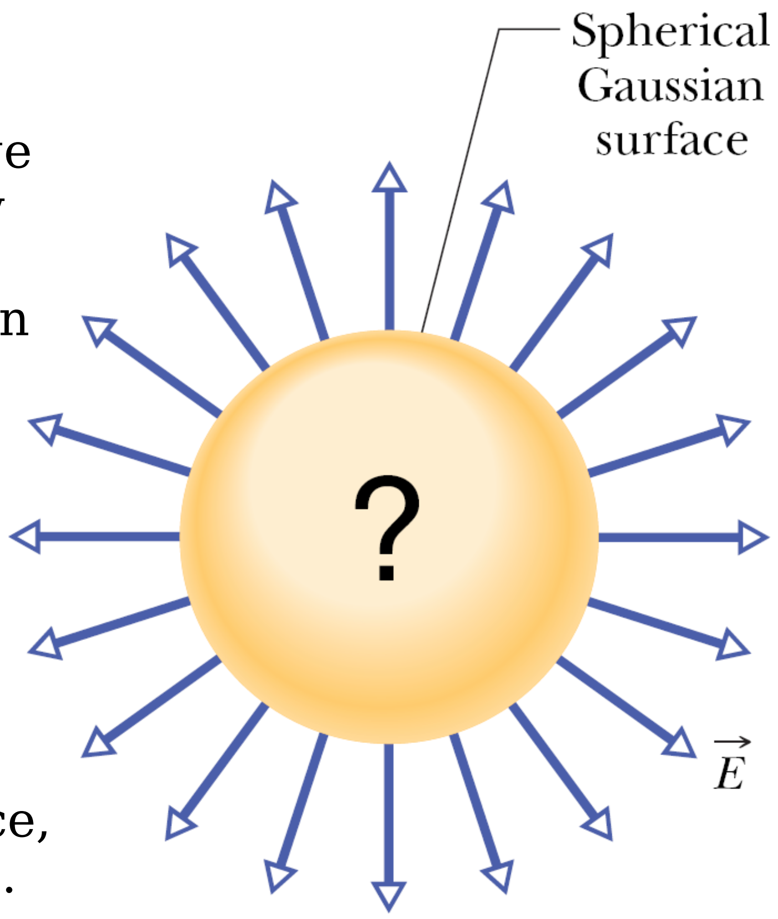
Chapter 23 Gauss' Law

- Instead of considering the electric fields of charge elements in a given charge distribution, Gauss' law considers a hypothetical closed surface enclosing the charge distribution. This **Gaussian surface** can have any shape.

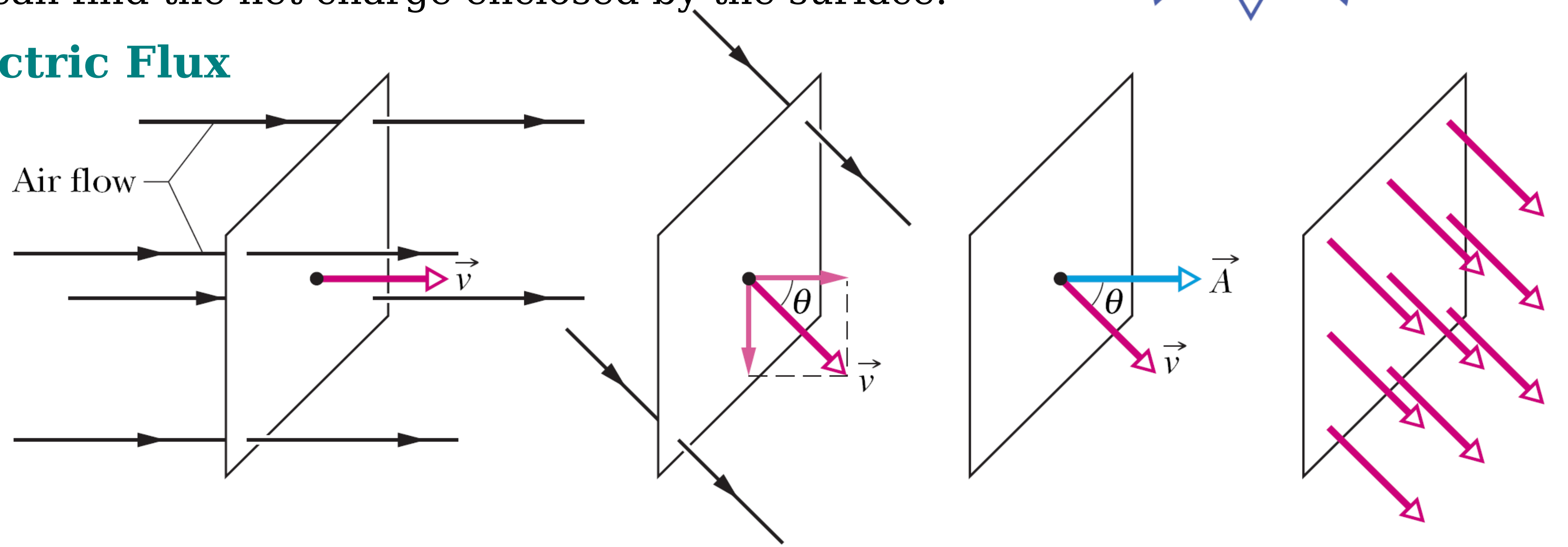


- Find the electric field on the surface by using the fact that

Gauss' law relates the electric fields at points on a (closed) Gaussian surface to the net charge enclosed by that surface.

- If we know the electric field on a Gaussian surface, we can find the net charge enclosed by the surface.

Electric Flux



- Let Φ represent the volume flow rate (volume per unit time) at which air flows through the loop.

