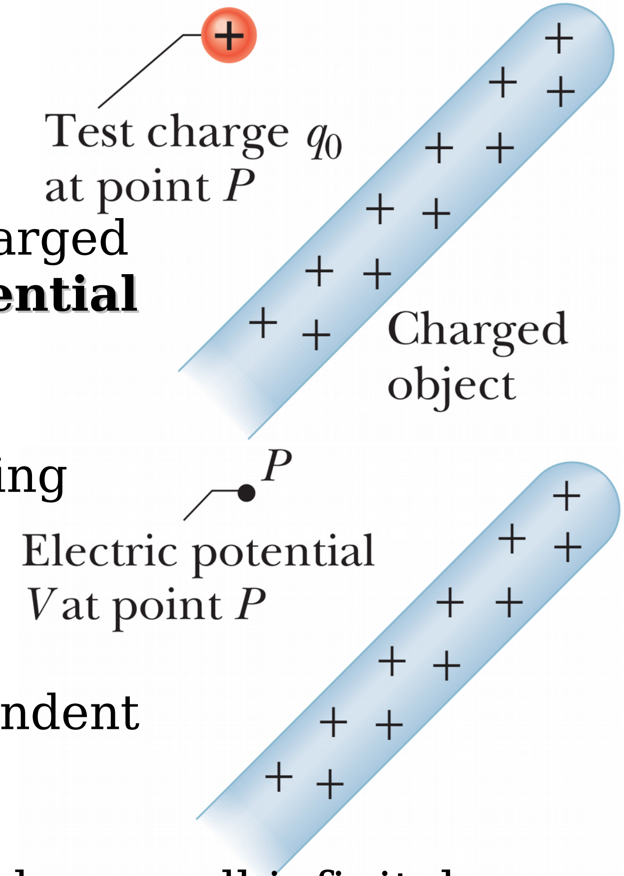
Chapter 24 Electric Potential

Electric Potential Energy



- When an electrostatic force acts between 2 or more charged particles within a system, we can assign an **electric potential energy** U to the system.

- If the system changes its configuration, then the resulting change ΔU in the potential energy of the system is

$$\Delta U = U_f - U_i = -W$$

- The work done by the electrostatic force is path independent since the electrostatic force is a conservative force.

- The *reference configuration* of a system is that the particles are all infinitely separated from one another. Therefore, the corresponding *reference potential energy* is 0.

- Let the initial potential energy $U_i = U_\infty$ be 0, and let W_∞ represent the work done by the electrostatic forces between the particles during the move in from infinity. Then the final potential energy of the system is

$$\Delta U = U_f - U_i = U_f - U_\infty = -W_\infty \quad \Rightarrow \quad U \equiv U_f = -W_\infty$$

Electric Potential

- The potential energy per unit charge, which can be symbolized as U/q, is independent of the charge q of the particle we happen to use and is characteristic only of the electric field we are investigating.

- The potential energy per unit charge at a point in an electric field is called the **electric potential** V at that point, $$V = \frac{U}{q}$$

- *An electric potential is a scalar, not a vector.*

- The electric potential difference ΔV between any 2 points i and f in an electric field
$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} = -\frac{W}{q} \quad \text{potential difference defined}$$

- If we set $U_i = U_\infty = 0$ at infinity as our reference potential energy, then the electric potential $V_i = V_\infty = 0$ there. Thus $$V = -\frac{W_\infty}{q} \quad \text{potential defined}$$

- The SI unit for potential (*volt*) is the joule per coulomb.

- The conversion between the unit of an electric potential and the unit for an electric field is
$$1\ \text{N/C} = 1\,\frac{\text{N}}{\text{C}}\,\frac{1\ \text{V}\cdot\text{C}}{1\ \text{J}}\,\frac{1\ \text{J}}{1\ \text{N}\cdot\text{m}} = 1\ \text{V/m}$$
$$1\ \text{eV} = e\,(1\ \text{V}) = 1.6\times10^{-19}\ \text{C}\,(1\ \text{J/C}) = 1.6\times10^{-19}\ \text{J}$$
therefore, we express values of the electric field in V/m rather than in N/C.

Work Done by an Applied Force

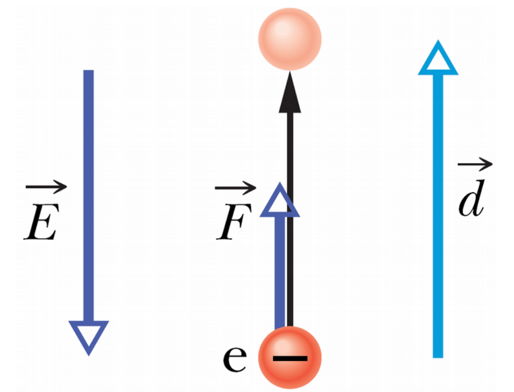
- Suppose we move a particle of charge q from point i to point f in an electric field by applying a force to it, then the change ΔK in the kinetic energy of the particle is

$$\Delta K = K_f - K_i = W_{app} + W$$

- Suppose the particle is stationary before and after the move, then $K_f = K_i = 0$,

$$W_{app} = -W$$

the work W_{app} done by the applied force is equal to the negative of the work W done by the electric field.



Problem 24-1

- Relate the work done by our applied force to the change in the potential energy $\Delta U = U_f - U_i = -W = W_{app}$

- Relate our work W_{app} to the electric potential difference ΔV: $W_{app} = q\,\Delta V$

- It is the work we must do to move a particle of charge q through a potential difference ΔV with no change in the particle's kinetic energy.

