For TE waves, $\mathbf{H}_t = \dfrac{\hat{\mathbf{z}} \times \mathbf{E}_t}{Z}$, $\mathbf{H}_\mu = \dfrac{\hat{\mathbf{z}} \times \mathbf{E}_\mu}{Z_\mu}$,

$$\mathbf{H}_\lambda \cdot \mathbf{H}_\mu = \frac{(\hat{\mathbf{z}} \times \mathbf{E}_\lambda) \cdot (\hat{\mathbf{z}} \times \mathbf{E}_\mu)}{Z_\lambda Z_\mu} = \frac{(E_{\lambda 1}\hat{\mathbf{y}} - E_{\lambda 2}\hat{\mathbf{x}}) \cdot (E_{\mu 1}\hat{\mathbf{y}} - E_{\mu 2}\hat{\mathbf{x}})}{Z_\lambda Z_\mu} = \frac{\mathbf{E}_\lambda \cdot \mathbf{E}_\mu}{Z_\lambda Z_\mu}$$

It has derived that $\displaystyle\int_A \mathbf{E}_\lambda \cdot \mathbf{E}_\mu \, \mathrm{d}a = \delta_{\lambda\mu}$ in the lecture,

$$\int_A \mathbf{H}_\lambda \cdot \mathbf{H}_\mu \, \mathrm{d}a = \int_A \frac{\mathbf{E}_\lambda \cdot \mathbf{E}_\mu}{Z_\lambda Z_\mu} \, \mathrm{d}a = \frac{\delta_{\lambda\mu}}{Z_\lambda Z_\mu} = \frac{\delta_{\lambda\mu}}{Z_\lambda^2}.$$

For TE waves, one can relate the transversal components with the longitudinal/z component by $\mathbf{H}_t = \dfrac{ik}{\gamma^2}\nabla_t H_z$, thus

$$\begin{aligned}
\int_A \mathbf{H}_\lambda \cdot \mathbf{H}_\mu \, \mathrm{d}a &= \int_A (\frac{ik}{\gamma^2}\nabla_t H_{z\lambda}) \cdot (\frac{ik}{\gamma^2}\nabla_t H_{z\mu}) \, \mathrm{d}a = -\frac{k_\lambda k_\mu}{\gamma_\lambda^2 \gamma_\mu^2} \int_A \nabla_t H_{z\lambda} \cdot \nabla_t H_{z\mu} \, \mathrm{d}a \\
&= -\frac{k_\lambda k_\mu}{\gamma_\lambda^2 \gamma_\mu^2} \left(\oint_C H_{z\lambda} \underbrace{\frac{\partial H_{z\mu}}{\partial n}}_{=0} \mathrm{d}\ell - \int_A H_{z\lambda} \underbrace{\nabla_t^2 H_{z\mu}}_{=-\gamma_\mu^2 H_{z\mu}} \mathrm{d}a \right) \\
&= -\frac{k_\lambda k_\mu}{\gamma_\lambda^2} \int_A H_{z\lambda} H_{z\mu} \, \mathrm{d}a.
\end{aligned}$$

Hence,

$$\frac{\delta_{\lambda\mu}}{Z_\lambda^2} = -\frac{k_\lambda k_\mu}{\gamma_\lambda^2} \int_A H_{z\lambda} H_{z\mu} \, \mathrm{d}a \;\Rightarrow\; \int_A H_{z\lambda} H_{z\mu} \, \mathrm{d}a = -\frac{\delta_{\lambda\mu}}{Z_\lambda^2} \cdot \frac{\gamma_\lambda^2}{k_\lambda k_\mu} = -\frac{\gamma_\lambda^2}{k_\lambda^2 Z_\lambda^2} \delta_{\lambda\mu}.$$