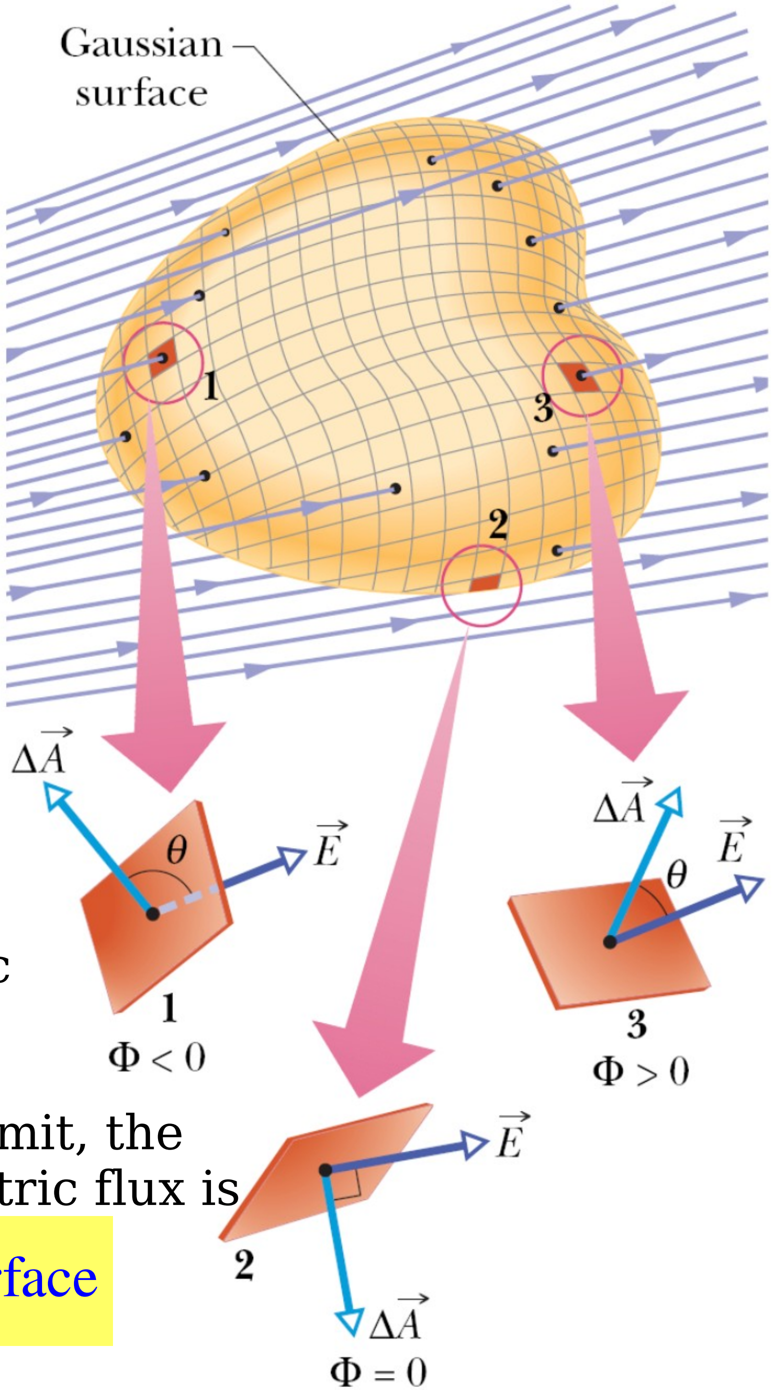
- the rate of volume flow through the loop is

$$\Phi = (v\cos\theta)\,A = v\,A\cos\theta = \vec{v}\cdot\vec{A}$$

where $\vec{A}$ is an area vector whose magnitude is equal to an area and whose direction is normal to the plane of the area.

- This rate of flow through an area is an example of a **flux** – a *volume flux* in this situation.

- In a more abstract way, the above equation is regarded as the flux of the velocity field through the loop. Thus flux means the product of an area and the field across that area.



Flux of an Electric Field

- A provisional definition for the flux of the electric field for the Gaussian surface is

$$\Phi = \sum \vec{E}\cdot\Delta\vec{A}$$

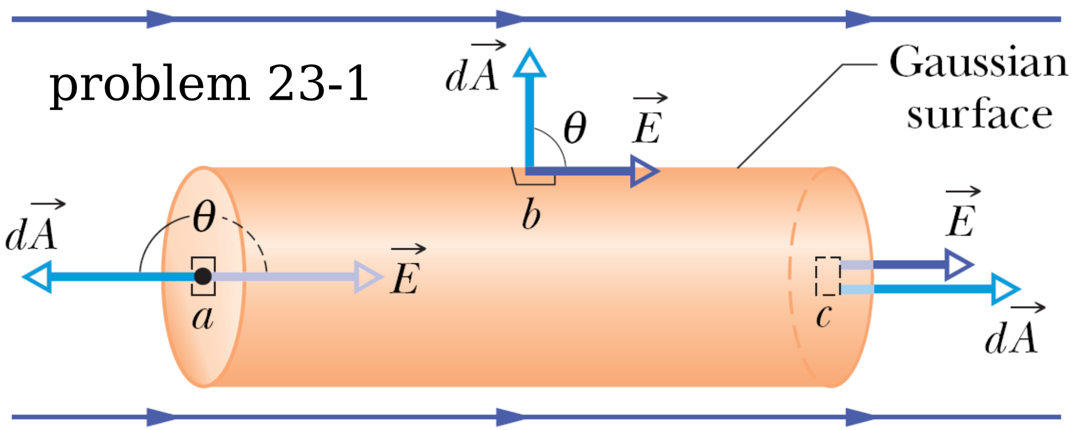
- For the area vectors approaching a differential limit, the sum becomes an integral and the definition of electric flux is

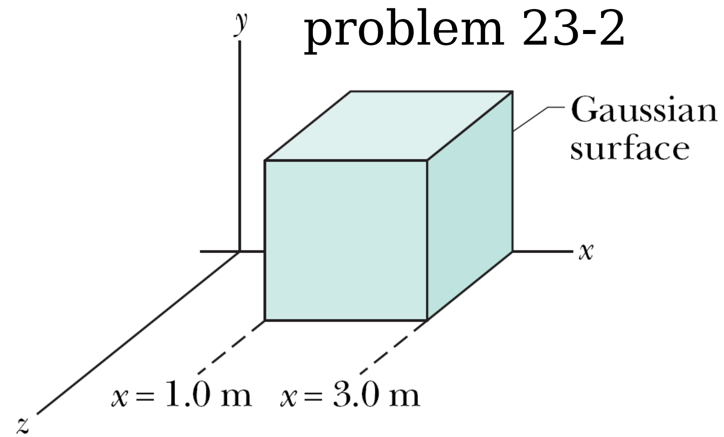
$$\Phi = \oint \vec{E}\cdot d\vec{A} \quad \text{electric flux through a Gaussian surface}$$

● The flux of the electric field is a scalar, and its SI unit is the newton – square-meter per coulomb ($N \cdot m^2/C$).

● The magnitude E is proportional to the number of electric field lines per unit area. Thus, the scalar product is proportional to the number of electric field lines passing through the area dA. Thus

The electric flux Φ through a Gaussian surface is proportional to the net number of electric field lines passing through that surface.