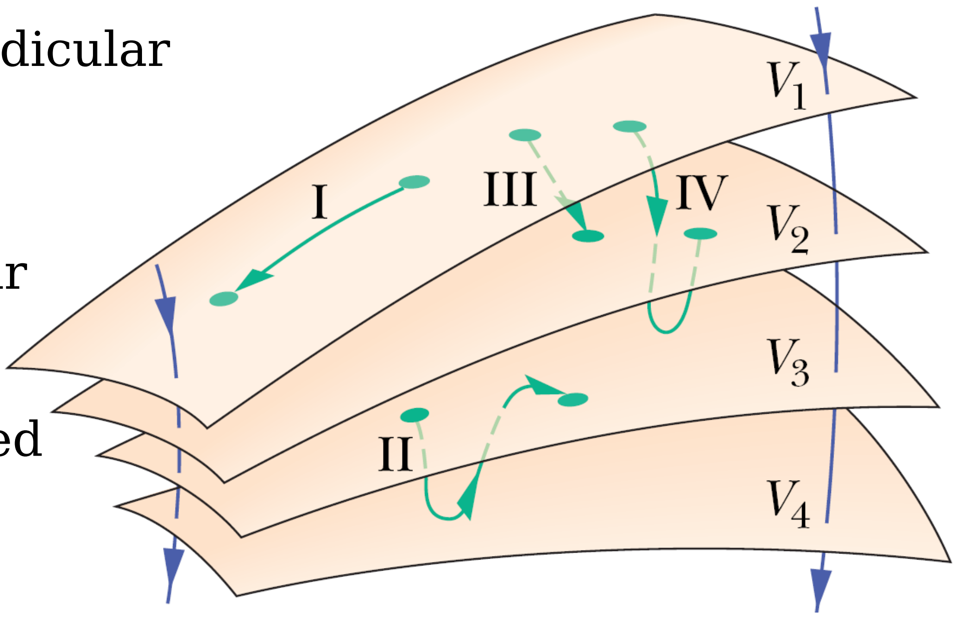
Equipotential Surfaces

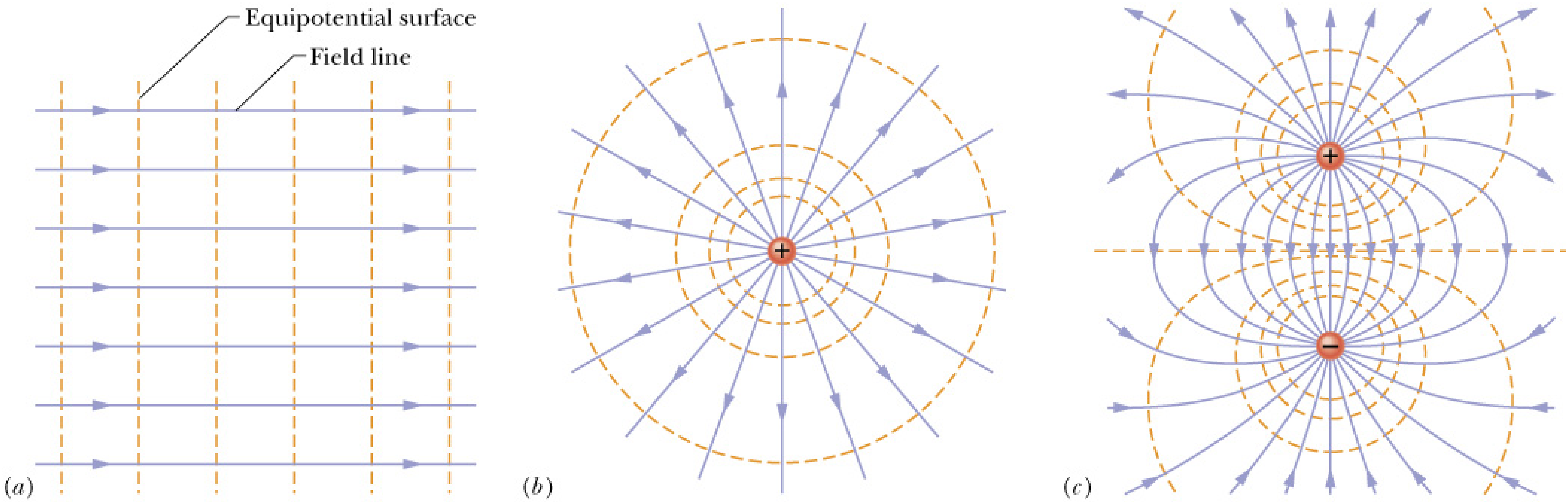
- Adjacent points that have the same electric potential form an **equipotential surface**.

- $W=0$ for any path connecting points on a given equipotential surface regardless of whether that path lies entirely on the surface.

- Equipotential surfaces are always perpendicular to electric field, which is always tangent to these lines.

- If the electric field were not perpendicular to an equipotential surface, it would have a component lying along that surface. This component would then do work on a charged particle as it moved along the surface.





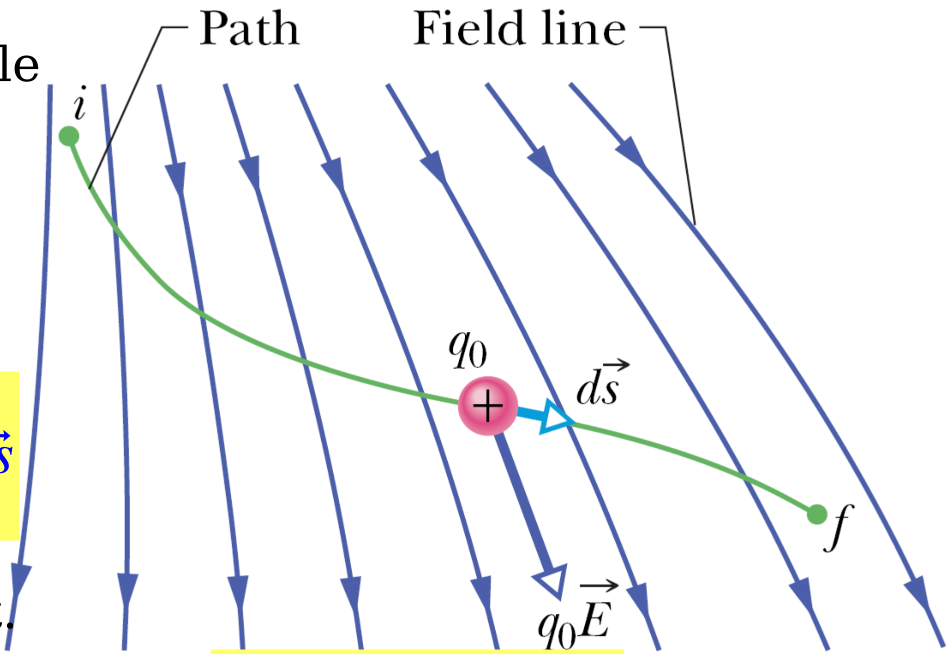
Calculating the Potential from the Field

- The differential work $\mathrm{d}W$ done on a particle by a force during a displacement is

$$\mathrm{d}\,W=\vec{F}\cdot\mathrm{d}\,\vec{s}=q_0\,\vec{E}\cdot\mathrm{d}\,\vec{s}$$

