GEOMETRY (1) HOMEWORK 8

(DUE: 9:50AM 11/10)

Write up the solutions to all problems.

Follow the notation used in class.

Problem A. *Let $A(t,p)$ be a solution to the ODE*

$$\partial_t A(t,p) = B(t,p)A(t,p), \text{ where } B(t,p) = (\mathrm{d}w)_{\varphi(t,p)}.$$

Show that

$$\|A(t,p)\| \leq e^{L|t|}.$$

Problem B. *Let V be a differentiable vector field on $U \subset \mathbf{R}^n$. Show that the flow $\varphi(t,p)$ satisfies the identity*

$$\varphi(0,p) = p \text{ , and } \varphi(t,\varphi(s,p)) = \varphi(s+t,p)$$

whenever $\varphi(t,p)$ is defined.

Problem C. *Let $U = \mathbf{R}^2 \setminus \{(0,0)\}$. Consider a vector field*

$$V\colon U \to \mathbf{R}^2 \text{ via } (x,y) \mapsto \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right).$$

Let $p \in U$. Find the flow $\varphi(t,p)$.

Problem D. *Show that every point in a regular surface S admits local orthogonal coordinates, i.e., for every $p \in S$ there exists a parameterization $\mathbf{x}\colon U \to S$ with $p \in \mathbf{x}(U)$ such that*

$$\langle \mathbf{x}_u, \mathbf{x}_v \rangle = 0.$$

Problem E (do Carmo)**.** *Section 3-4 Exercise #7.*

Problem F (do Carmo)**.** *Section 3-4 Exercise #11.*

Date: November 7, 2026.