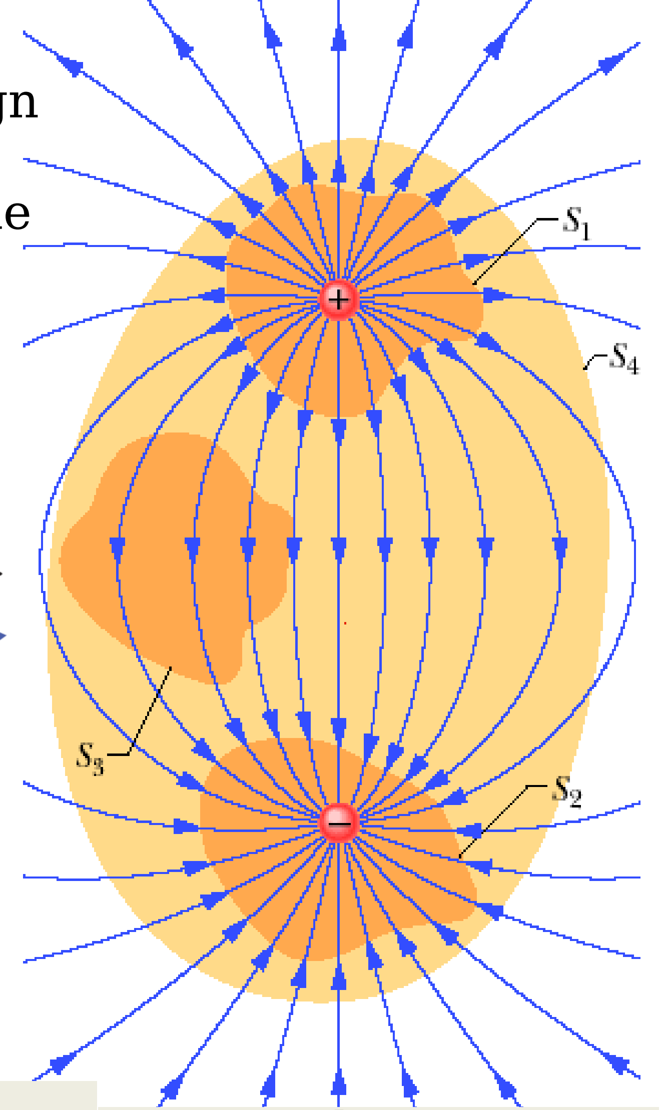
Gauss' Law

● Gauss' law relates the net flux Φ of an electric field through a closed surface (a Gaussian surface) to the *net* charge q_{enc} that is *enclosed* by that surface,

$$\epsilon_0 \Phi = \epsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \quad \text{Gauss's Law}$$

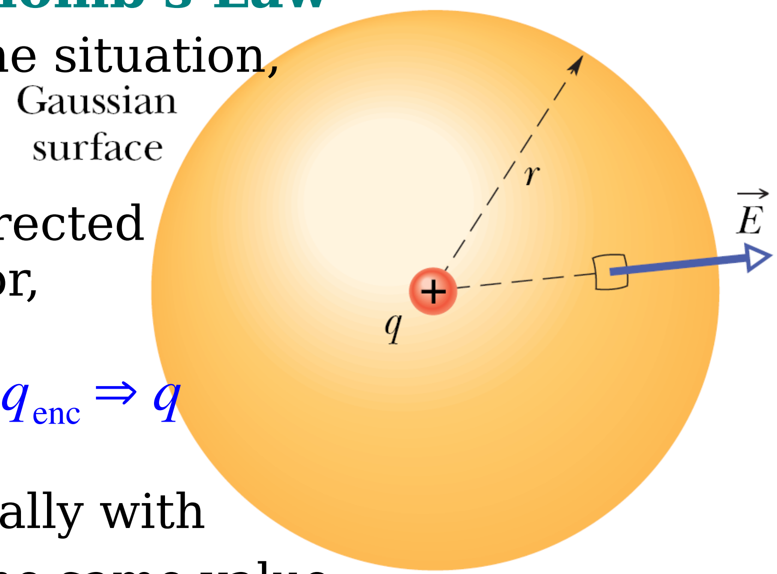
● If q_{enc} is *positive*, the net flux is *outward*; if q_{enc} is *negative*, the net flux is *inward*.

- The only things that matter are the magnitude and sign of the net enclosed charge. Charge outside the surface, and the exact form and location of the charges inside the Gaussian surface are of no concern.



Gauss' Law and Coulomb's Law

- For the symmetry of the situation, at any point the electric field is ⊥ the spherical Gaussian surface and directed outward from the interior,



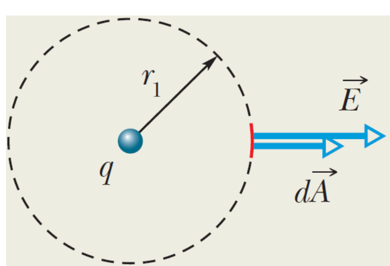
$$\epsilon_0 \oint E\, \mathrm{d}A \Leftarrow \epsilon_0 \oint \vec{E} \cdot \mathrm{d}\vec{A} = q_{enc} \Rightarrow q$$

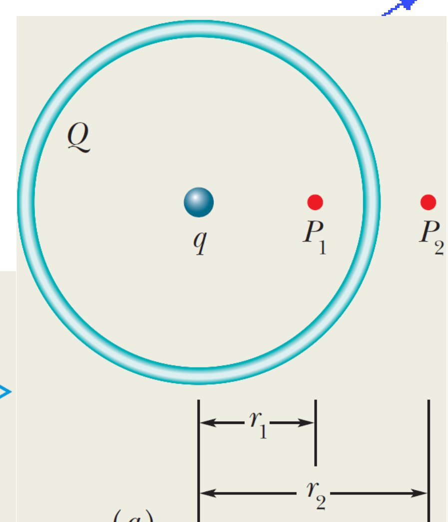
- Although E varies radially with distance from q, it has the same value everywhere on the spherical surface, this is exactly Coulomb's law.

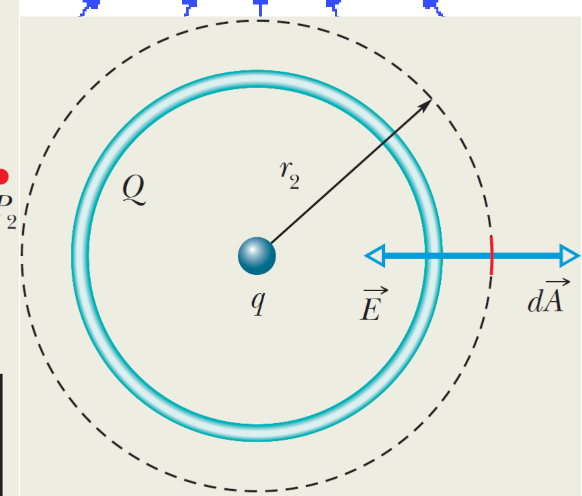
$$\epsilon_0 E \oint \mathrm{d}A = \epsilon_0 E \left(4\pi r^2\right) = q$$