- The total work W done on the particle by the field as the particle moves is

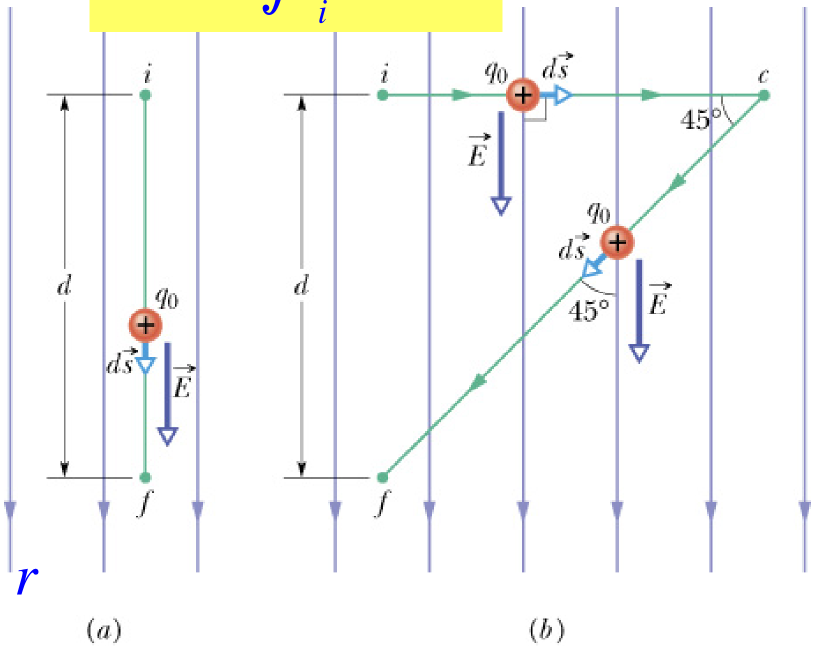
$$W=q_0\int_i^f\vec{E}\cdot\mathrm{d}\,\vec{s}\quad\Rightarrow\quad V_f-V_i=-\int_i^f\vec{E}\cdot\mathrm{d}\,\vec{s}$$



- Because the electrostatic force is conservative, all paths yield the same result.

- If we choose the potential V_i at point to be 0, then $V=-\int_i^f\vec{E}\cdot\mathrm{d}\,\vec{s}$

problem 24-2



Potential Due to a Point Charge

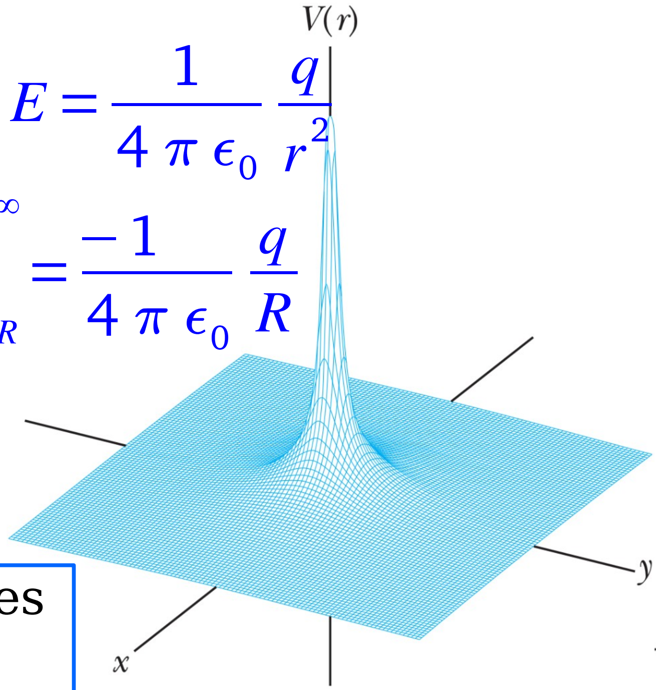
- Imagine that we move a positive test charge q_0 from point P to infinity. Because the path does not matter, let us choose the simplest one — a line that extends radially from the fixed particle through P to ∞. Then

$$\vec{E}\cdot\mathrm{d}\,\vec{s}=E\cos\theta\,\mathrm{d}\,s=E\,\mathrm{d}\,r\quad\Rightarrow\quad V_f-V_i=-\int_R^\infty E\,\mathrm{d}\,r$$

$$V_f = V(\infty) = 0\,, \quad V_i = V(R) = V\,, \quad E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$$

$$\Rightarrow \quad 0 - V = \frac{-q}{4\pi\epsilon_0}\int_R^{\infty}\frac{\mathrm{d}r}{r^2} = \frac{q}{4\pi\epsilon_0}\frac{1}{r}\Bigg|_R^{\infty} = \frac{-1}{4\pi\epsilon_0}\frac{q}{R}$$

$$\Rightarrow \quad V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}$$



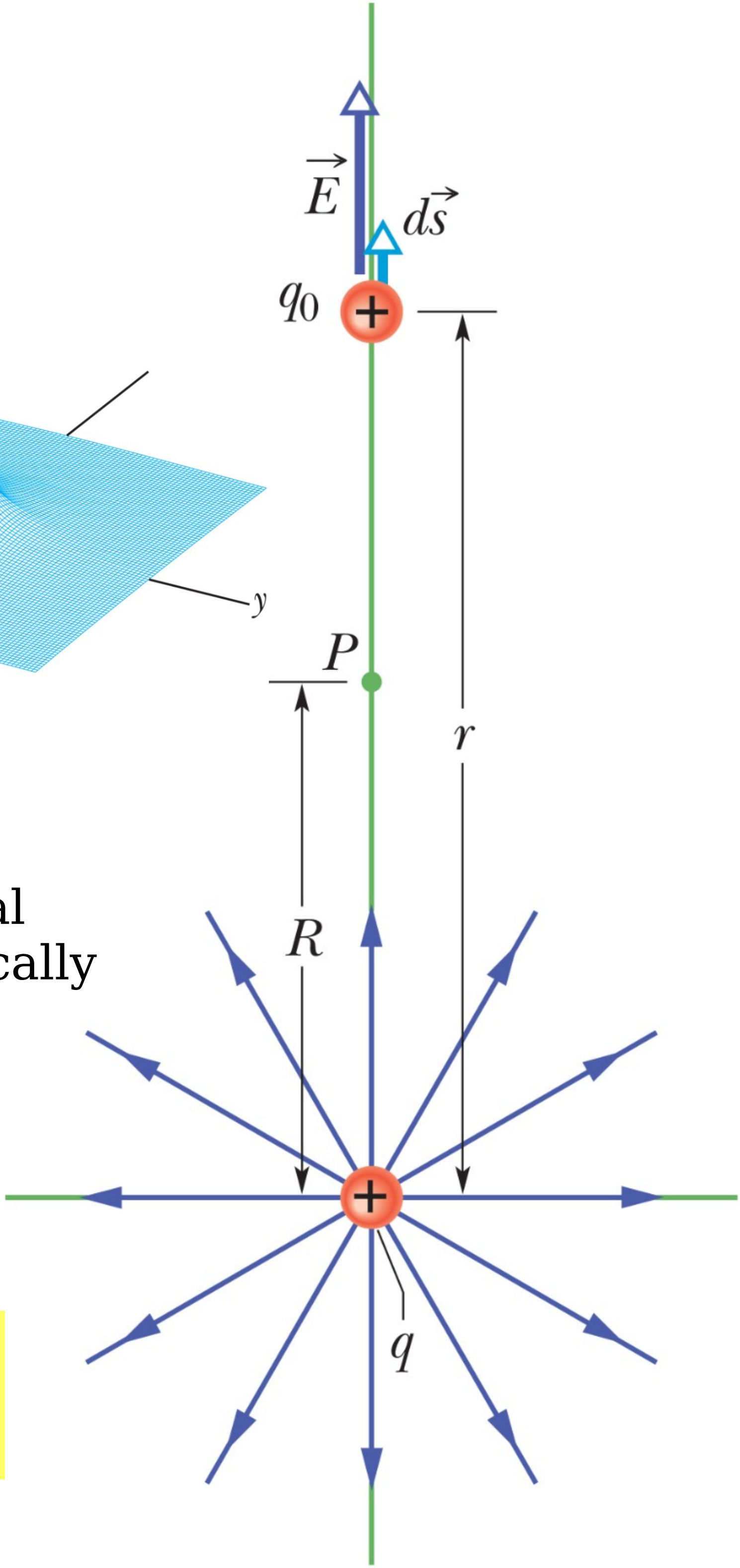
A positively charged particle produces a positive electric potential.
A negatively charged particle produces a negative electric potential.

