

By the dispersion relation $k = \frac{\omega_p}{c} \sqrt{\frac{\omega}{\omega_B}}$, the phase velocity is

$$v_p = \frac{\omega}{k} = \frac{\omega}{\frac{\omega_p}{c} \sqrt{\frac{\omega}{\omega_B}}} = \frac{\sqrt{\omega\omega_B}}{\omega_p} c.$$

In term of k , $\omega = \frac{c^2 k^2 \omega_B}{\omega_p^2}$. The group velocity is

$$v_g = \frac{d\omega}{dk} = \frac{2c^2 k \omega_B}{\omega_p^2} = \frac{2c^2 \omega_B}{\omega_p^2} \cdot \frac{\omega_p}{c} \sqrt{\frac{\omega}{\omega_B}} = 2 \frac{\sqrt{\omega\omega_B}}{\omega_p} c.$$

Hence, $v_g = 2v_p$.

Remark:

$\frac{d\omega}{dk} = \left(\frac{dk}{d\omega}\right)^{-1}$ may be used, since the dispersion relation is bijective (inverse function exists) and differentiable for the domain $\omega > 0$. Thus, for $k = f(\omega)$ and $\omega = f^{-1}(k) = f^{-1}(f(\omega))$,

$$\frac{d\omega}{dk} = 1 = \frac{df^{-1}(f(\omega))}{d\omega} = \underbrace{\frac{df^{-1}}{df}}_{=\frac{d\omega}{dk}} \cdot \underbrace{\frac{df(\omega)}{d\omega}}_{=\frac{dk}{d\omega}}.$$

and $\frac{d\omega}{dk} = \left(\frac{dk}{d\omega}\right)^{-1}$.