$$\Rightarrow \quad E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

Problem 23-3, 23-4

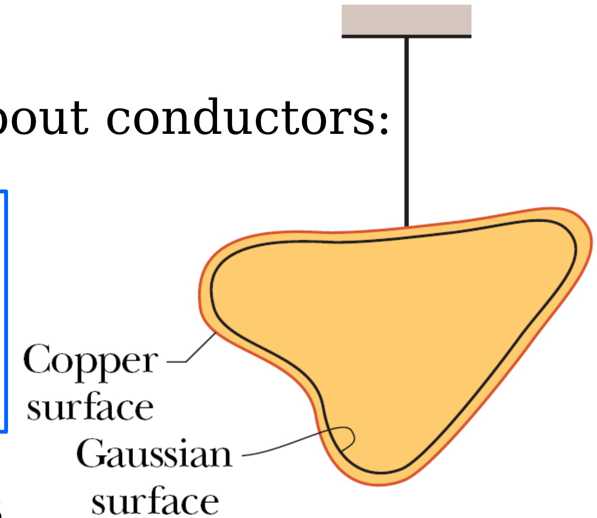






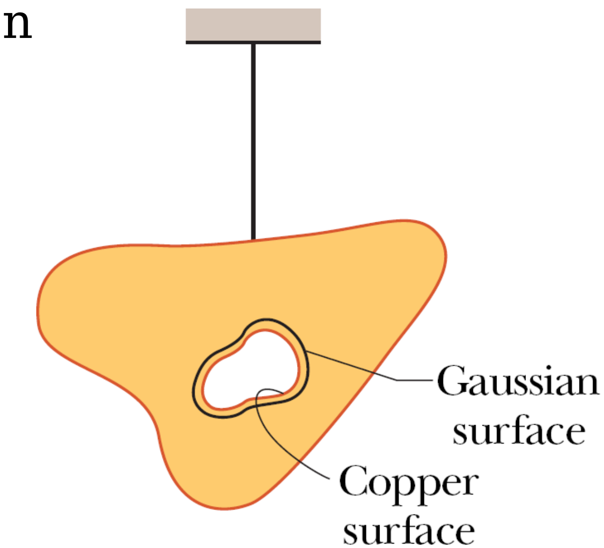
A Charged Isolated Conductor

- Gauss' law permits us to prove an important theorem about conductors:



If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

- The electric field inside this conductor must be 0. If this were not so, the field would exert forces on the conduction (free) electrons, which are always present in a conductor, and thus current would always exist within a conductor.



- Since there is no such perpetual current in an isolated conductor, and so the internal electric field is 0.

- The charges are then in *electrostatic equilibrium* if the net force on each charge is 0.

- If the electric field is 0 everywhere inside the copper conductor, it must be 0 for all points on the Gaussian surface because that surface is inside the conductor. And Gauss' law tells us that the net charge inside the Gaussian surface must also be 0.

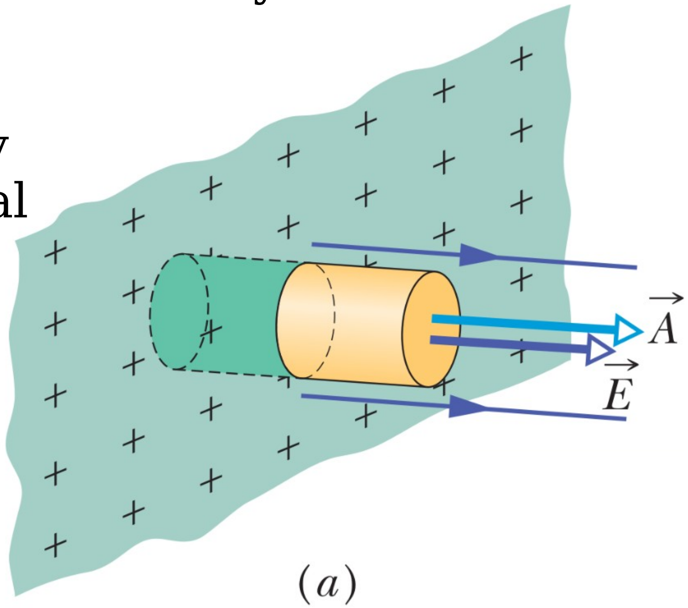
- The excess charge is not inside the Gaussian surface, it must be outside that surface, which means it must lie on the actual surface of the conductor.

An Isolated Conductor with a Cavity

- Because the electric field vanishes inside the conductor, there can be no flux through this new Gaussian surface. From Gauss' law, that surface can enclose no net charge. We conclude that there is no net charge on the cavity walls.

The Conductor Removed

- The electric field is set up by the charges and not by the conductor. The conductor simply provides an initial pathway for the charges to take up their positions.

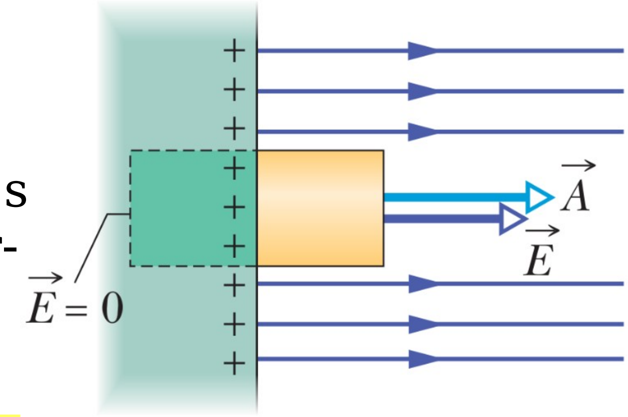


(a)

The External Electric Field

- The surface charge density varies over the surface of any nonspherical conductor. And this variation makes the determination of the electric field set up by the surface charges very difficult.