- The equation above also gives the electric potential either outside or on the external surface of a spherically symmetric charge distribution (shell theorem).

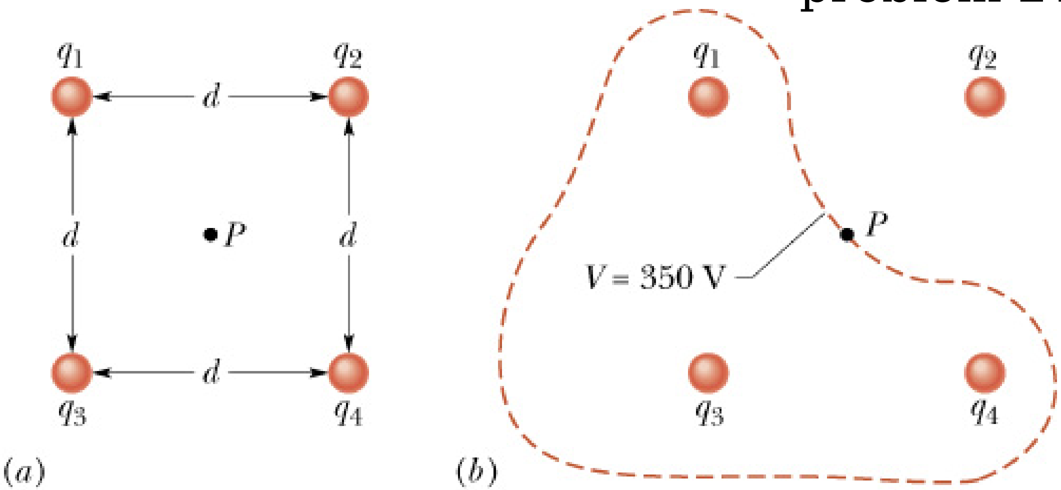
Potential Due to a Group of Point Charges

- Find the net potential at a point due to a group of point charges with the help of the superposition principle:

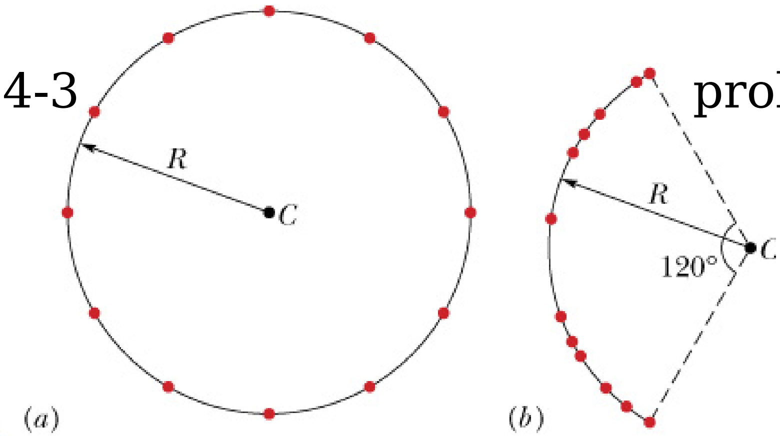
$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i} \quad n \text{ point charges}$$



- The sum is an algebraic sum. Therefore, it lies an important computational advantage of potential over electric field: It is a lot easier to sum several scalar quantities than to sum several vector quantities whose have directions and components.



