Since the two Lagrangians are not involved with second derivatives of x^λ, the equations of motion can be obtained by Euler-Lagrange equation, i.e.

$$\frac{\partial L}{\partial x^\lambda} - \frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{\partial L}{\partial \dot{x}^\lambda}\right) = 0$$

For simplicity, let's use $F = g_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}s}\frac{\mathrm{d}x^\beta}{\mathrm{d}s} = g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta$ in calculation and take $g_{\alpha\beta}$ to be Minkowski metric diag$(+1,-1,-1,-1)$ which makes L_1, L_2 only depend on $\dot{x}$.

For the Lagrangian $L_1 = -mc\sqrt{g_{\alpha\beta}\frac{\mathrm{d}x^\alpha}{\mathrm{d}s}\frac{\mathrm{d}x^\beta}{\mathrm{d}s}} = -mc\sqrt{g_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta} = \sqrt{F}$, we have that

$$\begin{aligned}
\frac{\partial L_1}{\partial x^\lambda} - \frac{\mathrm{d}}{\mathrm{d}s}\frac{\partial L_1}{\partial \dot{x}^\lambda} &= -mc\left[0 - \frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{1}{2}F^{-\frac{1}{2}}(\underbrace{g_{\alpha\beta}\delta^\alpha_\lambda\dot{x}^\beta}_{=g_{\lambda\gamma}\dot{x}^\gamma} + \underbrace{g_{\alpha\beta}\dot{x}^\alpha\delta^\beta_\lambda}_{=g_{\lambda\gamma}\dot{x}^\gamma})\right)\right] \\
&= mc\frac{\mathrm{d}}{\mathrm{d}s}\left(F^{-\frac{1}{2}}g_{\lambda\gamma}\dot{x}^\gamma\right) \qquad \leftarrow F = c^2 \\
&= mg_{\lambda\gamma}\ddot{x}^\gamma
\end{aligned}$$

Equation of motion for L_1 is $g_{\lambda\gamma}\ddot{x}^\gamma = 0$.

For the Lagrangian $L_2 = -mg_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta$, we have that

$$\begin{aligned}
\frac{\partial L_2}{\partial x^\lambda} - \frac{\mathrm{d}}{\mathrm{d}s}\frac{\partial L_2}{\partial \dot{x}^\lambda} &= -m\left[0 - \frac{\mathrm{d}}{\mathrm{d}s}(g_{\alpha\beta}\delta^\alpha_\lambda\dot{x}^\beta + g_{\alpha\beta}\dot{x}^\alpha\delta^\beta_\lambda)\right] \\
&= 2mg_{\lambda\gamma}\ddot{x}^\gamma
\end{aligned}$$

Equation of motion for L_2 is also $g_{\lambda\gamma}\ddot{x}^\gamma = 0$.

Remark:

If one treat the metric $g_{\alpha\beta}$ is function of *position*, the equations of motion for L_1 and L_2 are also the same.

$$\frac{\partial L_1}{\partial x^\lambda} - \frac{\mathrm{d}}{\mathrm{d}s}\frac{\partial L_1}{\partial \dot{x}^\lambda} = -mc\left[\frac{1}{2}F^{-\frac{1}{2}}\frac{\partial g_{\alpha\beta}}{\partial x^\lambda}\dot{x}^\alpha\dot{x}^\beta - \frac{\mathrm{d}}{\mathrm{d}s}\left(\frac{1}{2}F^{-\frac{1}{2}}(\underbrace{g_{\alpha\beta}\delta^\alpha_\lambda\dot{x}^\beta}_{=g_{\lambda\gamma}\dot{x}^\gamma} + \underbrace{g_{\alpha\beta}\dot{x}^\alpha\delta^\beta_\lambda}_{=g_{\lambda\gamma}\dot{x}^\gamma})\right)\right] \qquad \leftarrow F = c^2$$

$$
\begin{aligned}
&= -m\left[\frac{1}{2}(\partial_\lambda g_{\alpha\beta})\dot{x}^\alpha\dot{x}^\beta - \frac{\mathrm{d}}{\mathrm{d}s}\left(g_{\lambda\gamma}\dot{x}^\gamma\right)\right] \\
&= m\left(-\frac{1}{2}(\partial_\lambda g_{\alpha\beta})\dot{x}^\alpha\dot{x}^\beta + (\partial_\alpha g_{\lambda\gamma})\dot{x}^\alpha\dot{x}^\gamma + g_{\lambda\gamma}\ddot{x}^\gamma\right)
\end{aligned}
$$