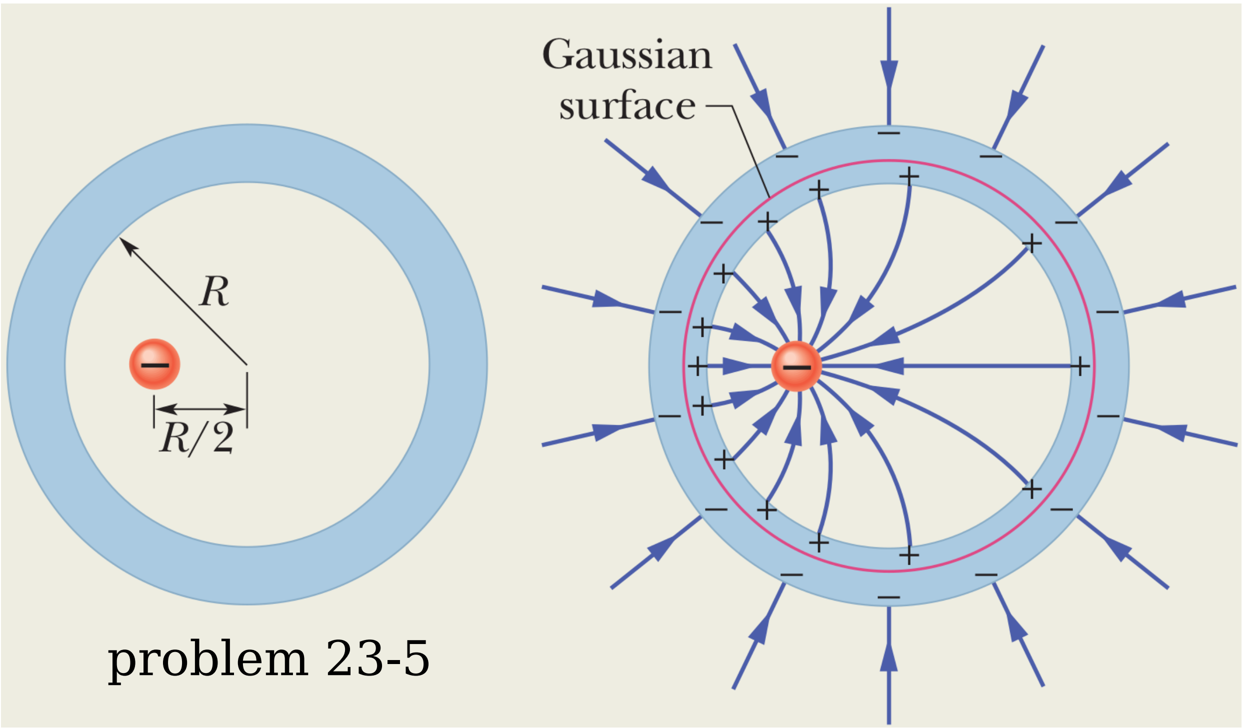
- However, the electric field just outside the surface of a conductor is easy to determine using Gauss' law.



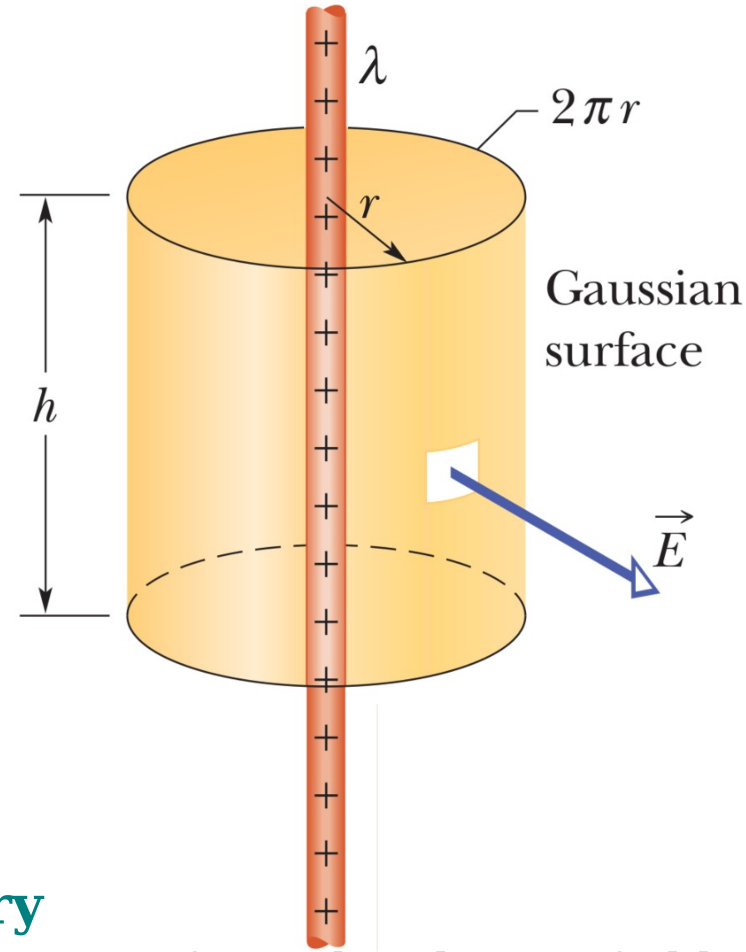
- The electric field at and just outside the conductor's surface must be perpendicular to that surface. Otherwise, then it exert forces on the surface charges, causing them to move. Thus

$$\epsilon_0 E A \Leftarrow \epsilon_0 \Phi = q_{\text{enc}} \Rightarrow \sigma A \quad \Rightarrow \quad \boxed{E = \frac{\sigma}{\epsilon_0} \quad \text{conducting surface}}$$

- The magnitude of the electric field just outside a conductor is proportional to the surface charge density on the conductor.



problem 23-5



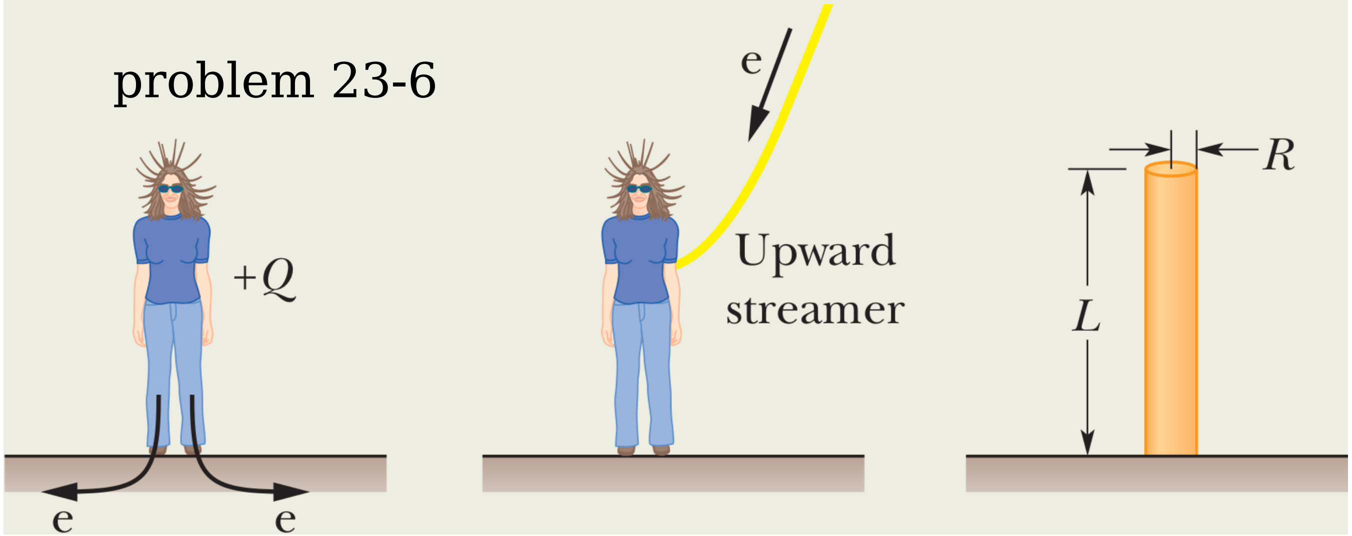
Applying Gauss' Law: Cylindrical Symmetry