problem 24-3



problem 24-4

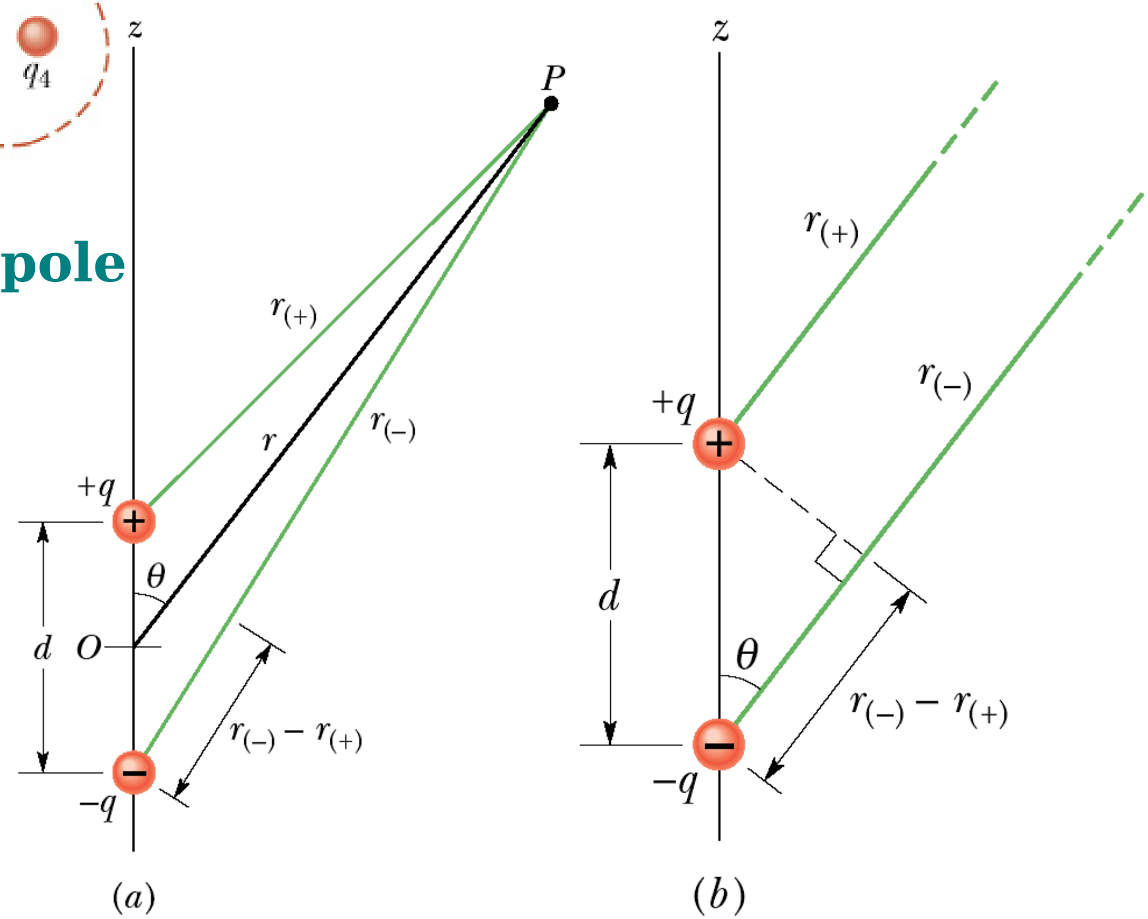
Potential Due to an Electric Dipole

- The net potential at P is

$$V = \sum_{i=1}^{2} V_i = V_+ + V_-$$

$$= \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r_+} + \frac{-q}{r_-}\right)$$

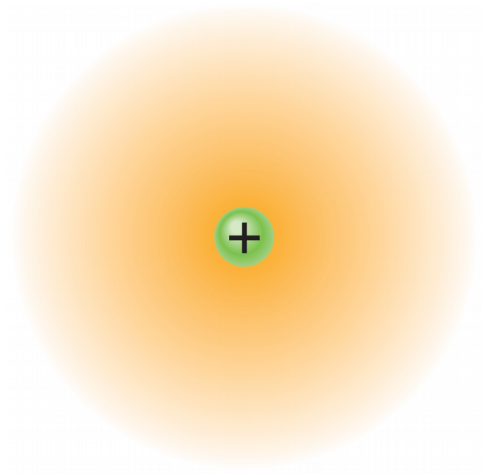
$$= \frac{q}{4\pi\epsilon_0}\,\frac{r_- - r_+}{r_- r_+}$$



- Naturally occurring dipoles are quite small; so we are usually interested only in points that are relatively far from the dipole, ie, $r \gg d$, thus

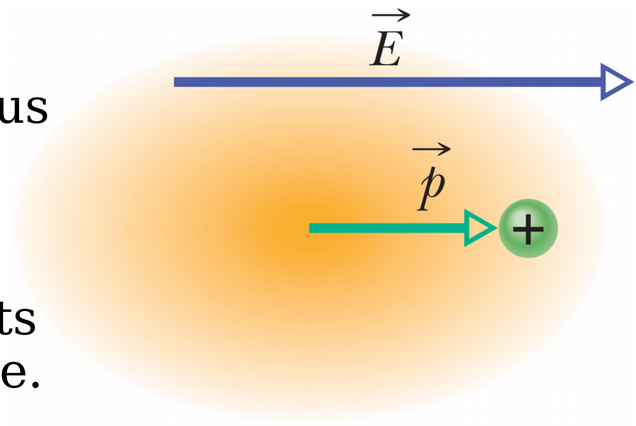
$$r_- - r_+ \approx d\cos\theta \quad \text{and} \quad r_+ r_- \approx r^2 \quad \Rightarrow \quad V \simeq \frac{q}{4\pi\epsilon_0}\frac{d\cos\theta}{r^2}$$

$$\Rightarrow \quad V \simeq \frac{1}{4\pi\epsilon_0}\frac{p\cos\theta}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2} \quad \text{electric dipole}$$



Induced Dipole Moment

- Many molecules, such as water, have *permanent* electric dipole moments.



- In *nonpolar molecules* and in every isolated atom, the centers of the positive and negative charges coincide, thus no dipole moment is set up.

- If an atom or a nonpolar molecule is placed in an external electric field, the field distorts the electron orbits and separates the centers of positive and negative charge.

- This shift sets up an *induced* dipole moment that points in the direction of the field. The atom or molecule is said to be *polarized* by the electric field.

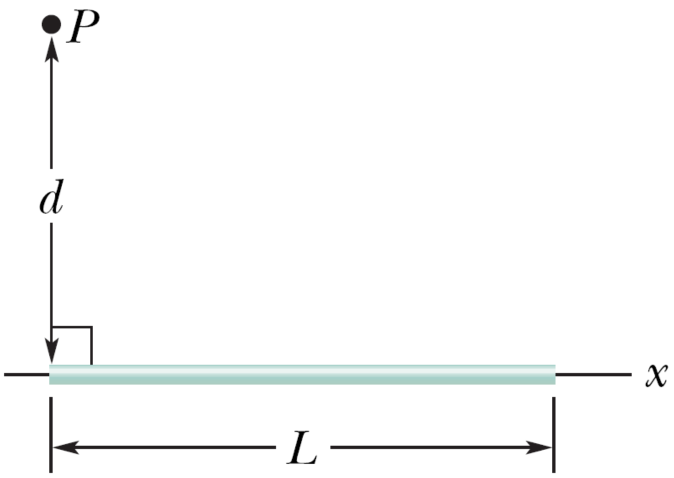
- When the field is removed, the induced dipole moment and the polarization disappear.

Potential Due to a Continuous Charge Distribution

- the potential dV at point P due to dq:

$$dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r} \quad \text{positive or negative } dq$$

$$\Rightarrow \quad V = \int dV = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$



(a)

Line of Charge

- An element of the rod dx has a differential charge:

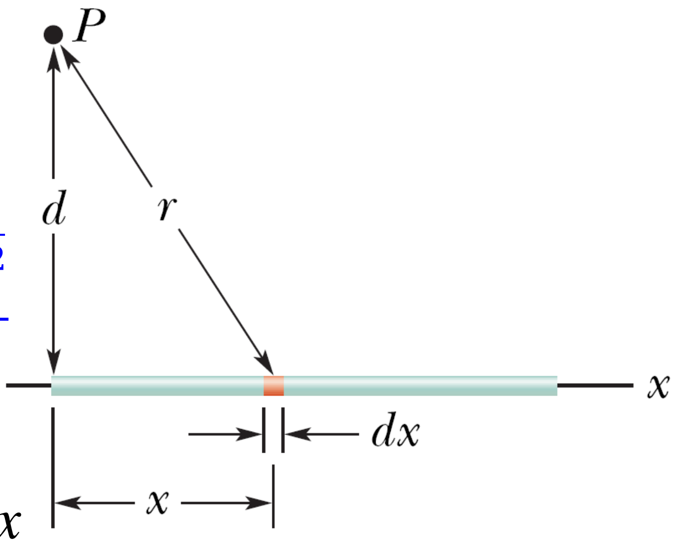
$$dq = \lambda\, dx$$

- The potential $dV = \frac{1}{4\pi\epsilon_0}\frac{dq}{r} = \frac{1}{4\pi\epsilon_0}\frac{\lambda\, dx}{\sqrt{x^2+d^2}}$