$\uparrow$ split the second term and rename the dummy index

$$
\begin{aligned}
&= m\left(-\frac{1}{2}(\partial_\lambda g_{\alpha\beta})\dot{x}^\alpha\dot{x}^\beta + \frac{1}{2}(\partial_\alpha g_{\beta\lambda})\dot{x}^\alpha\dot{x}^\beta + \frac{1}{2}(\partial_\beta g_{\lambda\alpha})\dot{x}^\alpha\dot{x}^\beta + g_{\lambda\gamma}\ddot{x}^\gamma\right) \\
&= m\left[g_{\lambda\gamma}\ddot{x}^\gamma + \frac{1}{2}\left(\partial_\alpha g_{\beta\lambda} + \partial_\beta g_{\lambda\alpha} - \partial_\lambda g_{\alpha\beta}\right)\dot{x}^\alpha\dot{x}^\beta\right]
\end{aligned}
$$

The equation of motion for L_1 becomes

$$
0 = g_{\lambda\gamma}\ddot{x}^\gamma + \frac{1}{2}\left(\partial_\alpha g_{\beta\lambda} + \partial_\beta g_{\lambda\alpha} - \partial_\lambda g_{\alpha\beta}\right)\dot{x}^\alpha\dot{x}^\beta
$$

Multiply both sides by $g^{\rho\lambda}$ and use $g^{\rho\lambda}g_{\lambda\gamma} = \delta^\rho_\gamma$

$$
0 = \delta^\rho_\gamma\ddot{x}^\gamma + \frac{1}{2}g^{\rho\lambda}\left(\partial_\alpha g_{\beta\lambda} + \partial_\beta g_{\lambda\alpha} - \partial_\lambda g_{\alpha\beta}\right)\dot{x}^\alpha\dot{x}^\beta \;\;\Rightarrow\;\; \ddot{x}^\rho + \Gamma^\rho_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta = 0
$$

where $\Gamma^\rho_{\alpha\beta} = \frac{1}{2}g^{\rho\lambda}\left(\partial_\alpha g_{\beta\lambda} + \partial_\beta g_{\lambda\alpha} - \partial_\lambda g_{\alpha\beta}\right)$ is the Christoffel symbol. The equation of motion is the so-called geodesic equation for the particle.

$$
\begin{aligned}
\frac{\partial L_2}{\partial x^\lambda} - \frac{\mathrm{d}}{\mathrm{d}s}\frac{\partial L_2}{\partial \dot{x}^\lambda} &= -m\left[\frac{\partial g_{\alpha\beta}}{\partial x^\lambda}\dot{x}^\alpha\dot{x}^\beta - \frac{\mathrm{d}}{\mathrm{d}s}\left(\underbrace{g_{\alpha\beta}\delta^\alpha_\lambda\dot{x}^\beta}_{=g_{\lambda\gamma}\dot{x}^\gamma} + \underbrace{g_{\alpha\beta}\dot{x}^\alpha\delta^\beta_\lambda}_{=g_{\lambda\gamma}\dot{x}^\gamma}\right)\right] \\
&= -m\left[(\partial_\lambda g_{\alpha\beta})\dot{x}^\alpha\dot{x}^\beta - \frac{\mathrm{d}}{\mathrm{d}s}\left(2g_{\lambda\gamma}\dot{x}^\gamma\right)\right]
\end{aligned}
$$

(the following is the same as those in L_1 except the constant coefficient.)

$$
= 2m\left(-\frac{1}{2}(\partial_\lambda g_{\alpha\beta})\dot{x}^\alpha\dot{x}^\beta + (\partial_\alpha g_{\lambda\gamma})\dot{x}^\alpha\dot{x}^\gamma + g_{\lambda\gamma}\ddot{x}^\gamma\right)
$$

$\uparrow$ split the second term and rename the dummy index

$$
\begin{aligned}
&= 2m\left(-\frac{1}{2}(\partial_\lambda g_{\alpha\beta})\dot{x}^\alpha\dot{x}^\beta + \frac{1}{2}(\partial_\alpha g_{\beta\lambda})\dot{x}^\alpha\dot{x}^\beta + \frac{1}{2}(\partial_\beta g_{\lambda\alpha})\dot{x}^\alpha\dot{x}^\beta + g_{\lambda\gamma}\ddot{x}^\gamma\right) \\
&= 2m\left[g_{\lambda\gamma}\ddot{x}^\gamma + \frac{1}{2}\left(\partial_\alpha g_{\beta\lambda} + \partial_\beta g_{\lambda\alpha} - \partial_\lambda g_{\alpha\beta}\right)\dot{x}^\alpha\dot{x}^\beta\right]
\end{aligned}
$$

With the same steps in L_1, one also can obtain the equation of motion for L_2 is also

$$\ddot{x}^\rho + \Gamma^\rho_{\alpha\beta}\dot{x}^\alpha\dot{x}^\beta = 0$$