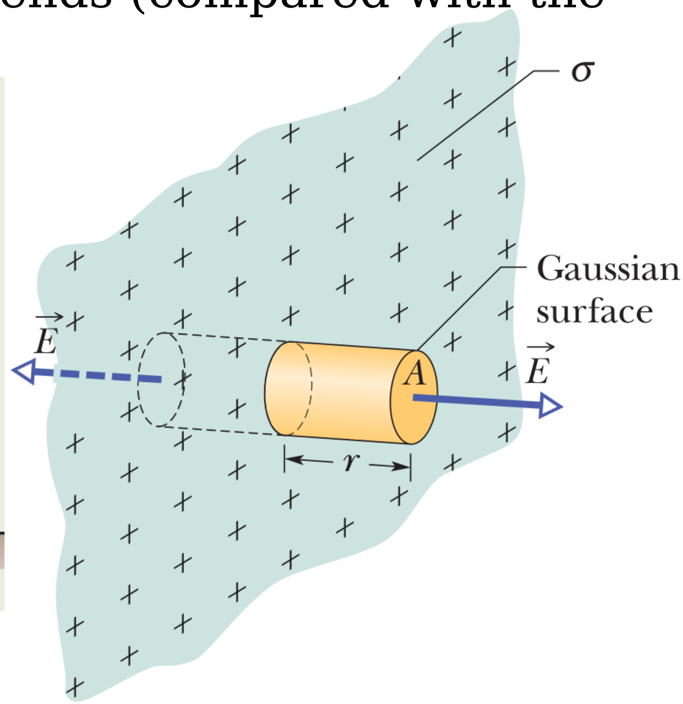
- At every point on the cylindrical part of the Gaussian surface, the electric field must have the same magnitude E and (for a positively charged rod) must be directed radially outward.

- The flux through this cylindrical surface is $\Phi = \oint \vec{E} \cdot \mathrm{d}\,\vec{A} = E\,(2\,\pi\,r\,h)$

- With Gauss' law, $\epsilon_0\,E\,(2\,\pi\,r\,h) \Leftarrow \epsilon_0\,\Phi = q_{enc} \Rightarrow \lambda\,h \quad \Rightarrow$ $\boxed{E = \dfrac{\lambda}{2\,\pi\,\epsilon_0\,r}}$ line of charge

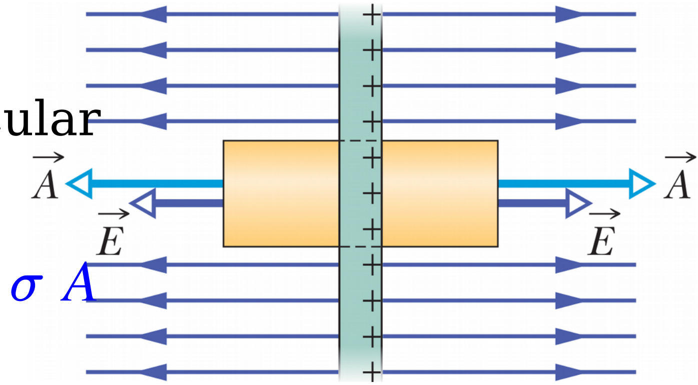
- This is the electric field due to an infinitely long, straight line of charge, at a point that is a radial distance r from the line. It also approximates the field of a finite line of charge at points that are not too near the ends (compared with the distance from the line).





Applying Gauss' Law: Planar Symmetry

Nonconducting Sheet



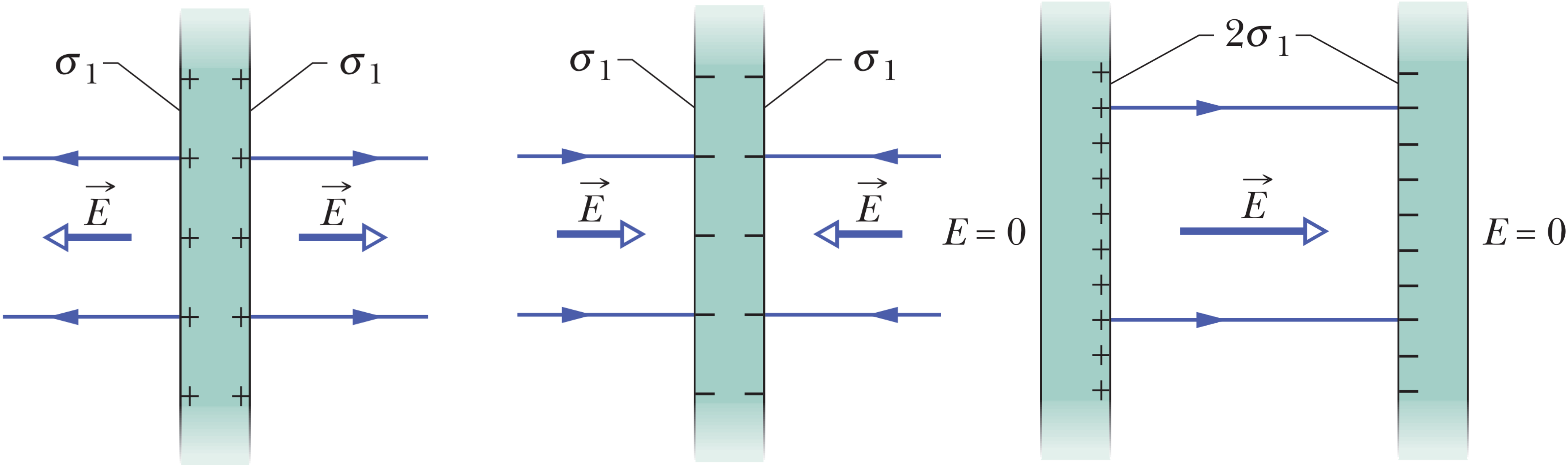
- From symmetry, the electric field must be perpendicular to the sheet and hence to the end caps.

- Use Gauss' law $\epsilon_0 \left(E\ A + E\ A\right) \Leftarrow \epsilon_0 \oint \vec{E}\cdot \mathrm{d}\,\vec{A} = q_{\text{enc}} \Rightarrow \sigma\ A$

$$\Rightarrow \boxed{E = \frac{\sigma}{2\,\epsilon_0} \quad \text{sheet of charge}}$$

- Since we are considering an infinite sheet with uniform charge density, this result holds for any point at a finite distance from the sheet.