- The total potential V is

$$V = \int dV = \frac{\lambda}{4\pi\epsilon_0}\int \frac{dx}{\sqrt{x^2+d^2}}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\ln\left(x+\sqrt{x^2+d^2}\right)\Big|_0^L = \frac{\lambda}{4\pi\epsilon_0}\ln\frac{L+\sqrt{L^2+d^2}}{d}$$



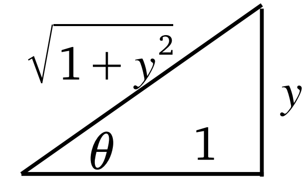
(b)

- Appendix E17:

$$\int \frac{dx}{\sqrt{x^2+a^2}} = \int \frac{d(x/a)}{\sqrt{(x/a)^2+1}} = \int \frac{dy}{\sqrt{y^2+1}} \quad \text{where } y \equiv \frac{x}{a}$$

$$\text{Define } \tan\theta \equiv y \quad \Rightarrow \quad \int \frac{\mathrm{d}\, y}{\sqrt{y^2+1}} = \int \frac{\mathrm{d}\tan\theta}{\sec\theta} = \int \sec\theta\, \mathrm{d}\,\theta \quad \Leftarrow \quad \mathrm{d}\tan\theta = \sec^2\theta\, \mathrm{d}\,\theta$$

$$\mathrm{d}\sec\theta = \tan\theta \sec\theta\, \mathrm{d}\,\theta \quad \Rightarrow \quad \mathrm{d}\left(\tan\theta + \sec\theta\right) = \sec\theta \left(\tan\theta + \sec\theta\right) \mathrm{d}\,\theta$$



$$\Rightarrow \quad \sec\theta\, \mathrm{d}\,\theta = \frac{\mathrm{d}\left(\tan\theta + \sec\theta\right)}{\tan\theta + \sec\theta} = \mathrm{d}\ln\left|\tan\theta + \sec\theta\right|$$

$$\Rightarrow \quad \int \sec\theta\, \mathrm{d}\,\theta = \int \mathrm{d}\ln\left|\tan\theta + \sec\theta\right| = \ln\left|\tan\theta + \sec\theta\right|$$

$$= \ln\left(x/a + \sqrt{(x/a)^2+1}\right) = \ln\left(x + \sqrt{x^2+a^2}\right) + \text{const}$$

$$\Rightarrow \quad \int \frac{\mathrm{d}\, x}{\sqrt{x^2+a^2}} = \ln\left(x + \sqrt{x^2+a^2}\right) + \text{const}$$

Charged Disk

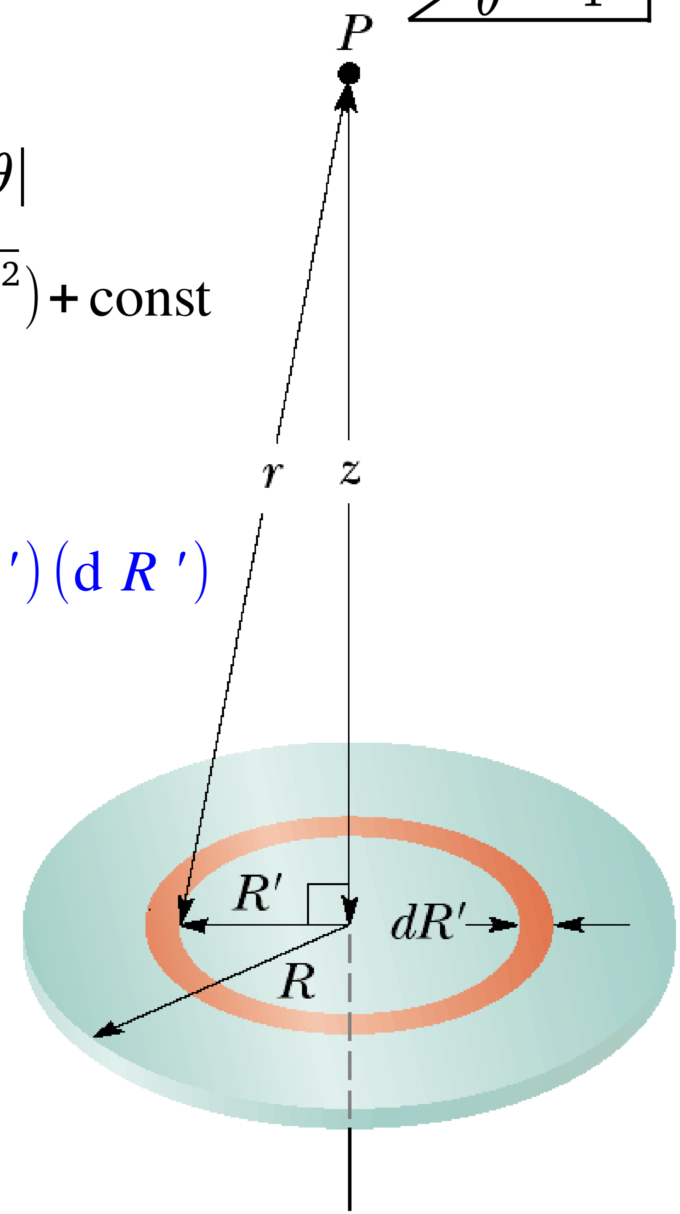
- A differential element has the charge $\mathrm{d}\, q = \sigma\left(2\pi R'\right)\left(\mathrm{d}\, R'\right)$
- Its contribution to the potential is

$$\mathrm{d}\, V = \frac{1}{4\pi\epsilon_0} \frac{\mathrm{d}\, q}{r} = \frac{1}{4\pi\epsilon_0} \frac{\sigma\left(2\pi R'\right)\left(\mathrm{d}\, R'\right)}{\sqrt{R'^2+z^2}}$$

