
As indicated in the lecture slide, the intensity of the field in the cavity satisfies

$$|E(\omega)|^2 \propto \frac{1}{\frac{\omega_0^2}{4Q^2} + (\omega - \omega_0 - \Delta\omega)^2}.$$

The intensity of the field reaches maximum when $\omega = \omega_0 + \Delta\omega$. The maximum intensity

is $\frac{k}{\frac{\omega_0^2}{4Q^2}}$, where k is a proportional constant.

When the intensity is half of the maximum, we have that

$$\begin{aligned} \frac{k}{\frac{\omega_0^2}{4Q^2} + (\omega - \omega_0 - \Delta\omega)^2} &= \frac{1}{2} \frac{k}{\frac{\omega_0^2}{4Q^2}} \Rightarrow \frac{\omega_0^2}{4Q^2} = (\omega - \omega_0 - \Delta\omega)^2 \\ \Rightarrow \omega &= \omega_0 + \Delta\omega \pm \sqrt{\frac{\omega_0^2}{4Q^2}} = \omega_0 + \Delta\omega \pm \frac{\omega_0}{2Q} = \omega_{\pm}. \end{aligned}$$

ω_{\pm} are the frequencies when half maximum intensity occurs.

Hence, the full width at half maximum (FWHM) Γ is

$$\Gamma = \omega_+ - \omega_- = 2 \cdot \frac{\omega_0}{2Q} = \frac{\omega_0}{Q}.$$