Two Conducting Plates

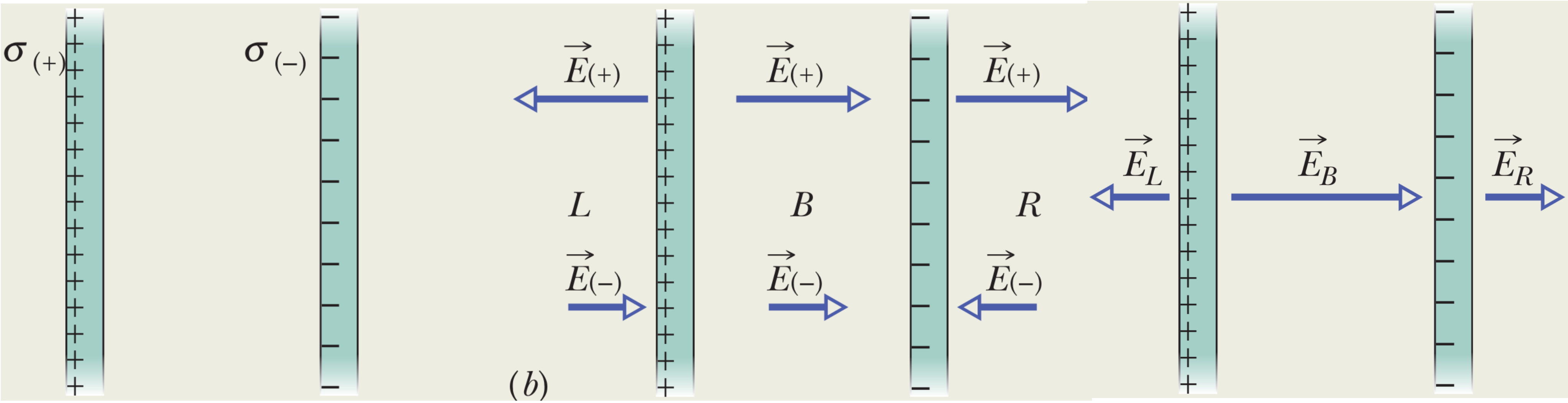


- Since the plates are conductors, when we bring them into this arrangement, the excess charge on one plate attracts the excess charge on the other plate, and all the excess charge moves onto the inner faces of the plates. Thus, the electric field at any point between the plates has the magnitude

$$E = \frac{\sigma}{\epsilon_0}$$

- Since no excess charge is left on the outer faces, the electric field to the left and right of the plates is 0.

- The charge distribution of the 2-plate system is not merely the sum of the charge distributions of the individual plates.



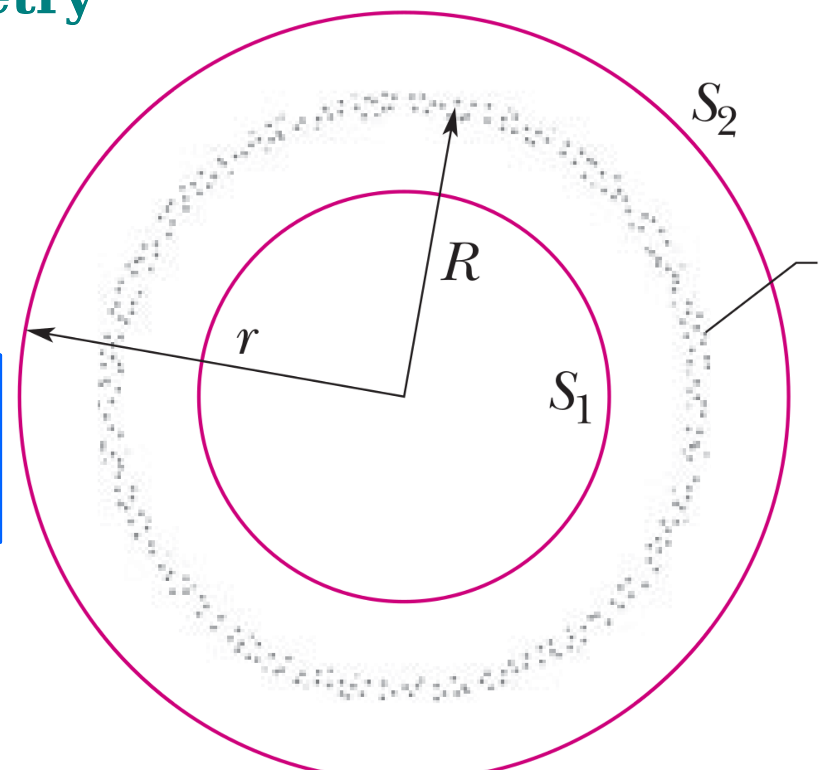
problem 23-7

Applying Gauss' Law: Spherical Symmetry

- Two shell theorems:

A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.

If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.



- Applying Gauss' law to surface S_2 ($r \geq R$)

$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \quad \text{spherical shell, field at } r \geqslant R$$

- Applying Gauss' law to surface S_1 ($r<R$)

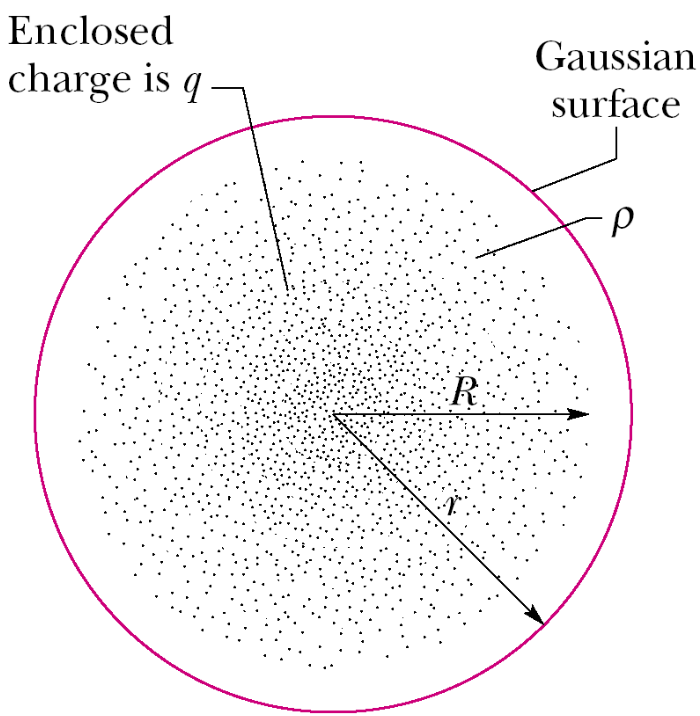
$$E = 0 \quad \text{spherical shell, field at } r < R$$

- Any spherically symmetric charge distribution can be constructed with a nest of concentric spherical shells.

- For purposes of applying the 2 shell theorems, the volume charge density should have a single value for each shell but need not be the same from shell to shell. Thus, for the charge distribution as a whole, ρ can vary, but only with r, the radial distance from the center.