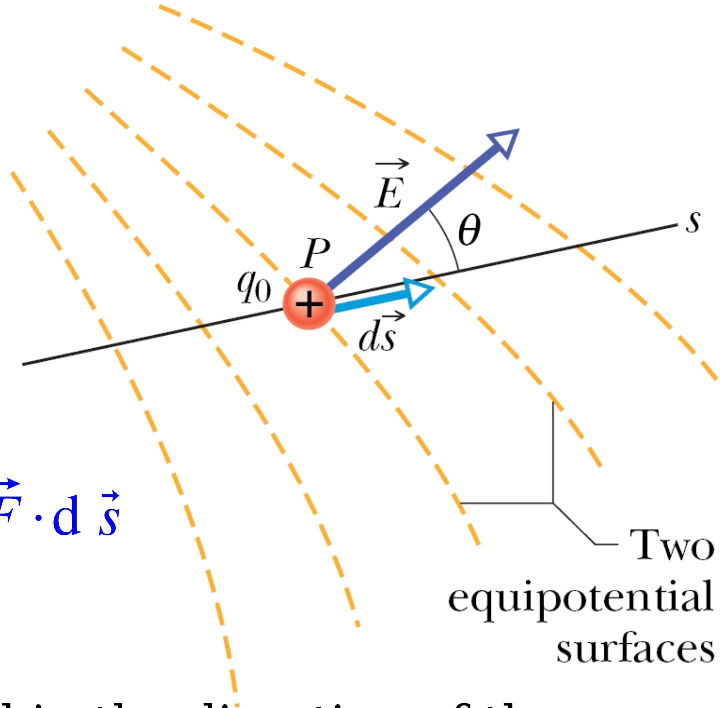
- The total potential is

$$V = \int \mathrm{d}\, V = \frac{\sigma}{2\epsilon_0} \int_0^R \frac{R'\,\mathrm{d}\, R'}{\sqrt{R'^2+z^2}} = \frac{\sigma}{4\epsilon_0} \int_0^R \frac{\mathrm{d}\left(R'^2+z^2\right)}{\sqrt{R'^2+z^2}}$$

$$= \frac{\sigma}{2\epsilon_0} \sqrt{R'^2+z^2}\,\Big|_0^R = \frac{\sigma}{2\epsilon_0}\left(\sqrt{R^2+z^2} - z\right)$$



Calculating the Field from the Potential



- The electric field at any point ⊥ the equipotential surface through that point:

$$\vec{E} \perp S_{EP}$$

- Suppose that a positive test charge q_0 moves through a displacement from one equipotential surface to the adjacent surface, then

$$-\mathrm{d}\,U \Rightarrow -q_0\,\mathrm{d}\,V = q_0\,E\,(\cos\theta)\,\mathrm{d}\,s \Leftarrow (q_0\,\vec{E})\cdot\mathrm{d}\,\vec{s} \Leftarrow \vec{F}\cdot\mathrm{d}\,\vec{s}$$

$$\Rightarrow \quad E\cos\theta = -\frac{\mathrm{d}\,V}{\mathrm{d}\,s}$$

- Since $E\cos\theta$ is the component of the electric field in the direction of the displacement, therefore

$$E_s = -\frac{\partial V}{\partial s}$$

- This equation states:

The component of an electric field in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

- If we take the s axis to be, in turn, the x, y, and z axes, then

$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

$$\Rightarrow \quad \vec{E} = -\left(\hat{\mathrm{i}}\frac{\partial V}{\partial x} + \hat{\mathrm{j}}\frac{\partial V}{\partial y} + \hat{\mathrm{k}}\frac{\partial V}{\partial z}\right) = -\left(\hat{\mathrm{i}}\frac{\partial}{\partial x} + \hat{\mathrm{j}}\frac{\partial}{\partial y} + \hat{\mathrm{k}}\frac{\partial}{\partial z}\right)V$$

- Define *gradient* operator: $\nabla = \hat{\mathrm{i}}\,\frac{\partial}{\partial x} + \hat{\mathrm{j}}\,\frac{\partial}{\partial y} + \hat{\mathrm{k}}\,\frac{\partial}{\partial z} \quad \Rightarrow \quad \vec{E} = -\nabla V$
- For the simple situation in which the electric field is uniform, $E = -\frac{\Delta V}{\Delta s}$

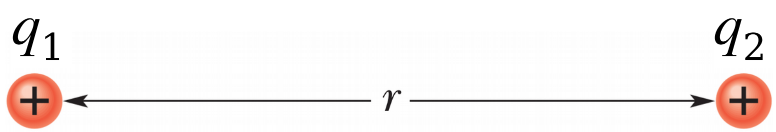
  where $s \perp$ the equipotential surfaces.

- The component of the electric field is 0 in any direction parallel to the equipotential surfaces.

problem 24-5

Electric Potential Energy of a System of Point Charges

- define the electric potential energy of a system of point charges, held in fixed positions by forces not specified, as follows:



The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

- When we bring q_1 in from infinity and put it in place, we do no work because no electrostatic force acts on q_1.

- When we next bring q_2 in from infinity and put it in place, we must do work because q_1 exerts an electrostatic force on q_2 during the move.

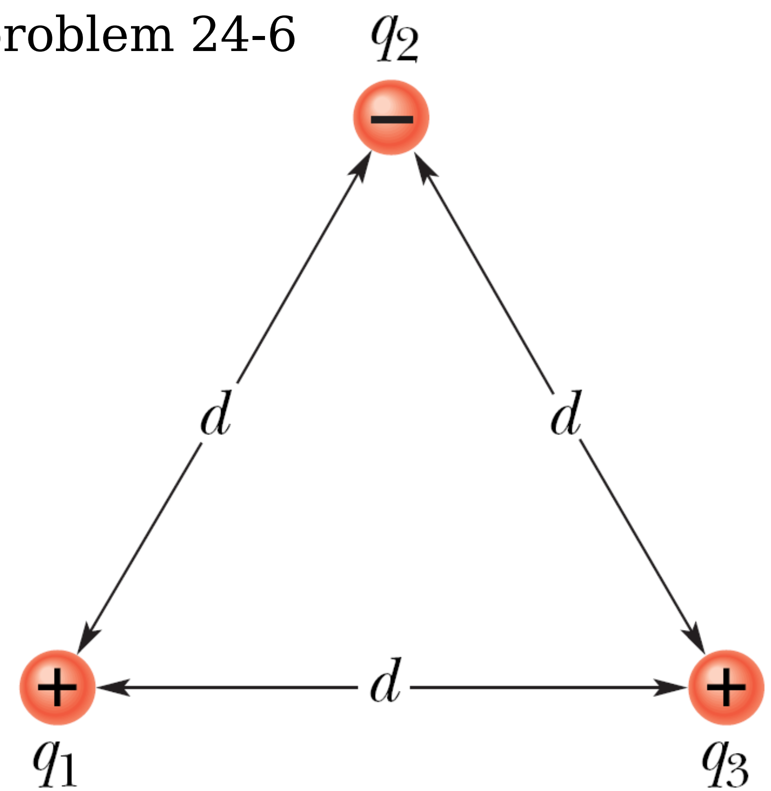
- To build up the potential energy, an external agent is needed to move q_2 in position, and the work is

