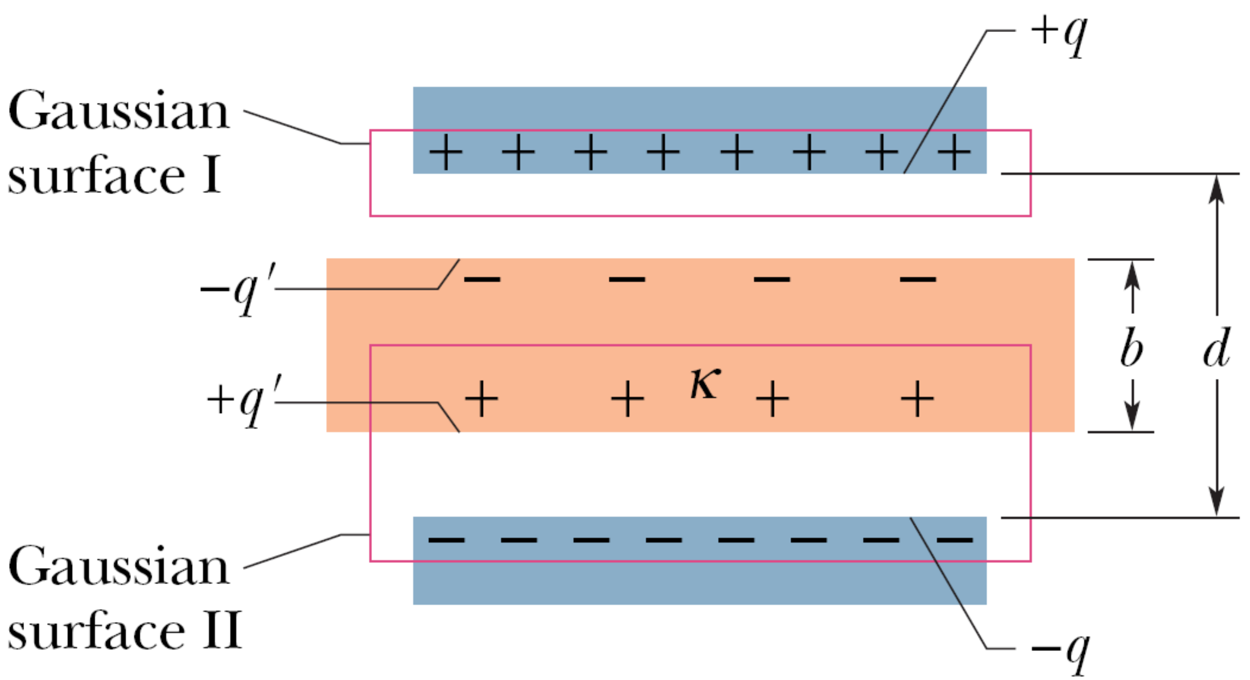
- The above equation stands for general cases.

- Note: (1) The flux integral now involves $\kappa \vec{E}$, not just $\vec{E}$.

  (2) The charge q enclosed by the Gaussian surface is now taken to be the *free charge only*.

(3) We keep κ inside the integral of the above equation to allow for cases in which κ is not constant over the entire Gaussian surface.

problem 25-6



Selected problems: 8, 22, 30, 40, 46