- For $r > R$, $E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{1}{4\pi\epsilon_0 r^2}\int_0^R \rho\,(4\pi r'^2\,\mathrm{d}r')$

- If the charge density is uniform, $E = \frac{1}{4\pi\epsilon_0 r^2}\frac{4\pi}{3}\rho R^3 = \frac{1}{3\epsilon_0}\frac{\rho R^3}{r^2}$



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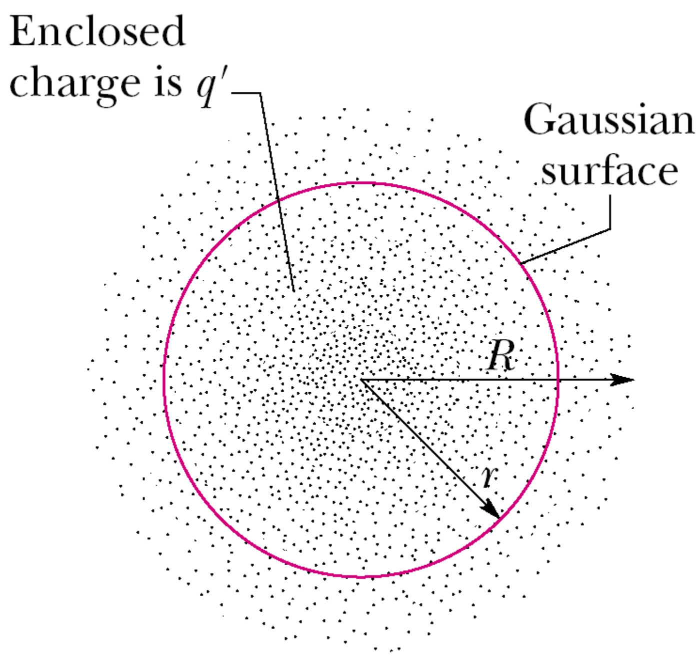


(b)

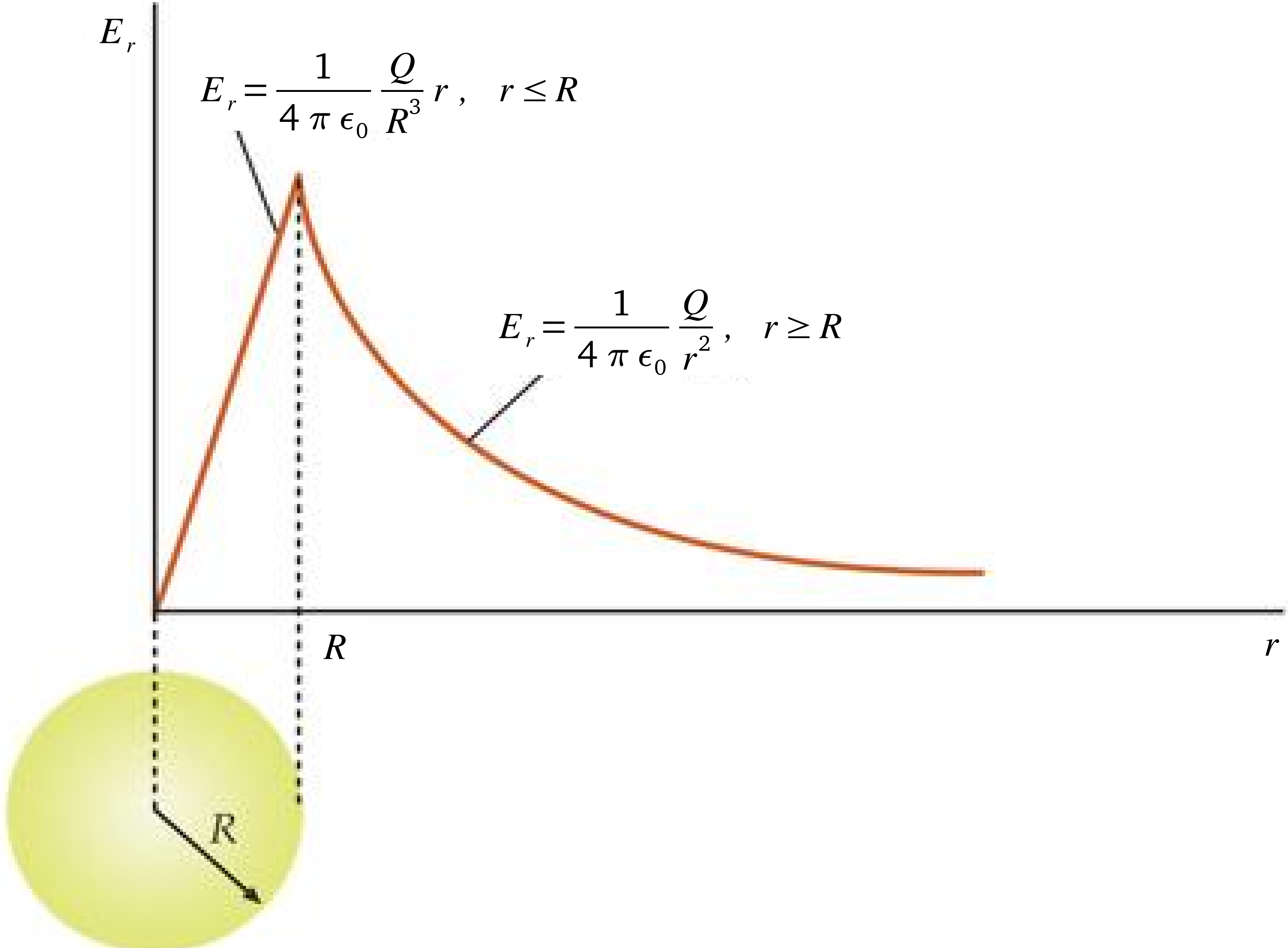
- Let q' represent that enclosed charge, $E = \dfrac{1}{4\pi\epsilon_0}\dfrac{q'}{r^2}$ spherical shell, field at $r \leqslant R$

- If the full charge q enclosed within radius R is uniform (ρ=const), then q' enclosed within radius r in Fig (b) is proportional to q:

$$\frac{\left(\begin{array}{c}\text{charge enclosed by}\\ \text{sphere of radius } r\end{array}\right)}{\left(\begin{array}{c}\text{volume enclosed by}\\ \text{sphere of radius } r\end{array}\right)} = \frac{\text{full charge}}{\text{full volume}} \quad \Rightarrow \quad \frac{q'}{4\pi r^3/3} = \frac{q}{4\pi R^3/3} \quad \Rightarrow \quad q' = q\,\frac{r^3}{R^3}$$

$$\Rightarrow \quad \boxed{E = \frac{q}{4\pi\epsilon_0 R^3}\, r = \frac{\rho}{3\epsilon_0}\, r \quad \text{uniform charge, field at } r \leqslant R}$$

Selected problems: 6, 12, 28, 40


E_r
$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{R^3}r, \quad r \le R$
$E_r = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}, \quad r \ge R$
R
r
R