$$W_a = U = q_2 V_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$$

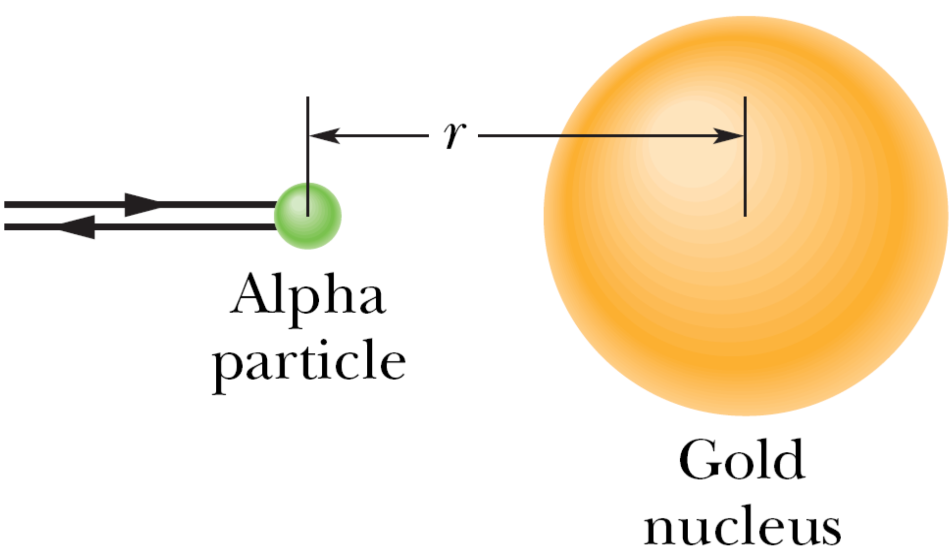
- For a system of n charged particles, the potential energy is

$$U = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} U_{ij} = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n}\frac{1}{4\pi\epsilon_0}\frac{q_i q_j}{r} \quad \text{for } i \neq j$$

problem 24-6



problem 24-7

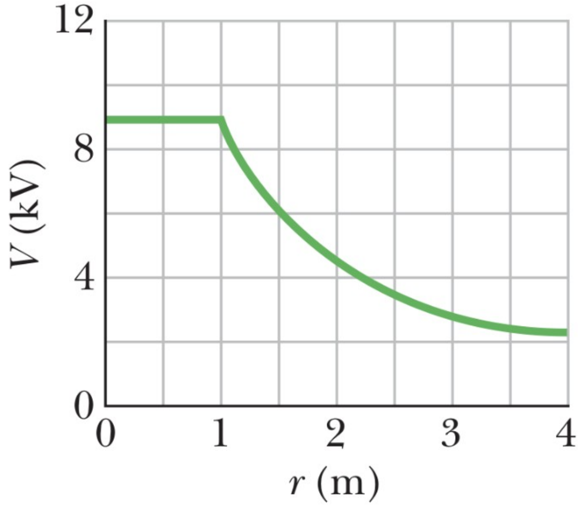


Potential of a Charged Isolated Conductor

An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor – whether on the surface or inside – come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.

Proof

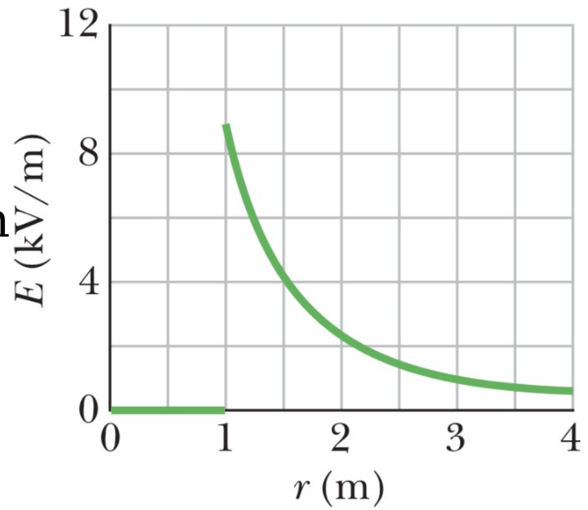
- We know $V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$
- Since $\vec{E} = 0$ for all points within a conductor, it follows directly that $V_f = V_i$ for all possible pairs of points i and f in the conductor.



(a)

Spark Discharge from a Charged Conductor

- At sharp points or sharp edges, the surface charge density and thus the external electric field, may reach very high values.
- In such circumstances, it is safe to enclose yourself in a cavity inside a conducting shell, eg, a car.
- Human body is a fairly good electrical conductor and can be easily charged if you move around or change clothing.
- It is better to discharge yourself before you touch some conducting objects, eg, computer, gas nozzle, etc.



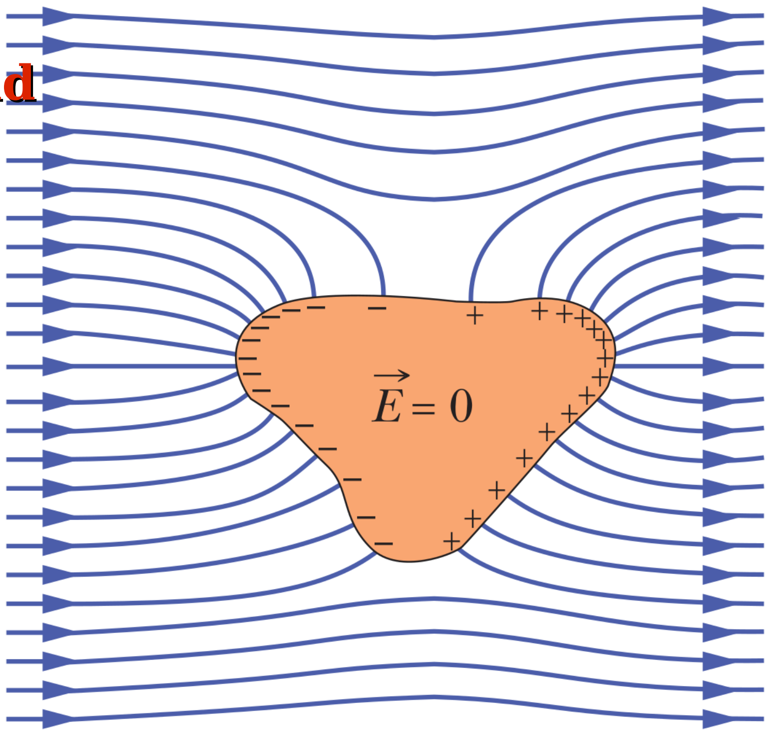
(b)

Isolated Conductor in an External Electric Field

- If an isolated conductor is placed in an *external electric field,* all points of the conductor still come to a single potential regardless of whether the conductor has an excess charge.

- The free conduction electrons distribute themselves on the surface in such a way that the electric field they produce at interior points cancels the external electric field.

- the electron distribution causes the net electric field at all points on the surface to be perpendicular to the surface.



Selected problems: 4, 30, 38, 44, 66